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7.7 Capacitor Charge & Discharge

PHYSICS

AQA A Level Revision Notes

A Level Physics AQA

7.7 Capacitor Charge & Discharge

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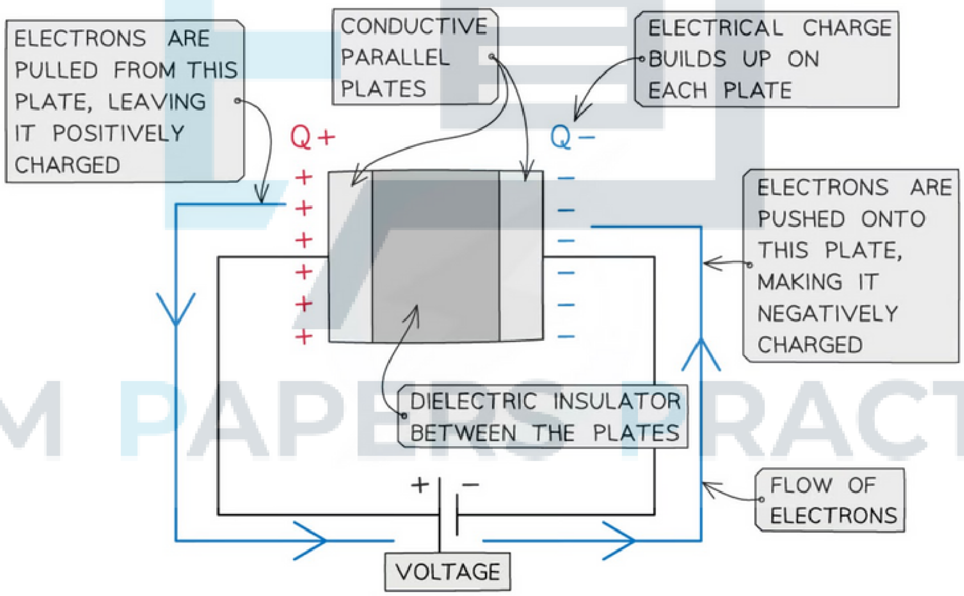
7.7.1 Charge & Discharge Graphs

Capacitor Charge & Discharge Graphs

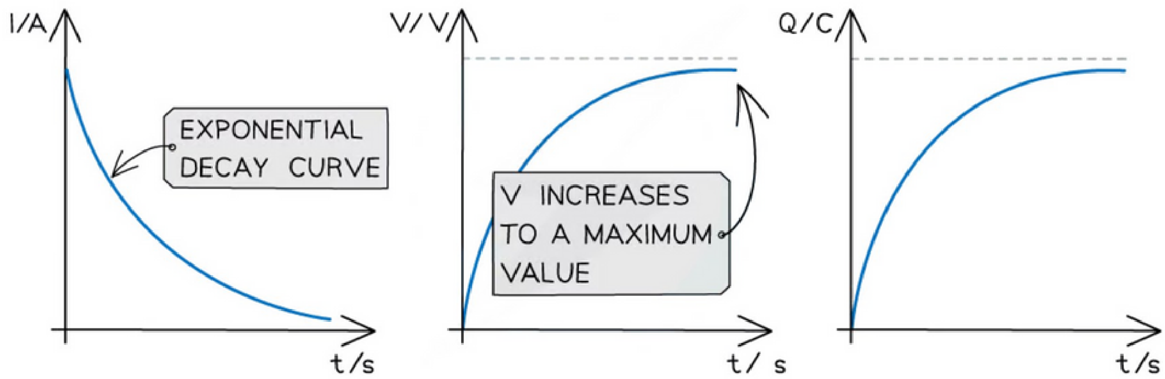
Charging

- Capacitors are charged by a **power supply** (eg. a battery)
- When charging, the electrons are pulled from the plate connected to the positive terminal of the power supply
  - Hence the plate nearest the positive terminal is **positively charged**
- They travel around the circuit and are pushed onto the plate connected to the negative terminal
  - Hence the plate nearest the negative terminal is **negatively charged**
- As the negative charge builds up, fewer electrons are pushed onto the plate due to electrostatic repulsion from the electrons already on the plate
- When no more electrons can be pushed onto the negative plate, the charging stops



A parallel plate capacitor is made up of two conductive plates with opposite charges building up on each plate

- At the start of charging, the current is large and gradually falls to zero as the electrons stop flowing through the circuit
  - The current decreases **exponentially**
  - This means the rate at which the current decreases is proportional to the amount of current it has left
- Since an equal but opposite charge builds up on each plate, the potential difference between the plates slowly increases until it is the same as that of the power supply
- Similarly, the charge of the plates slowly increases until it is at its maximum charge defined by the capacitance of the capacitor

