AQA

Please write clearly in block capitals.

Centre number | | | | | |

Candidate number | | | | |

Surname ______________________________

Forename(s) ______________________________

Candidate signature ______________________________
I declare this is my own work.

A-level
FURTHER MATHEMATICS
Paper 2

Friday 6 June 2025 | Afternoon | Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

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Answer **all** questions in the spaces provided.

1 The vectors $\begin{bmatrix} a \\ b \end{bmatrix}$ and $\begin{bmatrix} c \\ 1 \end{bmatrix}$ are perpendicular.

Which **one** of the following statements must be true?

Tick (✓) **one** box.

[1 mark]

$a = bc$ ☐

$b = ac$ ☐

$a = -bc$ ☐

$b = -ac$ ☐

2 The quadrilateral Q_1 has an area of 5 cm^2

The matrix $\begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix}$ represents the transformation T

The transformation T acts on Q_1 to give the quadrilateral Q_2

Find the area of Q_2

Circle your answer.

[1 mark]

5 cm^2 10 cm^2 30 cm^2 180 cm^2

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3 Find $\frac{\mathrm{d}}{\mathrm{d}x}\left(\sin^{-1} x - 2\cos^{-1} x\right)$

Circle your answer.

[1 mark]

$\frac{-3}{\sqrt{1-x^2}}$ $\qquad$ $\frac{-1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{1}{\sqrt{1-x^2}}$ $\qquad$ $\frac{3}{\sqrt{1-x^2}}$

4 The function f is defined by $\mathrm{f}(x) = 16 - x^2 \qquad (x \in \mathbb{R})$

On which of the following intervals is the mean value of f the greatest?

Tick (✓) **one** box.

[1 mark]

$0 \leq x \leq 1$ ☐

$0 \leq x \leq 2$ ☐

$0 \leq x \leq 3$ ☐

$0 \leq x \leq 4$ ☐

Turn over for the next question

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5 The complex number $z = (3 + 4\mathrm{i})(5 + c\mathrm{i})$, where c is an integer.

It is given that $\mathrm{Re}(z) = 7$

Find the value of c

[2 marks]

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6 A curve passes through the point $(2, k)$, where $k > 1$

The curve satisfies the differential equation

$$\frac{dy}{dx} = \frac{x^2 - 3y}{xy}$$

Using Euler's step by step method once, with starting point $(2, k)$ and a step length of 0.1, gives an estimate of $y = 6.069$ when $x = 2.1$

Find the value of k

Give your answer to **three** decimal places.

[3 marks]

Turn over for the next question

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7 The matrix **A** is defined by $\mathbf{A} = \begin{bmatrix} 4 & -2 \\ -6 & 3 \end{bmatrix}$

Find a non-zero 2×2 matrix **B** such that $\mathbf{AB} = 0$

[3 marks]

Turn over for the next question

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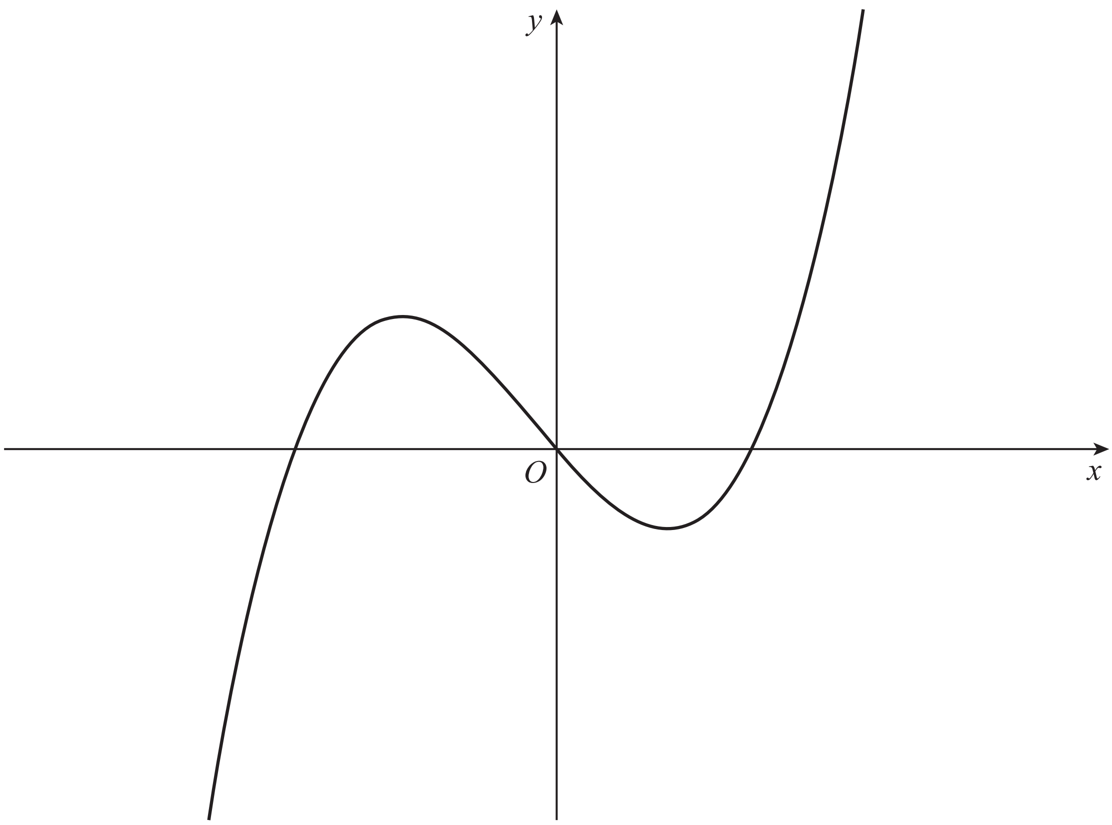
8 The function f is defined by

