AQA

Please write clearly in block capitals.

Centre number | | | | | |

Candidate number | | | | |

Surname ____________________

Forename(s) ____________________

Candidate signature ____________________
I declare this is my own work.

A-level
FURTHER MATHEMATICS
Paper 1

Thursday 22 May 2025 Afternoon Time allowed: 2 hours

Materials
- You must have the AQA Formulae and statistical tables booklet for A-level Mathematics and A-level Further Mathematics.
- You should have a graphical or scientific calculator that meets the requirements of the specification.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space for your answer(s), use the lined pages at the end of this book. Write the question number against your answer(s).
- Do **not** write outside the box around each page or on blank pages.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

For Examiner's Use	
Question	Mark
1	
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TOTAL	

Answer **all** questions in the spaces provided.

1 The equation $x^2 + 4x + c = 0$ has non-real roots.

Find the range of possible values of c

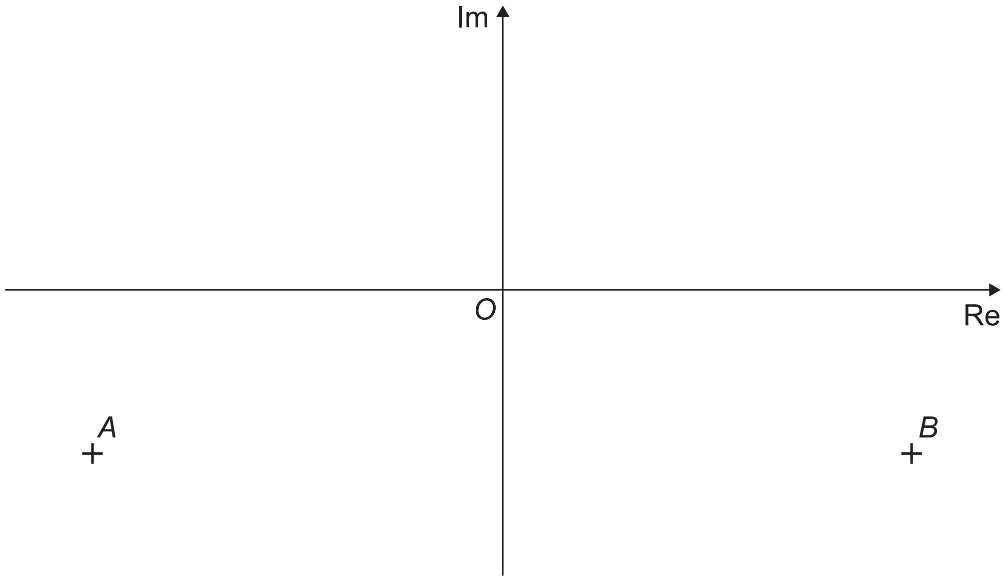
Circle your answer.

[1 mark]

$c < 4$ $\quad$ $c \leq 4$ $\quad$ $c \geq 4$ $\quad$ $c > 4$

2 The point *A* on the Argand diagram represents the complex number z

The point *B* is the reflection in the imaginary axis of the point *A*



Which complex number does the point *B* represent?

Circle your answer.

[1 mark]

$-z$ $\quad$ z^* $\quad$ $-z^*$ $\quad$ $\dfrac{1}{z}$

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3 Which **one** of the following expressions **cannot** be evaluated by l'Hôpital's rule?

Circle your answer.

[1 mark]

$\lim_{x \to 0}\left(\frac{\sqrt{x}}{\cos x}\right)$ $\lim_{x \to 0}\left(\frac{x^2}{\sin x}\right)$ $\lim_{x \to 0}\left(\frac{\tanh x}{2x}\right)$ $\lim_{x \to 0}\left(\frac{e^x - 1}{\ln(1+x)}\right)$

4 Which **one** of the following statements is always correct?

Tick (✓) **one** box.