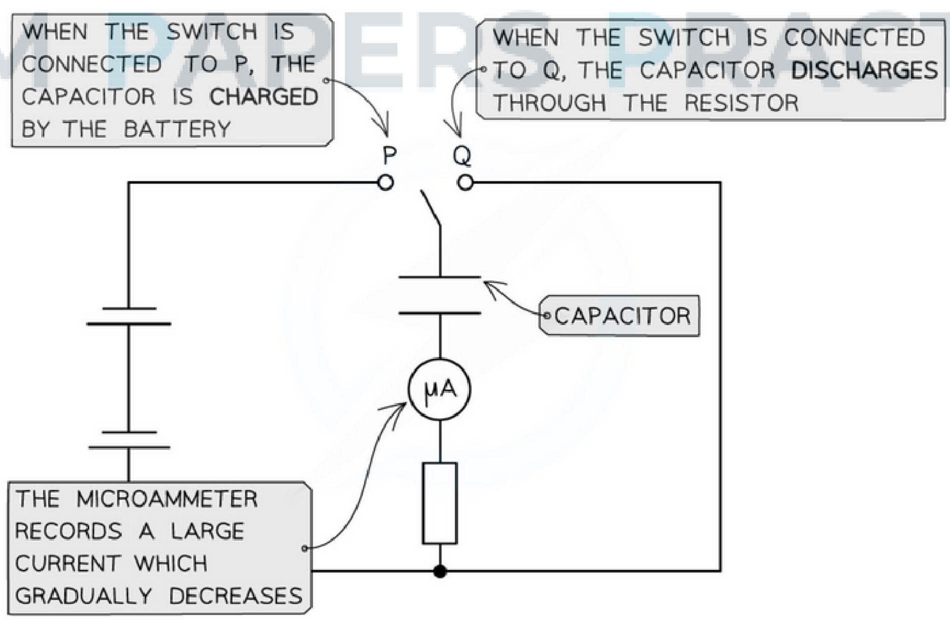


Graphs of variation of current, p.d and charge with time for a capacitor charging through a battery

- **The key features of the charging graphs are:**
  - The shapes of the p.d. and charge against time graphs are identical
  - The current against time graph is an **exponential decay** curve
  - The initial value of the current starts on the y axis and decreases exponentially
  - The initial value of the p.d and charge starts at 0 up to a maximum value

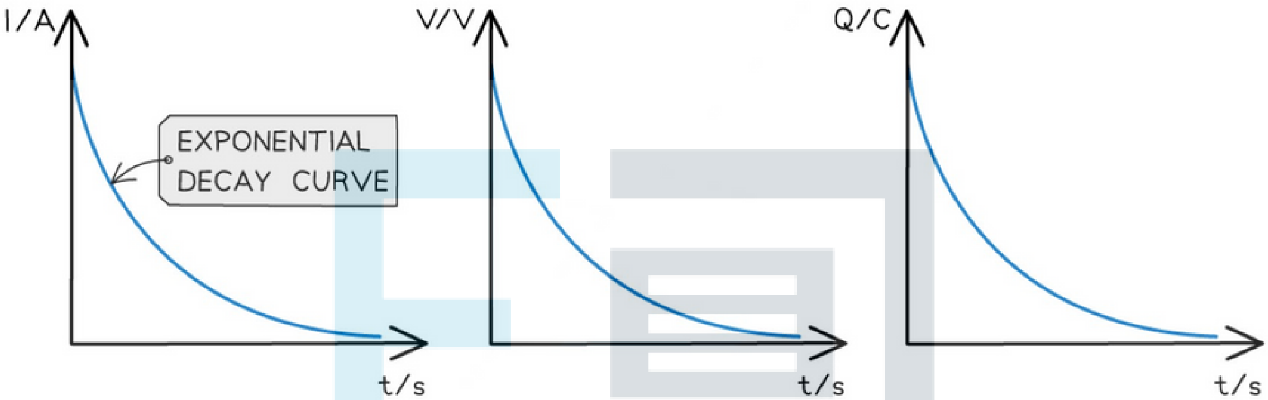
Discharging

- Capacitors are **discharged** through a resistor with **no** power supply present
- The electrons now flow back from the negative plate to the positive plate until there are equal numbers on each plate and no potential difference between them
- Charging and discharging is commonly achieved by moving a switch that connects the capacitor between a power supply and a resistor



The capacitor charges when connected to terminal P and discharges when connected to terminal Q

- At the start of discharge, the current is **large** (but in the opposite direction to when it was charging) and gradually falls to zero
- As a capacitor discharges, the current, p.d and charge all decrease **exponentially**
  - This means the rate at which the current, p.d or charge decreases is proportional to the amount of current, p.d or charge it has left
- The graphs of the variation with time of current, p.d and charge are all identical and follow a pattern of **exponential decay**



Graphs of variation of current, p.d and charge with time for a capacitor discharging through a resistor

- **The key features of the discharge graphs are:**
  - The shape of the current, p.d. and charge against time graphs are identical
  - Each graph shows exponential decay curves with decreasing gradient
  - The initial values (typically called I_0, V_0 and Q_0 respectively) start on the y axis and decrease exponentially
- The rate at which a capacitor discharges depends on the **resistance** of the circuit
  - If the resistance is **high**, the current will decrease and charge will flow from the capacitor plates more slowly, meaning the capacitor will take longer to discharge
  - If the resistance is **low**, the current will increase and charge will flow from the capacitor plates quickly, meaning the capacitor will discharge faster

Exam Tip

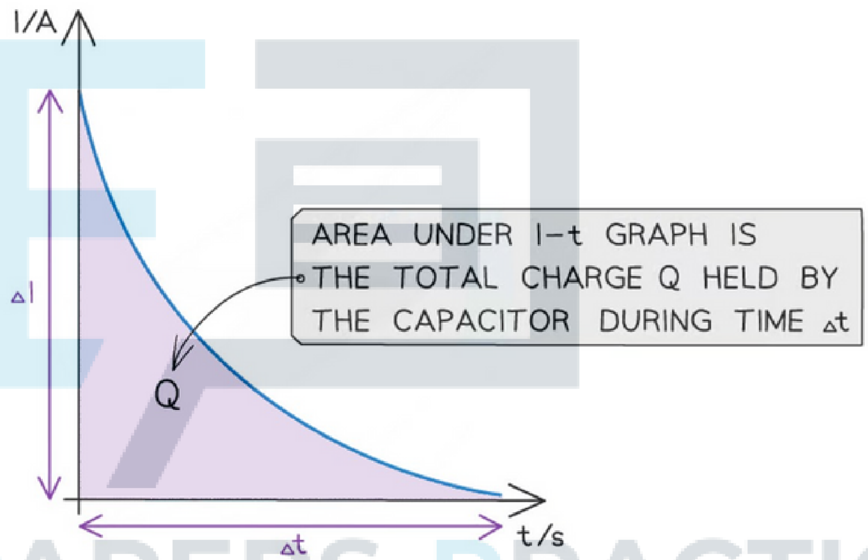
Make sure you're comfortable with sketching and interpreting charging and discharging graphs, as these are common exam questionsRemember that conventional current flow is in the **opposite** direction to the electron flow