$$\mathrm{f}(x) = x^3 + x^2 - 12x \qquad (x \in \mathbb{R})$$

Figure 1 shows the graph of $y = \mathrm{f}(x)$

Figure 1



8 (a) The graph of $y = \mathrm{f}(x)$ is transformed by a stretch, scale factor 2, parallel to the x-axis with the y-axis fixed, to give the graph of $y = \mathrm{g}(x)$

On **Figure 1**, sketch the graph of $y = \mathrm{g}(x)$, showing the values of x where the graph crosses the x-axis.

[3 marks]

8(b) Find the set of values of x such that the conditions $f(x) > 0$ **and** $g(x) < 0$ are **both** satisfied.

[2 marks]

Turn over for the next question

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9 (a) It is given that, for the complex number z,

$$\left|\frac{z}{z+1}\right| = 1$$

Find $\mathrm{Re}(z)$

[3 marks]

9 (b) Show that the only solutions of the equation

$$\left(\frac{w}{w+1}\right)^3 = 1$$

are $w = \dfrac{e^{\frac{2\pi i}{3}}}{1 - e^{\frac{2\pi i}{3}}}$ and $w = \dfrac{e^{-\frac{2\pi i}{3}}}{1 - e^{-\frac{2\pi i}{3}}}$

[4 marks]

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9 (c) Use the results of part **(a)** and part **(b)** to find $\mathrm{Re}\left(\dfrac{e^{\frac{2\pi i}{3}}}{1-e^{\frac{2\pi i}{3}}}\right)$

Fully justify your answer.

[2 marks]

10 The function f is defined by

$$\mathrm{f}(x) = \frac{2\left(x^2 - 4\right)}{x^2 + 6x + 9}$$

10 (a) Write down the equations of the asymptotes to the graph of $y = \mathrm{f}(x)$

[2 marks]

10 (b) Without using calculus, show that the range of f is $\left\{k : k \geq -\frac{8}{5}\right\}$

[4 marks]

