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7.6 Capacitance

PHYSICS

AQA A Level Revision Notes

A Level Physics AQA

7.6 Capacitance

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7.6.1 Capacitance

Capacitance

- Capacitors are electrical devices used to store energy in electronic circuits, commonly for a backup release of energy if the power fails
- They are in the form of two conductive metal plates connected to a voltage supply (parallel plate capacitor)
    - There is commonly a **dielectric** in between the plates, this is to ensure charge does not freely flow between the plates
- The capacitor circuit symbol is:

The capacitor circuit symbol is two parallel lines

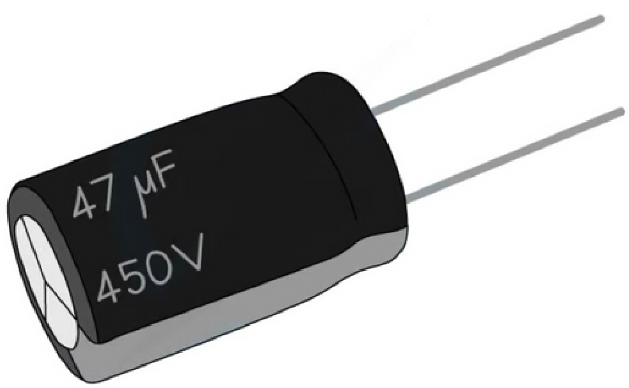
- Capacitors are marked with a value of their **capacitance.** This is defined as:

The charge stored per unit potential difference (between the plates)

- The greater the **capacitance**, the greater the **energy stored** in the capacitor
- The capacitance of a capacitor is defined by the equation:

$$C = \frac{Q}{V}$$

- Where:
    - C = capacitance (F)
    - Q = charge (C)
    - V = potential difference (V)



A capacitor used in small circuits

- Capacitance is measured in the unit **Farad (F)**
    - In practice, 1 F is a very large unit

- Often it will be quoted in the order of micro Farads (μF), nanofarads (nF) or picofarads (pF)
- If the capacitor is made of parallel plates, Q is the charge on the plates and *V* is the potential difference across the capacitor
  - The charge Q is **not** the charge of the capacitor itself, it is the charge stored **on** the plates
- This capacitance equation shows that an object's capacitance is the **ratio of the charge stored by the capacitor to the potential difference between the plates**

Worked Example

A parallel plate capacitor has a capacitance of 1 nF and is connected to a voltage supply of 0.3 kV. Calculate the charge on the plates.

Step 1: Write down the known quantities

- Capacitance, $C = 1\text{ nF} = 1 \times 10^{-9}\text{ F}$
- Potential difference, $V = 0.3\text{ kV} = 0.3 \times 10^{3}\text{ V}$