Properties of Capacitor Discharge Graphs

- From electricity, the charge is defined:

$$\Delta Q = I\Delta t$$

- Where:
  - I = current (A)
  - ΔQ = change in charge (C)
  - Δt = change in time (s)
- This means that the **area** under a current-time graph for a charging (or discharging) capacitor is the **charge stored** for a certain time interval

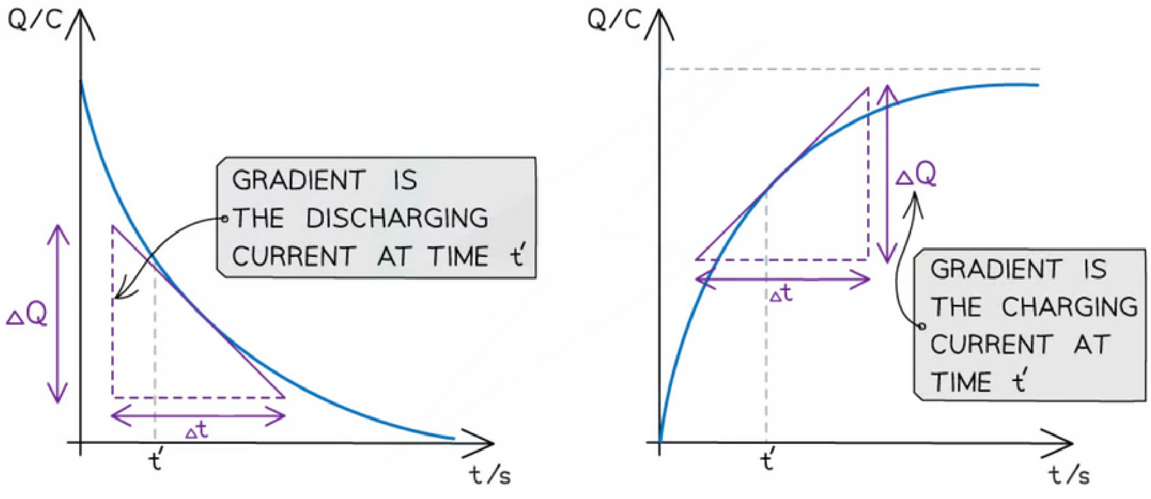


The area under the I-t graph is the total charge stored in the capacitor in the time interval Δt

- Rearranging for the current:

$$I = \frac{\Delta Q}{\Delta t}$$

- This means that the **gradient** of the charge-time graph is the **current** at that time

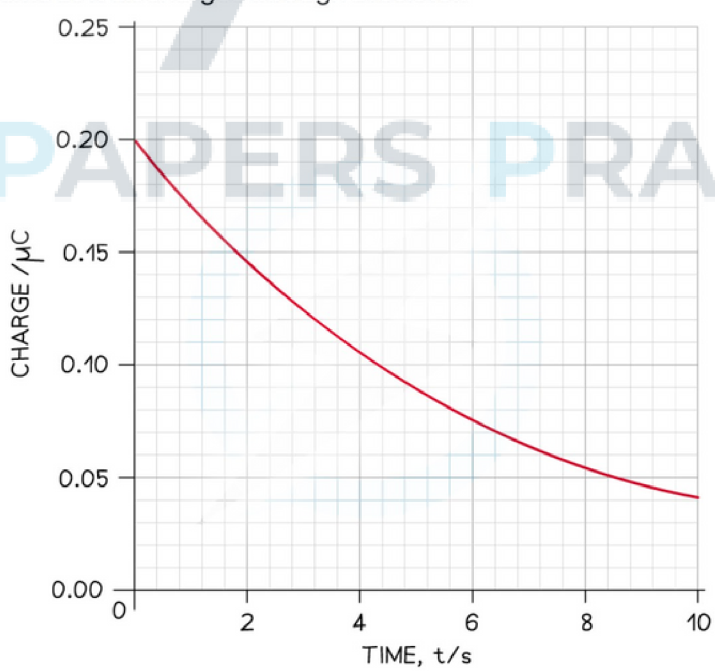


The gradient of a discharging and charging Q-t graph is the current

- In the **discharging** graph, this is the **discharging** current at that time
- In the **charging** graph, this is the **charging** current at that time
- To calculate the gradient of a curve, draw a tangent at that point and calculate the gradient of that tangent