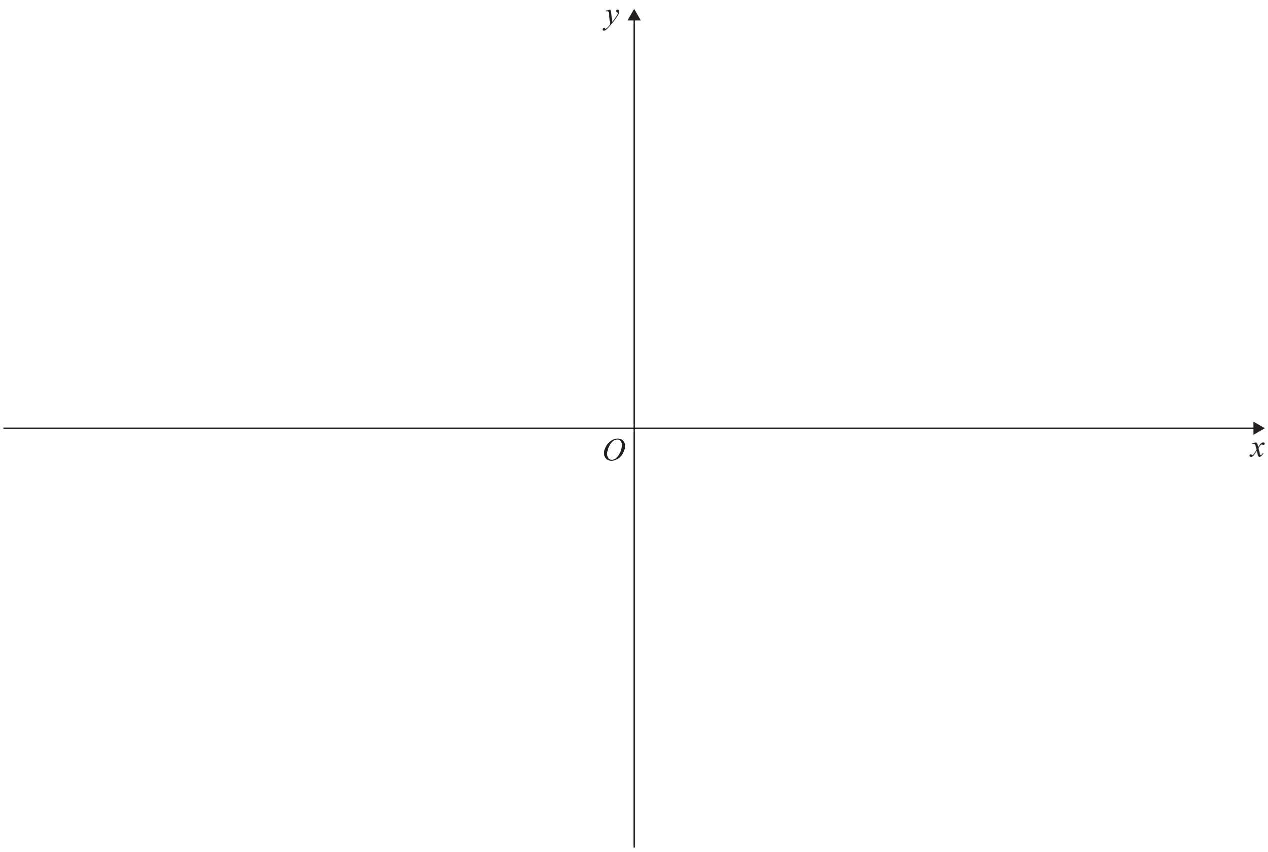
10 (c) The graph of $y = \mathrm{f}(x)$ has one stationary point.

Without using calculus, find the coordinates of this stationary point.

[3 marks]

10 (d) Sketch the graph of $y = \mathrm{f}(x)$ on the axes below.

[4 marks]



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11 The line L has vector equation

$$\mathbf{r} = \begin{bmatrix} 5 \\ 2 \\ 11 \end{bmatrix} + \lambda \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

The point A has coordinates $(1, -1, -6)$

11 (a) Find the coordinates of the point on L which is closest to the point A

[5 marks]

11(b) The point B has coordinates $(3, -2, -1)$

The point C has coordinates $(4, 0, 1)$

The points A, B and C all lie in the plane Π

Find a Cartesian equation of Π

[4 marks]

11 (c) Find the coordinates of the point where the line L meets the plane Π

[3 marks]

12 Find the value of

$$\int_1^5 \frac{1}{\sqrt{x^2+6x+5}}\,\mathrm{d}x$$

Give your answer in the form

$$\ln\left(8+a\sqrt{3}+b\sqrt{5}+c\sqrt{15}\right)$$

where a, b and c are integers.

[5 marks]

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13 The matrix **M** is defined by $\mathbf{M}=\begin{bmatrix}1 & -\sqrt{3}\\ \sqrt{3} & 1\end{bmatrix}$

13 (a) The matrix **M** represents an anticlockwise rotation about the origin through an angle θ, where $0 \le \theta \le 2\pi$, followed by an enlargement, scale factor r, with centre at the origin where r is a positive integer.

Find the value of r and the value of θ

[3 marks]

13 (b) It is given that $\begin{bmatrix}u\\ v\end{bmatrix}=\mathbf{M}\begin{bmatrix}x\\ y\end{bmatrix}$

Using the value of r and the value of θ which you obtained in part **(a)**, verify that

$$r\,\mathrm{e}^{\mathrm{i}\theta}\left(x+\mathrm{i}y\right)=u+\mathrm{i}v$$

[3 marks]

13 (c) Hence, find the value of x and the value of y such that

$$\mathbf{M}^8 \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix}$$

Give your answers in an exact form.

[4 marks]

Turn over for the next question

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14 A children's play area in a park contains a paddling pool.