Step 2: Write out the equation for capacitance

$$C = \frac{Q}{V}$$

Step 3: Rearrange for charge Q

$$Q = CV$$

Step 4: Substitute in values

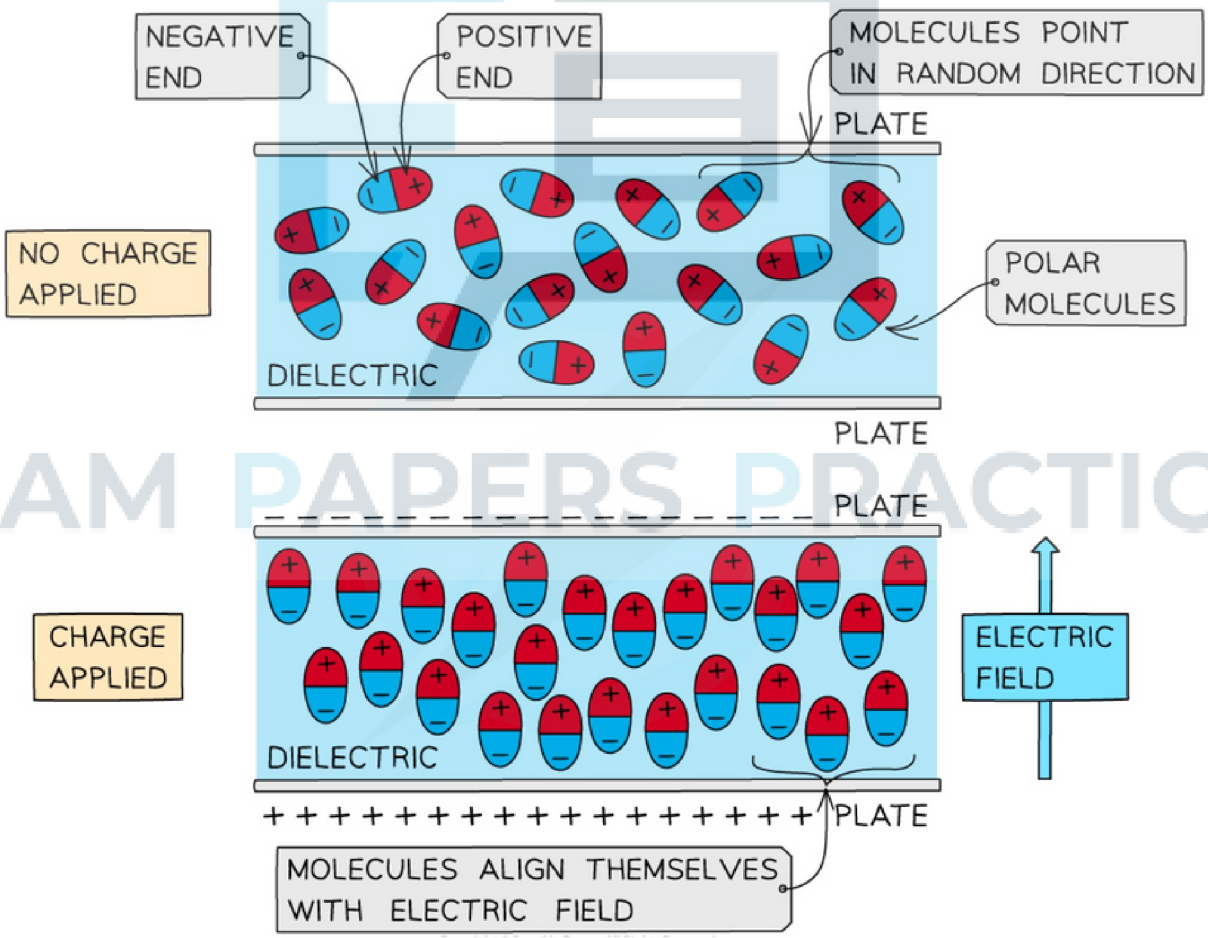
$$Q = (1 \times 10^{-9}) \times (0.3 \times 10^{3}) = 3 \times 10^{-7}\text{ C} = 300\text{ nC}$$

Exam Tip

The 'charge stored' by a capacitor refers to the magnitude of the charge stored **on** each plate in a parallel plate capacitor or **on** the surface of a spherical conductor. The capacitor itself **does not** store charge. The letter 'C' is used both as the symbol for capacitance as well as the unit of charge (coulombs). Take care not to confuse the two!

Polar Molecule in an Electric Field

- A dielectric is made up of many **polar molecules**
  - These are molecules that have a 'positive' and 'negative' end (poles)
- When **no charge** is applied to the capacitor:
  - There is no electric field between the parallel plates and the molecules are aligned in **random** directions
- When there is a **charge applied**:
  - One of the parallel plates becomes positively charged and the other negatively charged hence an electric field is generated between the plates (from positive to negative)
  - The negative ends of the polar molecules are attracted to the positive plate and vice versa
  - This means all the molecules rotate and align themselves **parallel to the electric field**



Polar molecules align themselves when an electric field is between two parallel plates