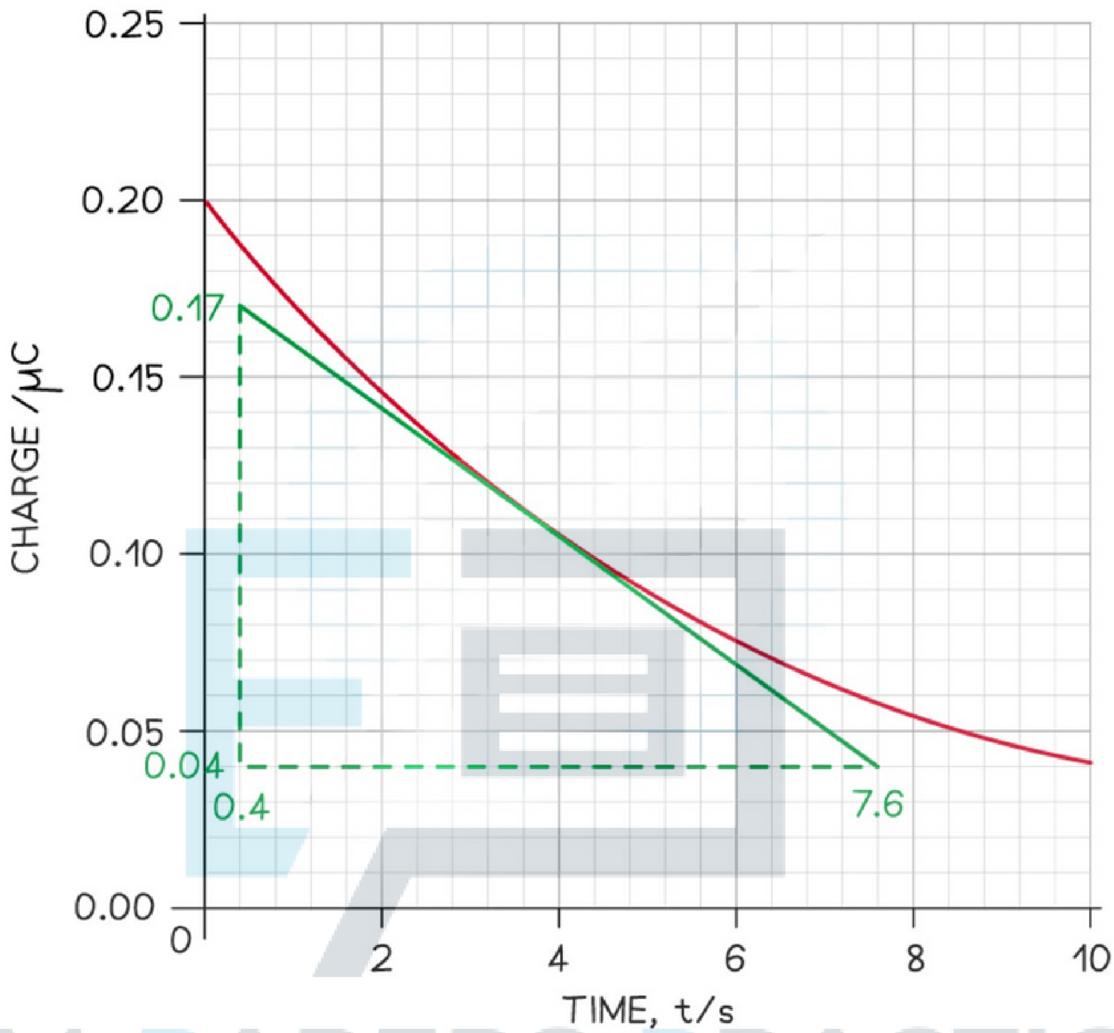
Worked Example

The graph below shows how the charge stored on a capacitor with capacitance C varies with time as it discharges through a resistor.



Calculate the current through the circuit after 4 s.

Step 1: Draw a tangent at *t* = 4



Step 2: Calculate the gradient of the tangent to find the current *I*

$$\text{gradient} = I = \frac{\Delta Q}{\Delta t} = \frac{(0.04 - 0.17) \times 10^{-6}}{(7.6 - 0.4)} = \frac{-0.13 \times 10^{-6}}{7.2}$$

$$I = \mathbf{-1.8 \times 10^{-8}\ A}$$

7.7.2 The Time Constant

The Time Constant

- The time constant of a capacitor discharging through a resistor is a measure of how long it takes for the capacitor to discharge
- The definition of the time constant is:

  The time taken for the charge, current or voltage of a discharging capacitor to decrease to 37% of its original value

- Alternatively, for a **charging** capacitor:

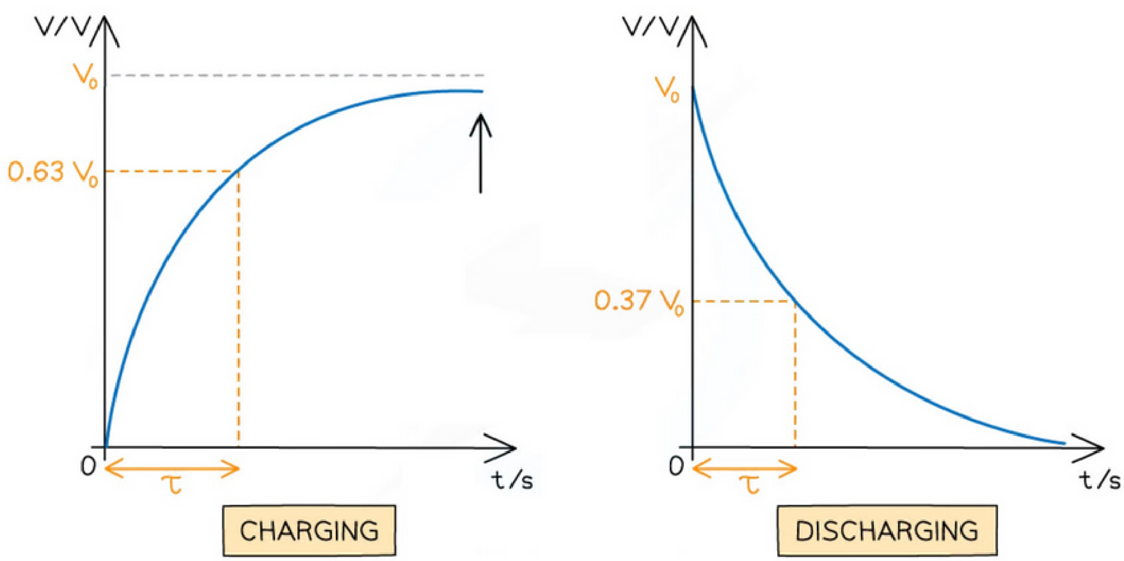
  The time taken for the charge or voltage of a charging capacitor to rise to 63% of its maximum value