[1 mark]

$\sinh^2 x = \frac{1}{2}(1 - \cosh 2x)$ ☐

$\text{sech}^2 x + \tanh^2 x = 1$ ☐

$\text{cosech}^2 x + \coth^2 x = 1$ ☐

$\cosh^2 x = \frac{1}{2}(\cosh 2x - 1)$ ☐

Turn over for the next question

Do not write outside the box

5 The complex number z_1 has argument $\tan^{-1}\left(\frac{1}{5}\right)$

The complex number $z_2 = -2 - 4\mathrm{i}$

Find $\arg\left(\frac{z_1}{z_2}\right)$

Give your answer as a number in the range $-\pi < \alpha < \pi$, to **two** decimal places.

[3 marks]

6 Use Simpson's rule with 5 ordinates to find an approximation to

$$\int_0^2 \frac{1}{\sqrt{1+x^4}}\,\mathrm{d}x$$

Give your answer to **three** decimal places.

[3 marks]

Turn over for the next question

Do not write outside the box

7 The cubic equation

$$5z^3 + 4z^2 - z + 3 = 0$$

has roots α, β and γ

Find an equation, with integer coefficients, that has roots $2\alpha - 1$, $2\beta - 1$ and $2\gamma - 1$

[4 marks]

8 The curve C_1 has equation

$$\frac{x^2}{4} + \frac{y^2}{25} = 1$$

The curve C_1 is translated by the vector $\begin{bmatrix} 1 \\ 3 \end{bmatrix}$ to give the curve C_2

Find the coordinates of the points where the curve C_2 intersects the x-axis.

[3 marks]

Turn over for the next question

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9 The vectors **a** and **b** are such that

$$|\mathbf{a} \times \mathbf{b}| = 12$$

$$\mathbf{a} \bullet \mathbf{b} = -\sqrt{3}$$

$$|\mathbf{a}| = 7$$

Find the value of $|\mathbf{b}|$

[4 marks]

Turn over for the next question

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10 Astrid is solving this mathematics problem:

> The series S_n is defined by
>
> $$S_n = 2 + 4 + 6 + \ldots + 2n \quad (n \in \mathbb{Z}, n \geq 1)$$
>
> Prove by induction that
>
> $$S_n = n(n+1)$$

Astrid's solution is as follows:

> Assume the result is true for $n = k$
>
> Then
>
> $S_k = k(k+1)$
> $S_{k+1} = S_k + 2(k+1)$
> $S_{k+1} = k(k+1) + 2(k+1)$
> $S_{k+1} = (k+2)(k+1)$
> $S_{k+1} = (k+1)((k+1)+1)$
>
> So the result is also true for $n = k+1$
>
> The result is true for $n = 1$
>
> It is true for $n = k$, and also true for $n = k + 1$
>
> Hence, by induction $S_n = n(n+1)$ for all integers $n \geq 1$

10 (a) (i) Chloe says that Astrid missed out an essential part of the proof, which could have been written at the start.

Explain what Astrid missed out.

[1 mark]

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10 (a) (ii) Write down the working that Astrid missed out.

[1 mark]

____

____

____

10 (b) (i) One statement in the last three lines of Astrid's solution is written incorrectly.

Which statement is written incorrectly?

Tick (✓) **one** box.

[1 mark]

The result is true for $n = 1$ ☐

It is true for $n = k$, and also true for $n = k + 1$ ☐

Hence, by induction $S_n = n(n+1)$ for all integers $n \geq 1$ ☐

10 (b) (ii) Write out a correct statement which should replace the incorrect statement identified in part **(b)(i)**

[1 mark]

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Turn over for the next question

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11 The function f is defined by

$$\mathrm{f}(x) = \frac{1}{1+\mathrm{e}^x} \qquad (x \in \mathbb{R})$$

11 (a) Show that

$$\mathrm{f}''(x) = \frac{-\mathrm{e}^x + \mathrm{e}^{2x}}{\left(1+\mathrm{e}^x\right)^3}$$

[3 marks]

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11 (b) Hence, find the Maclaurin expansion of $\mathrm{f}(x)$ up to and including the term in x^3

[4 marks]

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12 (a) Find the eigenvalues and corresponding eigenvectors of the matrix

$$\mathbf{M} = \frac{1}{20}\begin{bmatrix} 19 & 3 \\ 3 & 11 \end{bmatrix}$$

[5 marks]