7.6.2 Parallel Plate Capacitor

Relative Permittivity & Dielectric Constant

- Permittivity is the measure of how easy it is to generate an electric field in a certain material
- The relativity permittivity ε_r is sometimes known as the **dielectric constant**
- For a given material, it is defined as:

The ratio of the permittivity of a material to the permittivity of free space

- This can be expressed as:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

- Where:
  - ε_r = relative permittivity
  - ε = permittivity of a material (F m^{-1})
  - ε_0 = permittivity of free space (F m^{-1})

- The relative permittivity has **no** units because it is a ratio of two values with the same unit

Worked Example

Calculate the permittivity of a material that has a relative permittivity of 4.5×10^{11}. State an appropriate unit for your answer.

Step 1: Write down the relative permittivity equation

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0}$$

Step 2: Rearrange for permittivity of the material ε

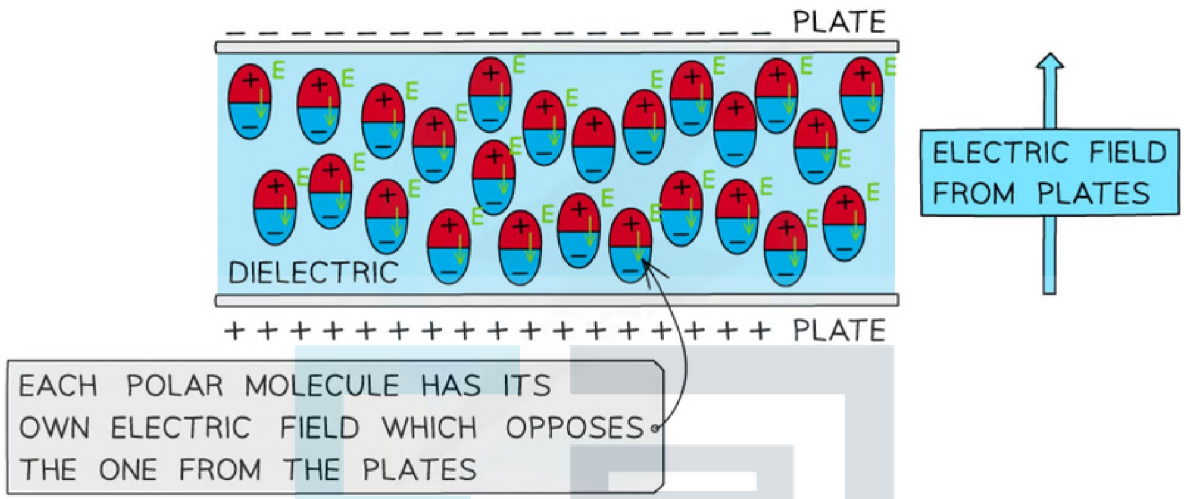
$$\varepsilon = \varepsilon_r \varepsilon_0$$

Step 3: Substitute in the values

$$\varepsilon = (4.5 \times 10^{11}) \times (8.85 \times 10^{-12}) = 3.9825 = 4 \text{ F m}^{-1}$$

Dielectric Action in a Capacitor

- When the polar molecules in a **dielectric** align with the applied electric field from the plates, they each produce their own electric field
  - This electric field **opposes** the electric field from the plates