- 37% is 0.37 or 1 / e (where e is the exponential function) multiplied by the original value (I_0, Q_0 or V_0)
- This is represented by the Greek letter tau, τ, and measured in units of **seconds** (s)
- The time constant provides an easy way to compare the rate of change of similar quantities eg. charge, current and p.d.
- It is defined by the equation:

$$\tau = RC$$

- Where:
  - τ = time constant (s)
  - R = resistance of the resistor (Ω)
  - C = capacitance of the capacitor (F)

- For example, to find the time constant from a voltage-time graph, calculate $0.37V_0$ and determine the corresponding time for that value



The time constant shown on a charging and discharging capacitor

- The time to half, $t_{1/2}$ (half-life) for a discharging capacitor is:

  The time taken for the charge, current or voltage of a discharging capacitor to reach half of its initial value

- This can also be written in terms of the time constant:

$$t_{1/2} = 0.69\,\tau = 0.69RC$$

Worked Example

A capacitor of 7 nF is discharged through a resistor of resistance R. The time constant of the discharge is 5.6×10^{-3} s. Calculate the value of *R*.

Step 1: Write the known quantities

Capacitance, $C = 7\ \text{nF} = 7 \times 10^{-9}$ F

Time constant, $\tau = 5.6 \times 10^{-3}$ s

Step 2: Write down the time constant equation

$$\tau = RC$$

Step 3: Rearrange for R

$$R = \frac{\tau}{C}$$

Step 4: Substitute in values and calculate

$$R = \frac{5.6 \times 10^{-3}}{7 \times 10^{-9}} = \mathbf{800\ k\Omega}$$

Exam Tip

Remember to check the context of an exam question, i.e., whether the capacitor is **charging** or **discharging**. The definition of the time constant depends on it!

- For a **charging** capacitor, the time constant refers to the time taken to reach 63% of its **maximum** potential difference or charge stored
- For a **discharging** capacitor, the time constant refers to the time take to discharge to 37% of its **initial** potential difference or charge stored

7.7.3 Charge & Discharge Equations

Capacitor Discharge Equation

- The time constant is used in the exponential decay equations for the current, charge or potential difference (p.d) for a capacitor discharging through a resistor
  - These can be used to determine the **amount** of current, charge or p.d left **after a certain amount of time** for a discharging capacitor

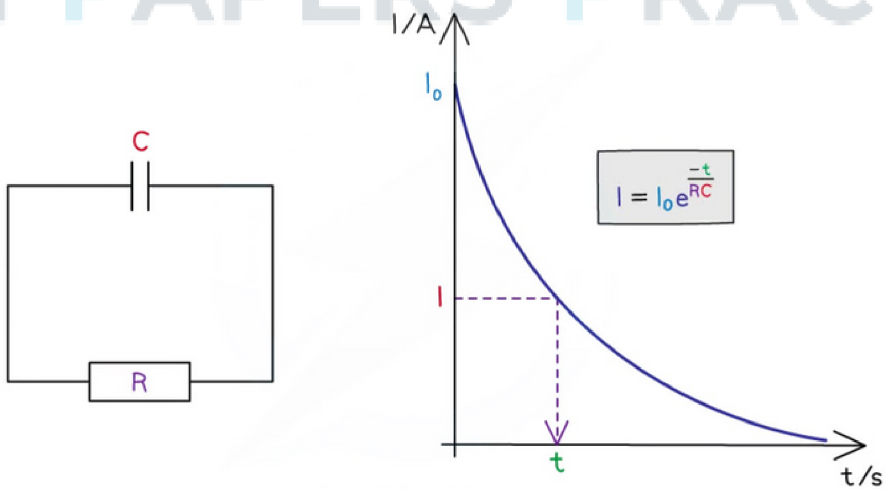
- This exponential decay means that no matter how much charge is initially on the plates, the amount of time it takes for that charge to **halve** is the **same**
- The exponential decay of current on a discharging capacitor is defined by the equation:

$$I = I_0 e^{-\frac{t}{RC}}$$

- Where:
  - I = current (A)
  - I_0 = initial current before discharge (A)
  - e = the exponential function
  - t = time (s)
  - RC = resistance (Ω) × capacitance (F) = the time constant τ (s)

- This equation shows that the **smaller** the time constant τ, the **quicker** the exponential decay of the current when discharging
- Also, how big the initial current is affects the **rate** of discharge
  - If I_0 is large, the capacitor will take **longer** to discharge

- **Note:** during capacitor discharge, I_0 is always larger than I, as the current I will always be decreasing



Values of the capacitor discharge equation on a graph and circuit

- The current at any time is directly proportional to the p.d across the capacitor and the charge across the parallel plates

- Therefore, this equation also describes the charge on the capacitor after a certain amount of time:

$$Q = Q_0 e^{-\frac{t}{RC}}$$

- Where:
  - Q = charge on the capacitor plates (C)
  - Q_0 = initial charge on the capacitor plates (C)

- As well as the p.d after a certain amount of time:

$$V = V_0 e^{-\frac{t}{RC}}$$

- Where:
  - V = p.d across the capacitor (C)
  - V_0 = initial p.d across the capacitor (C)

The Exponential Function e

- The symbol e represents the exponential constant, a number which is approximately equal to e = 2.718...
- On a calculator, it is shown by the button e^x
- The inverse function of e^x is ln(y), known as the natural logarithmic function
  - This is because, if $e^x = y$, then $x = \ln(y)$

- The 0.37 in the definition of the **time constant** arises as a result of the exponential constant, the true definition is:

The time taken for the charge of a capacitor to decrease to $\frac{1}{e}$ of its original value

- Where $\frac{1}{e}$ = 0.3678...