12 (b) State, with a reason, the Cartesian equation of the line of invariant points of the matrix **M**

[2 marks]

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12 (c) Find matrices **U**, **D** and $\mathbf{U}^{-1}$, such that **D** is diagonal and $\mathbf{M} = \mathbf{UDU}^{-1}$

[3 marks]

12(d) Hence, find the matrix **L** such that $\mathbf{M}^n \to \mathbf{L}$ as $n \to \infty$

[3 marks]

Turn over for the next question

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13 The function f is defined by

$$\mathrm{f}(z) = 4z^3 + rz^2 + 92z + s$$

where r and s are real numbers.

One of the roots of the equation $\mathrm{f}(z) = 0$ is $-4 + 3\mathrm{i}$

13 (a) Find the other two roots of the equation $\mathrm{f}(z) = 0$

[4 marks]

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13 (b) Find the value of r and the value of s

[3 marks]

Turn over for the next question

Do not write outside the box

14 (a) Using the exponential definitions of sinh x and cosh x, prove that

$$\coth^{-1}(x) = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$$

[3 marks]

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14 (b) Hence, solve the equation

$$\coth^{-1}(x) = -\ln 5$$

Give your answer in an exact form.

[3 marks]

Turn over for the next question

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15 Three planes have equations

$$\begin{aligned} x + 2y - z &= 9 \\ x - 3y + 3z &= t \\ 3x + y + z &= 4t \end{aligned}$$

where t is a constant.

The planes meet along a line of intersection.

15 (a) Find the value of t

[4 marks]

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15 (b) Find a vector equation of the line of intersection.

Fully justify your answer.

[5 marks]

Turn over for the next question

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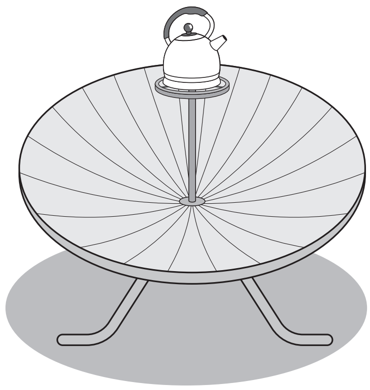
16 The picture shows a solar cooker. Part of the solar cooker is a parabolic dish.



The shape of the internal surface of the dish is formed by rotating the part of the parabola $y^2 = 0.8x$ between $x = 0$ and $x = 0.25$ through 2π radians about the x-axis, where x and y are measured in metres.

Use integration to show that the internal surface area of the dish, to **three** decimal places, is 0.796 square metres.

Fully justify your answer.

[7 marks]

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Turn over for the next question

Turn over ▶

17 In this question use $g = 10$ m s^{-2}

A particle P of mass 0.6 kg is attached to one end of each of two light elastic strings, AP and BP

The other ends of the strings, A and B, are attached to fixed points which are 7 metres apart, with A vertically above B

The natural length of the string AP is 2 metres.

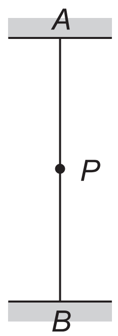
When the extension of the string AP is e metres, the tension in the string AP is $5e$ newtons.

The natural length of the string BP is 3 metres.

When the extension of the string BP is e metres, the tension in the string BP is $3e$ newtons.

The whole system is in a large tub of oil.

The diagram shows the particle P, the strings and the points A and B



The particle P is held at the point between A and B which is 0.5 metres vertically below its equilibrium position.

The particle is then released from rest.

During the subsequent motion the oil causes a resistive force of magnitude $\frac{4}{\sqrt{5}}v$ newtons to act on the particle, where v m s^{-1} is the speed of the particle.

At time t seconds after P is released, its displacement towards B from its equilibrium position is x metres.

17 (a) Show that during the subsequent motion the particle satisfies the differential equation

$$0.6\frac{\mathrm{d}^2x}{\mathrm{d}t^2}+\frac{4}{\sqrt{5}}\frac{\mathrm{d}x}{\mathrm{d}t}+8x=0$$

Fully justify your answer.