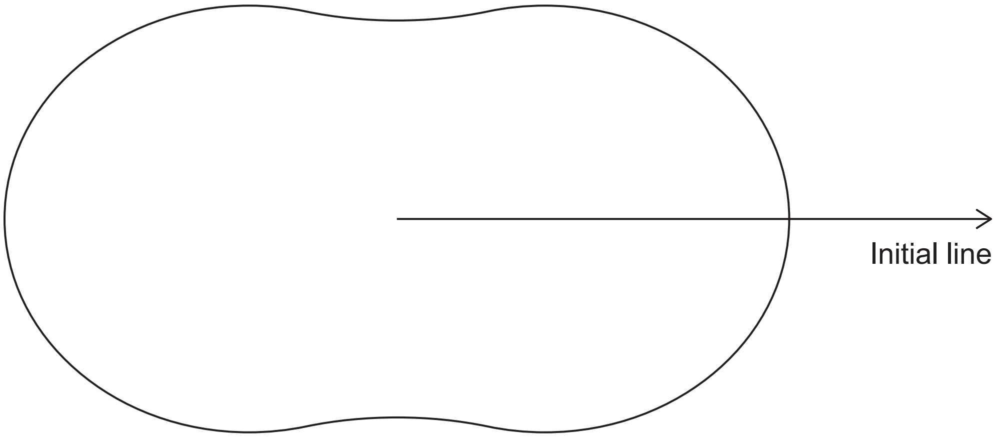
The outline of the paddling pool is modelled by the polar curve

$$r = 6 + 2\cos(2\theta)$$

where r is measured in metres.

Figure 2 shows the outline of the paddling pool.

Figure 2



There is a solid concrete island inside the paddling pool.

The boundary of the island is modelled by the polar curve

$$r = 2 + \cos\theta$$

where r is measured in metres.

14 (a) Sketch the boundary of the island on **Figure 2**

[2 marks]

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14 (b) The paddling pool has a constant depth of 0.3 metres.

Find the volume of water in the paddling pool.

Give your answer to **four** significant figures.

Fully justify your answer.

[6 marks]

15 Prove that

$$\int_{5}^{\infty} \frac{1}{(x+1)(2x+3)}\,\mathrm{d}x = \ln\left(\frac{13}{12}\right)$$

Show the limiting process clearly.

[6 marks]

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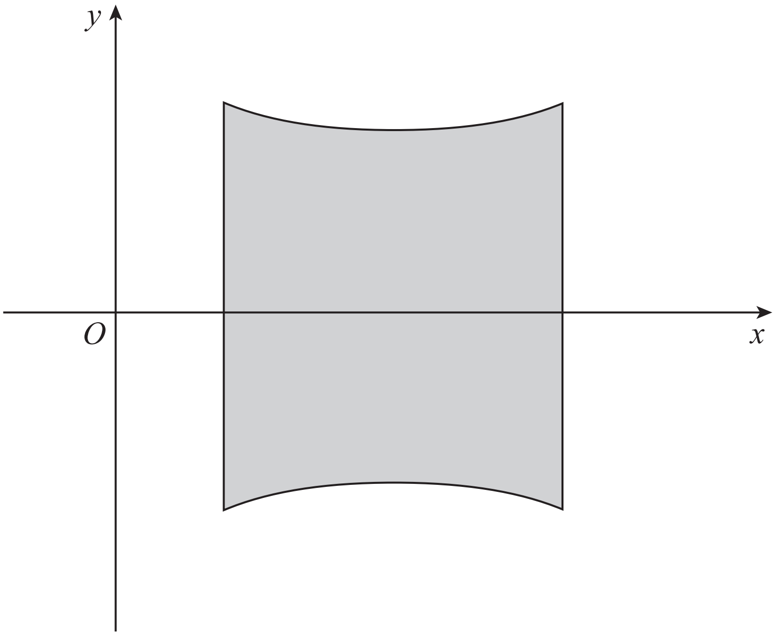
16 The diagram shows a design for a table top.



The table top is modelled as being bounded by the lines $x = 0.5$ and $x = 1.5$,

and the curves defined by $y^2 = \dfrac{0.27}{2x - x^2}$, where x and y are measured in metres.

Find the area of the table top according to this model.

Give your answer in the form $\dfrac{\pi\sqrt{p}}{q}$ square metres, where p and q are integers.

Fully justify your answer.

[5 marks]

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17 A sample of biological material is placed in a freezer, and the freezer is then turned on.

The initial temperature of the sample is 38 °C

The temperature u °C of the freezer, at time t minutes after it is turned on, is modelled by

$$u = 18 - 2t$$

The temperature y °C of the sample is modelled as decreasing at a rate which is proportional to the difference between the temperature of the sample and the temperature of the freezer.

Initially, the temperature of the sample is decreasing at a rate of 0.8 °C per minute.

17(a) Show that y satisfies the differential equation

$$\frac{dy}{dt} + 0.04y = 0.72 - 0.08t$$

[3 marks]

17 (b) Find an expression for y in terms of t

[7 marks]

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Question 17 continues on the next page

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17 (c) Find the time at which the freezer is 28 °C colder than the sample.

Give your answer in minutes and seconds.

[4 marks]

17 (d) State **one** limitation of the model used.

[1 mark]

END OF QUESTIONS

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