Worked Example

The initial current through a circuit with a capacitor of 620 μF is 0.6 A.The capacitor is connected across the terminals of a 450 Ω resistor.Calculate the time taken for the current to fall to 0.4 A.

Step 1: Write out the known quantities

Initial current before discharge, I_0 = 0.6 A

Current, I = 0.4 A

Resistance, R = 450 Ω

Capacitance, C = 620 μF = 620 × 10^{-6} F

Step 2: Write down the equation for the exponential decay of current

$$I = I_0 e^{-\frac{t}{RC}}$$

Step 3: Rearrange for t

$$\frac{I}{I_0} = e^{-\frac{t}{RC}}$$

The exponential is removed by taking the natural log (ln) of both sides

$$\ln\left(\frac{I}{I_0}\right) = -\frac{t}{RC}$$

$$t = -RC\ln\left(\frac{I}{I_0}\right)$$

Step 4: Substitute in the values

$$t = -450 \times (620 \times 10^{-6}) \times \ln\left(\frac{0.4}{0.6}\right) = 0.1131 = \mathbf{0.1\ s}$$

Exam Tip

The equation for Q will be given on the data sheet, however you will be expected to remember that it is similar for *I* and *V*.

Capacitor Charge Equation

- When a capacitor is charging, the way the charge Q and potential difference *V* increases stills shows exponential decay
  - Over time, they continue to increase but at a slower rate
- This means the equation for Q for a **charging** capacitor is:

$$Q = Q_0(1 - e^{-\frac{t}{RC}})$$

- Where:
  - Q = charge on the capacitor plates (C)
  - Q_0 = maximum charge stored on capacitor when fully charged (C)
  - e = the exponential function
  - *t* = time (s)
  - *RC* = resistance (Ω) × capacitance (F) = the time constant τ (s)

- Similarly, for *V*:

$$V = V_0(1 - e^{-\frac{t}{RC}})$$

- Where:
  - *V* = p.d across the capacitor (V)
  - V_0 = maximum potential difference across the capacitor when fully charged (V)

- The charging equation for the current *I* is the **same** as its discharging equation since the current still decreases exponentially
- The key difference with the charging equations is that Q_0 and V_0 are now the **final** (or maximum) values of Q and *V* that will be on the plates, rather than the initial values

Worked Example

A capacitor is to be charged to a maximum potential difference of 12 V between its plate. Calculate how long it takes to reach a potential difference 10 V given that it has a time constant of 0.5 s.

Step 1: Write down the known values

Final potential difference, V_0 = 12 A

Potential difference, V = 10 V

Time constant, RC = 0.5 s

Step 2: Write down the potential difference charging equation

$$V = V_0(1 - e^{-\frac{t}{RC}})$$

Step 3: Rearrange for t

$$\frac{V}{V_0} = 1 - e^{-\frac{t}{RC}}$$

Rearrange so the exponential is on its own on the right-hand side

$$\frac{V}{V_0} - 1 = -e^{-\frac{t}{RC}}$$

Divide by -1 on both sides

$$1 - \frac{V}{V_0} = e^{-\frac{t}{RC}}$$

The exponential is removed by taking the natural log (ln) of both sides

$$\ln(1 - \frac{V}{V_0}) = \ln(e^{-\frac{t}{RC}})$$

$$\ln(1 - \frac{V}{V_0}) = -\frac{t}{RC}$$

$$t = -RC\ln(1 - \frac{V}{V_0})$$

Step 4: Substitute in the values

$$t = -(0.5) \times \ln(1 - \frac{10}{12}) = 0.89588 = \mathbf{0.9\ s}$$

EXAM PAPERS PRACTICE

Exam Tip

Make sure you're confident in rearranging equations with natural logs (ln) and the exponential function (e) for both charging and discharging equations. To refresh your knowledge of this, have a look at the AS Maths revision notes on Exponentials & Logarithms.

Required Practical: Charging & Discharging Capacitors

Aim of the Experiment

- The overall aim of this experiment is to calculate the capacitance of a capacitor. This is just one example of how this required practical might be carried out

Variables

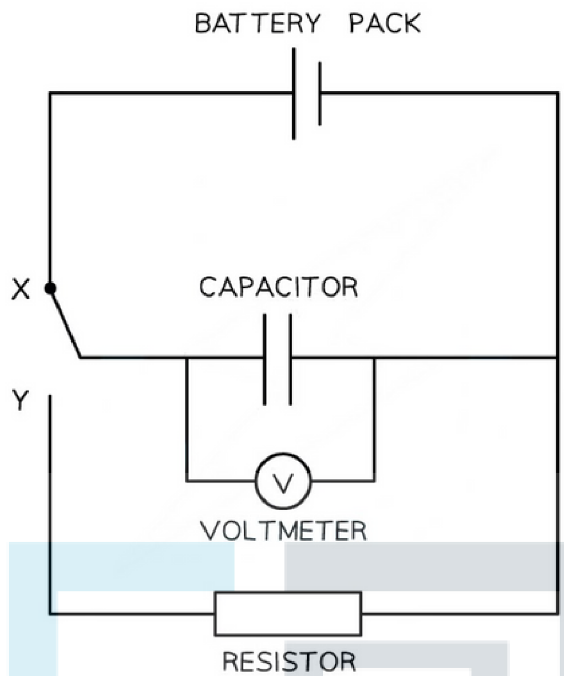
- Independent variable = time, t
- Dependent variable= potential difference, V
- Control variables:
  - Resistance of the resistor
  - Current in the circuit