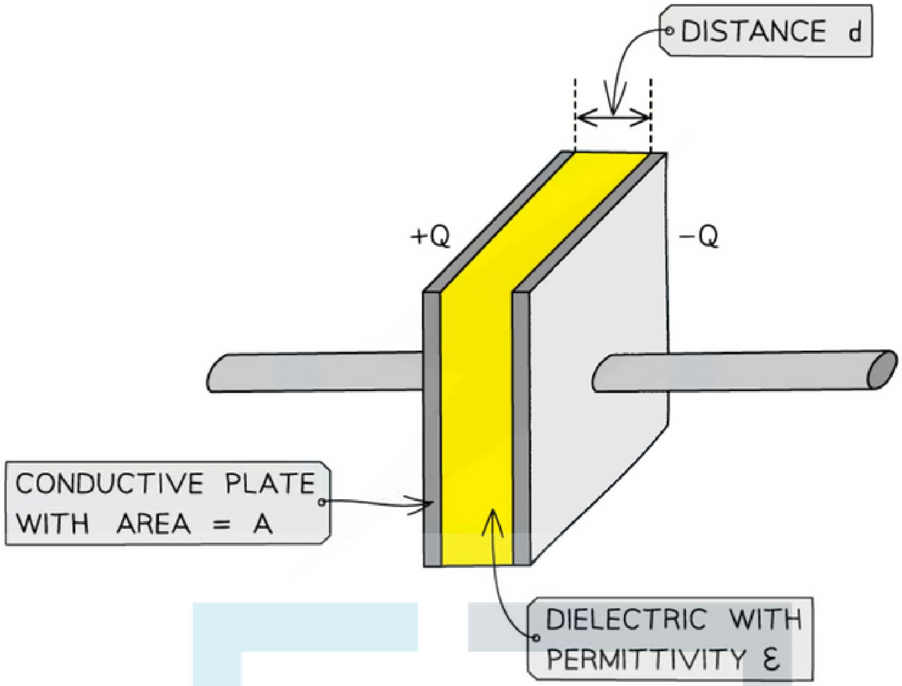


The electric field of the polar molecules opposes that of the electric field produced by the parallel plates

- The **larger** the opposing electric field from the polar molecules in the dielectric, the **larger** the permittivity
  - In other words, the permittivity is how well the polar molecules in a dielectric align with an applied electric field
- The opposing electric field **reduces** the **overall** electric field, which **decreases** the potential difference between the plates
  - Therefore, the capacitance of the plates **increases**
- The capacitance of a capacitor can also be written in terms of the relative permittivity:

$$C = \frac{A\varepsilon_0\varepsilon_r}{d}$$

- Where:
  - C = capacitance (F)
  - A = cross-sectional area of the plates (m^2)
  - d = separation of the plates (m)
  - ε_r = relative permittivity of the dielectric between the plates
  - ε_0 = permittivity of free space ($F\ m^{-1}$)



A parallel plate capacitor consists of conductive plates each with area A, a distance d apart and a dielectric ε between them

- Capacitor plates are general square, therefore if they have a length *L* on all sides then their cross-sectional area is L^2

Worked Example

A parallel-plate capacitor has square plates of length *L* separated by distance *d* and is filled with a dielectric. A second capacitor has square plates of length *3L* separated by distance 3*d* and has air as its dielectric. Both capacitors have the same capacitance.

Determine the relative permittivity of the dielectric in the first capacitor.

Step 1: Write down the capacitance equation with the relativity permittivity

$$C = \frac{A\varepsilon_0\varepsilon_r}{d}$$

Step 2: Write the known values for each capacitor

Capacitor 1:

$C = C$

$A = L^2$

$\varepsilon_r = \varepsilon_r$

$\varepsilon_0 = \varepsilon_0$

$d = d$

Capacitor 2:

$C = C$

$A = (3L)^2 = 9L^2$

$\varepsilon_r = 1$

$\varepsilon_0 = \varepsilon_0$

$d = 3d$

Since the dielectric for capacitor 2 is air, and air has a permittivity of ε_0

therefore using the equation:

$$\varepsilon_r = \frac{\varepsilon}{\varepsilon_0} = \frac{\varepsilon_0}{\varepsilon_0} = 1$$

for capacitor 2

Step 3: Substitute them into the capacitance equation

$$C_1 = \frac{L^2\varepsilon_0\varepsilon_r}{d}$$

$$C_2 = \frac{9L^2\varepsilon_0}{3d}$$

Step 4: Equate the capacitances

Both capacitors have the same capacitance

$$\frac{L^2\varepsilon_0\varepsilon_r}{d} = \frac{9L^2\varepsilon_0}{3d}$$

Step 5: Cancel out d, L^2 and ε_0 from both sides to find the value of ε_r

$$\varepsilon_r = \frac{9}{3} = \mathbf{3}$$

Exam Tip

Remember that *A*, the cross-sectional area, is only for **one** of the parallel plates. Don't multiply this by 2 for both the plates for the capacitance equation!