[5 marks]

Question 17 continues on the next page

17 (b) Find x in terms of t, giving your answer in exact form.

[6 marks]

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Turn over for the next question

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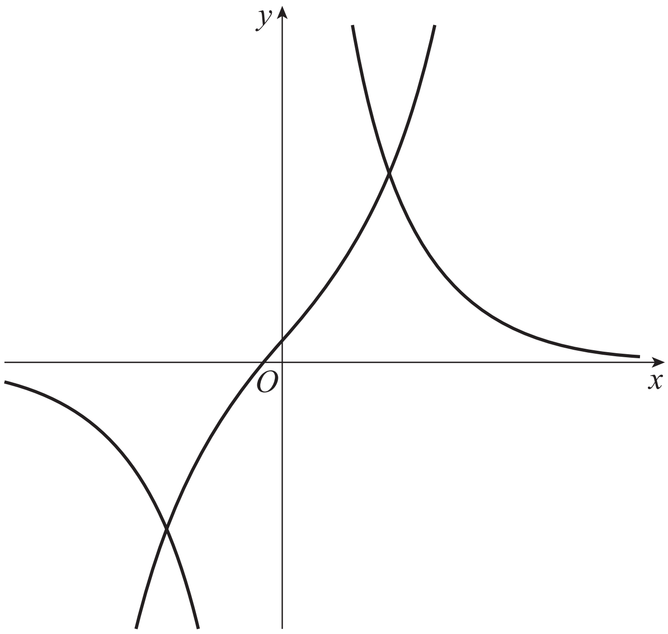
18 The diagram shows part of the graph of $y = 15\,\text{cosech}\,x$ and part of the graph of $y = 4\sinh x + \dfrac{1}{2}$



18 (a) Solve the inequality

$$15\,\text{cosech}\,x < 4\sinh x + \frac{1}{2}$$

Give your answer in logarithmic form.

[4 marks]

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18 (b) Given that

$$f(x) = \ln\left(\tanh\left(\frac{1}{2}x\right)\right) \qquad (x > 0)$$

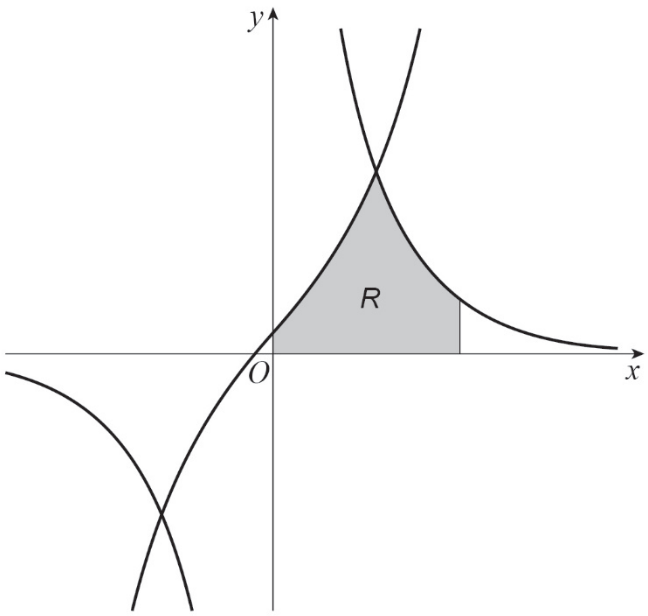
Show that

$$f'(x) = \operatorname{cosech} x$$

[4 marks]

Question 18 continues on the next page

18 (c) The shaded region R is enclosed by the positive x-axis, the positive y-axis, the graph of $y = 4\sinh x + \frac{1}{2}$, the graph of $y = 15\operatorname{cosech} x$ and the line $x = \ln 9$



Find the area of R

Give your answer in the form $\frac{p}{q} + \ln r + s\ln\left(\frac{t}{3}\right)$ where p, q, r, s and t are integers.

[7 marks]

END OF QUESTIONS

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