Equipment List

Apparatus	Purpose
Switch	To switch between the charging and discharging circuit
Capacitor	To measure the capacitance
10 kΩ Resistor	To discharge the capacitor
Battery pack (power supply)	To provide the potential difference across the capacitor
Voltmeter	To measure the potential difference across the capacitor
Stopwatch	To measure the time taken for the capacitor to discharge

- **Resolution** of measuring equipment:
  - Voltmeter = 0.1 V
  - Stopwatch = 0.01 s

Method



1. Set up the apparatus like the circuit above, making sure the switch is not connected to **X** or **Y** (no current should be flowing through)
2. Set the battery pack to a potential difference of 10 V and use a 10 kΩ resistor. The capacitor should initially be fully discharged
3. Charge the capacitor fully by placing the switch at point **X**. The voltmeter reading should read the same voltage as the battery (10 V)
4. Move the switch to point **Y**
5. Record the voltage reading every 10 s down to a value of 0 V. A total of 8–10 readings should be taken

- An example table might look like this:

FROM STOPWATCH → TIME t / s

FROM VOLTMETER → POTENTIAL DIFFERENCE / V

TIME t / s	POTENTIAL DIFFERENCE / V
0.00	
10.00	
20.00	
30.00	
40.00	
50.00	
60.00	
70.00	
80.00	

Analysing the Results

- The potential difference (p.d) across the capacitance is defined by the equation:

$$V = V_0 e^{-\frac{t}{RC}}$$

- Where:
  - V = p.d across the capacitor (V)
  - V_0 = initial p.d across the capacitor (V)
  - t = time (s)
  - e = exponential function
  - R = resistance of the resistor (Ω)
  - C = capacitance of the capacitor (F)

- Rearranging this equation for ln(V) by taking the natural log (ln) of both sides:

$$\ln\left(\frac{V}{V_0}\right) = -\frac{t}{RC}$$

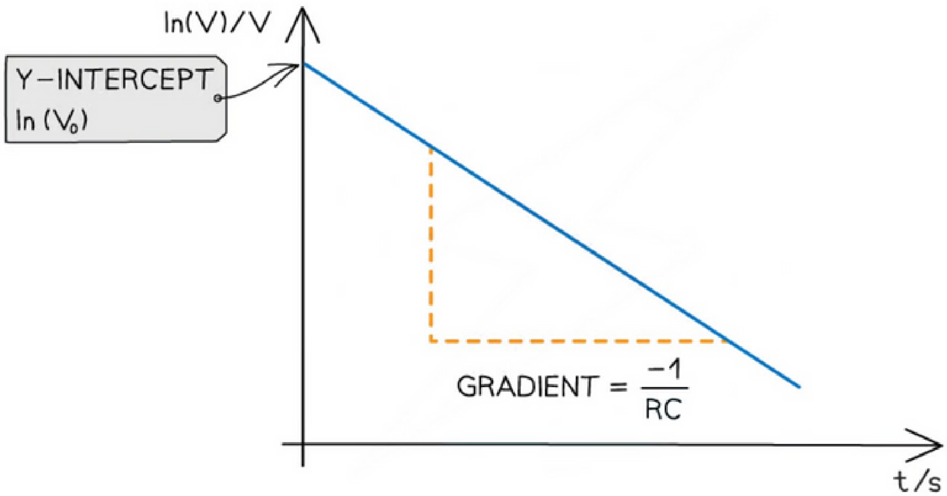
$$\ln(V) - \ln(V_0) = -\frac{t}{RC}$$

$$\ln(V) = -\frac{t}{RC} + \ln(V_0)$$

- Comparing this to the equation of a straight line: y = mx + c
  - y = ln(V)
  - x = t
  - gradient = -1/RC
  - c = ln(V_0)

1. Plot a graph of ln(*V*) against *t* and draw a line of best fit
2. Calculate the gradient (this should be negative)
3. The capacitance of the capacitor is equal to:

$$C = -\frac{1}{R \times \text{gradient}}$$



Evaluating the Experiment

Systematic Errors:

- If a digital voltmeter is used, wait until the reading is settled on a value if it is switching between two
- If an analogue voltmeter is used, reduce parallax error by reading the p.d at eye level to the meter
- Make sure the voltmeter starts at zero to avoid a zero error

Random Errors:

- Use a resistor with a large resistance so the capacitor discharges slowly enough for the time to be taken accurately at p.d intervals
- Using a datalogger will provide more accurate results for the p.d at a certain time. This will reduce the error in the speed of the reflex needed to stop the stopwatch at a certain p.d
- The experiment could be repeated by measuring the time for the capacitor to charge instead

Safety Considerations

- Keep water or any fluids away from the electrical equipment
- Make sure no wires or connections are damaged and contain appropriate fuses to avoid a short circuit or a fire
- Using a resistor with too low a resistance will not only mean the capacitor discharges too quickly but also that the wires will become very hot due to the high current
- Capacitors can still retain charge after power is removed which could cause an electric shock. These should be fully discharged and removed after a few minutes

Worked Example

A student investigates the relationship between the potential difference and the time it takes to discharge a capacitor. They obtain the following results:

Time t/s	Potential Difference / V
0.00	10.0
10.00	8.95
20.00	7.58
30.00	6.13
40.00	4.90
50.00	4.12
60.00	3.00
70.00	2.03

The capacitor is labelled with a capacitance of 4200 μF. Calculate:

(i) The value of the capacitance of the capacitor discharged.