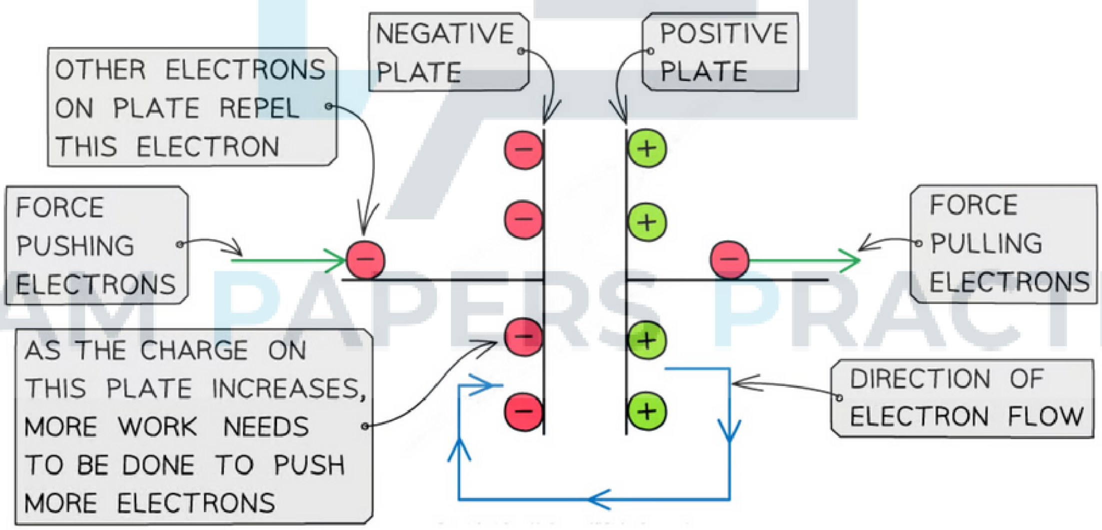
Energy Stored by a Capacitor

- When charging a capacitor, the power supply pushes electrons from the positive to the negative plate
  - It therefore does **work** on the electrons and **electrical energy** becomes stored on the plates

- At first, a small amount of charge is pushed from the positive to the negative plate, then gradually, this builds up
  - Adding more electrons to the negative plate at first is relatively easy since there is little repulsion

- As the charge of the negative plate increases ie. becomes more negatively charged, the force of repulsion between the electrons on the plate and the new electrons being pushed onto it increases
- This means a greater amount of work must be done to increase the charge on the negative plate or in other words:

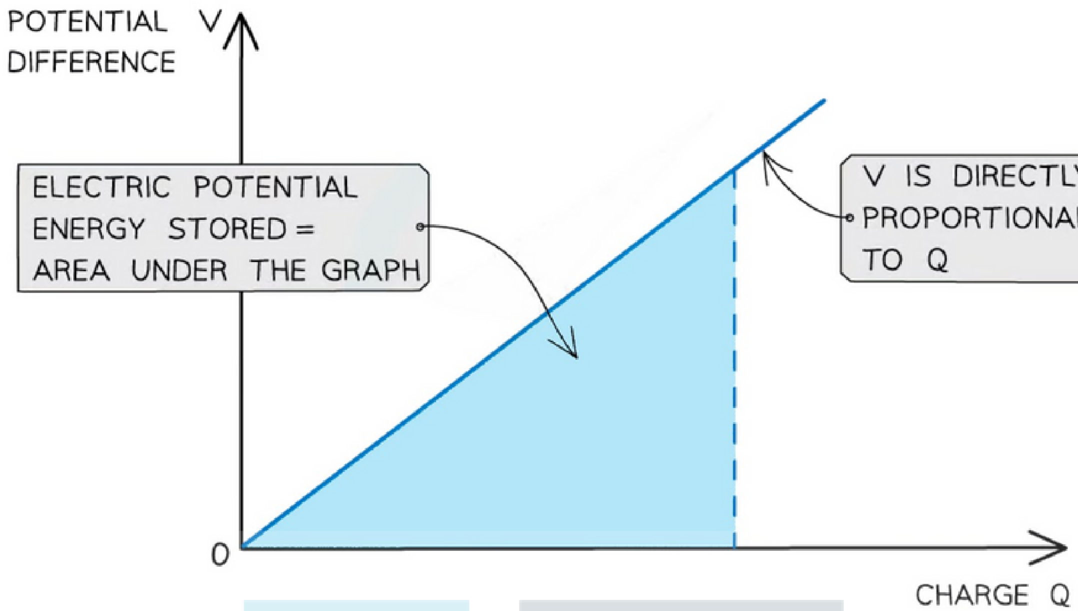
The potential difference across the capacitor increases as the amount of charge increases



As the charge on the negative plate builds up, more work needs to be done to add more charge

- The charge Q on the capacitor is **directly proportional** to its potential difference *V*
- The graph of charge against potential difference is therefore a straight line graph through the origin
- The electrical (potential) energy stored in the capacitor can be determined from the **area under the potential-charge graph** which is equal to the **area** of a right-angled triangle:

Area = 0.5 × base × height



The electric energy stored in the capacitor is the area under the potential-charge graph

- Therefore the work done, or **energy stored** in a capacitor is defined by the equation:

$$E = \frac{1}{2}QV$$

- Where:
  - E = work done or energy stored (J)
  - Q = charge (C)
  - V = potential difference (V)
- Substituting the charge with the **capacitance** equation **Q = CV**, the energy stored can also be defined as:

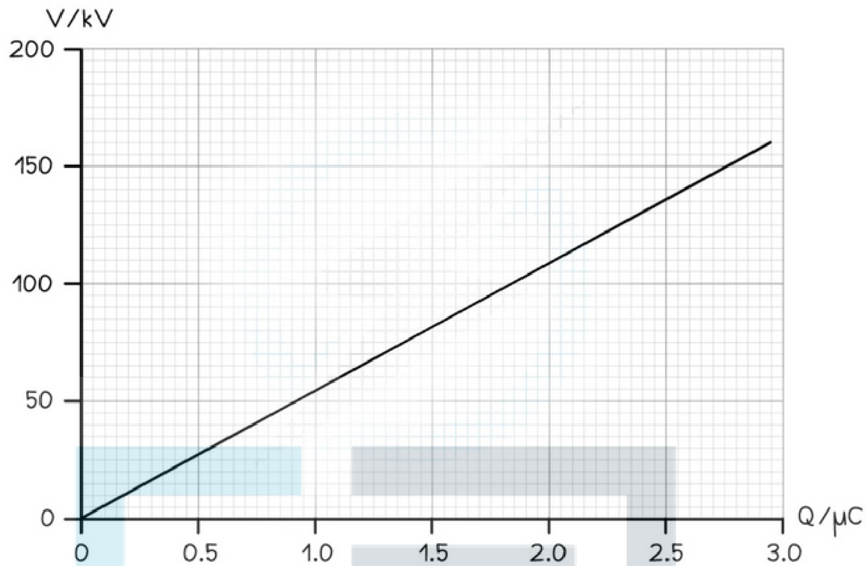
$$E = \frac{1}{2}CV^2$$

- By substituting the potential V, the energy stored can also be defined in terms of just the charge, Q and the capacitance, C:

$$E = \frac{Q^2}{2C}$$

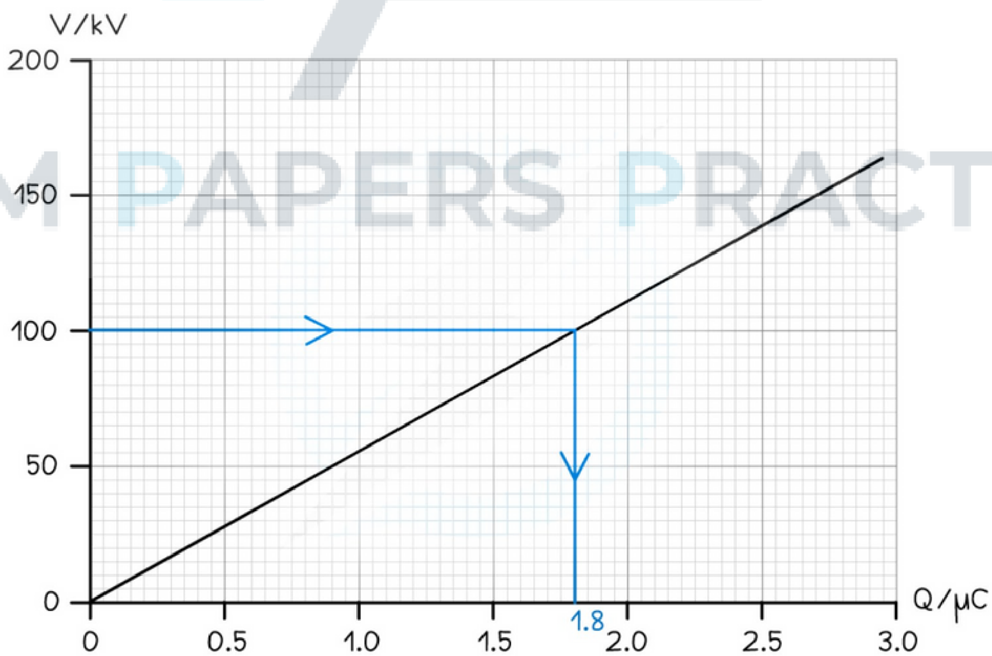
Worked Example

The variation of the potential *V* of a charged isolated metal sphere with surface charge Q is shown on the graph below.



Using the graph, determine the electric potential energy stored on the sphere when charged to a potential of 100 kV.

Step 1: Determine the charge on the sphere at the potential of 100 kV



- From the graph, the charge on the sphere at 100 kV is **1.8 μC**

Step 2: Calculate the electric potential energy stored

- The energy stored is equal to the area under the graph at 100 kV
- The area is equal to a right-angled triangle, so, can be calculated with the equation:

$$\text{Area} = 0.5 \times \text{base} \times \text{height}$$

$$\text{Area} = 0.5 \times 1.8\ \mu\text{C} \times 100\ \text{kV}$$

$$\text{Energy } E = 0.5 \times (1.8 \times 10^{-6}) \times (100 \times 10^{3}) = \mathbf{0.09\ J}$$

Worked Example

Calculate the change in the energy stored in a capacitor of capacitance 1500 μF when the potential difference across the capacitor changes from 10 V to 30 V.

Step 1: Write down the equation for energy stored, in terms of C and V and list the known values

$$E = \frac{1}{2} CV^2$$

Capacitance, C = 1500μF

Final p.d, $V_2 = 30$ V

Initial p.d $V_1 = 10$ V

Step 2: The change in energy stored is proportional to the change in p.d

$$\Delta E = \frac{1}{2} C(\Delta V)^2 = \frac{1}{2} C(V_2^2 - V_1^2)$$

Step 3: Substitute in the values

$$\Delta E = \frac{1}{2} (1500 \times 10^{-6})(30^2 - 10^2) = 0.6\ \text{J}$$

Exam Tip

All 3 equations for the energy stored will be given on your data sheet. To figure out which to use, check what variables (C, Q or V) have already been given in the question.