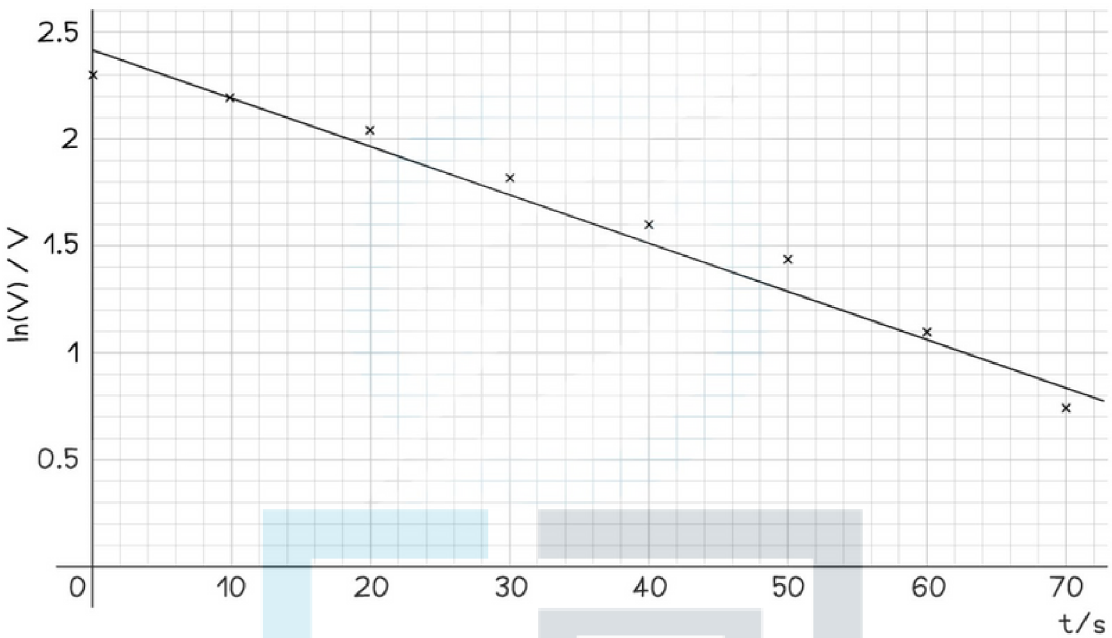
(ii) The relative percentage error of the value obtained from the graph and this true value of the capacitance.

Step 1: Complete the table

- Add an extra column ln(V) and calculate this for each p.d

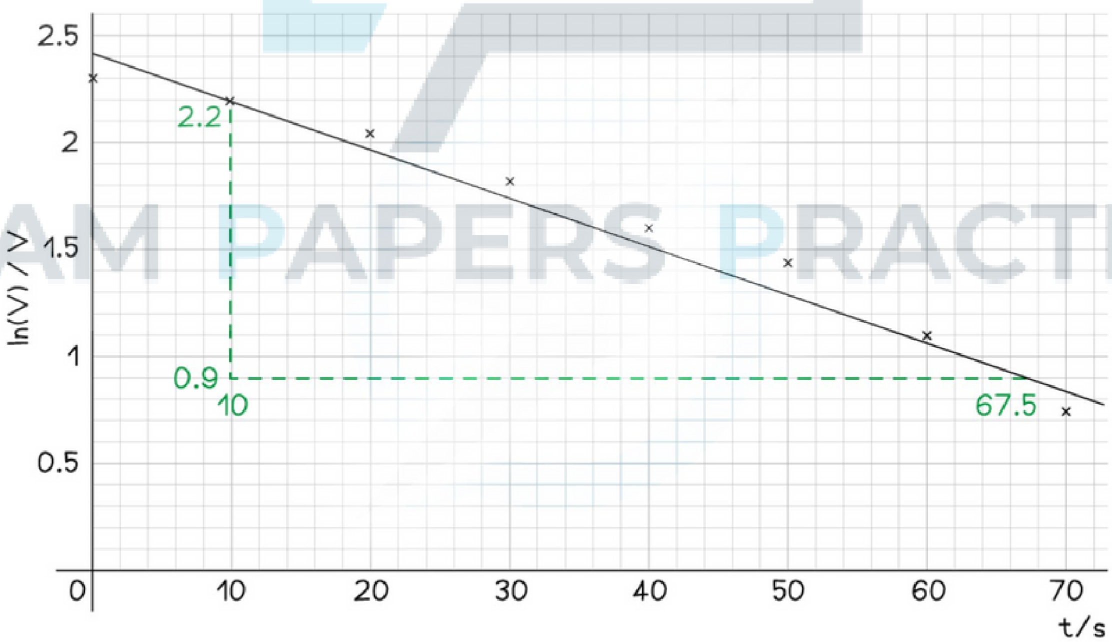
Time t/s	Potential Difference / V	ln(V) / V
0.00	10.0	2.30
10.00	8.95	2.19
20.00	7.58	2.03
30.00	6.13	1.81
40.00	4.90	1.59
50.00	4.12	1.42
60.00	3.00	1.10
70.00	2.03	0.71

Step 2: Plot the graph of ln(*V*) against average time *t*



- Make sure the axes are properly labelled and the line of best fit is drawn with a ruler

Step 3: Calculate the gradient of the graph



- The gradient is calculated by:

$$\text{gradient} = \frac{0.9 - 2.2}{67.5 - 10} = -0.0226$$

Step 4: Calculate the capacitance, C

$$C = -\frac{1}{R \times \text{gradient}}$$

$$C = -\frac{1}{10\,000 \times -0.0226} = 4.42 \times 10^{-3}\text{ F} = 4400\ \mu\text{F}$$

Step 5: Calculate the relative percentage error of the value obtained

$$\text{Relative percentage error} = \frac{\text{Measured value} - \text{Accepted value}}{\text{Accepted value}} \times 100\ \%$$

$$\text{Relative percentage error} = \frac{(4.42 \times 10^{-3}) - (4200 \times 10^{-6})}{4200 \times 10^{-6}} \times 100\ \% = 5.2\ \%$$