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7.3 Orbits of Planets & Satellites

XVIII

PHYSICS

AQA A Level Revision Notes

A Level Physics AQA

7.3 Orbits of Planets & Satellites

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7.3.1 Circular Orbits in Gravitational Fields

Circular Orbits in Gravitational Fields

- Since most planets and satellites have a near circular orbit, the gravitational force F_G between the Sun and another planet provides the centripetal force needed to stay in an orbit
- The gravitational force is centripetal, therefore it is **perpendicular** to the direction of travel of the planet
- Consider a satellite with mass *m* orbiting Earth with mass *M* at a distance *r* from the centre travelling with linear speed *v*

$$F_G = F_{centripetal}$$

- Equating the gravitational force to the centripetal force for a planet or satellite in orbit gives:

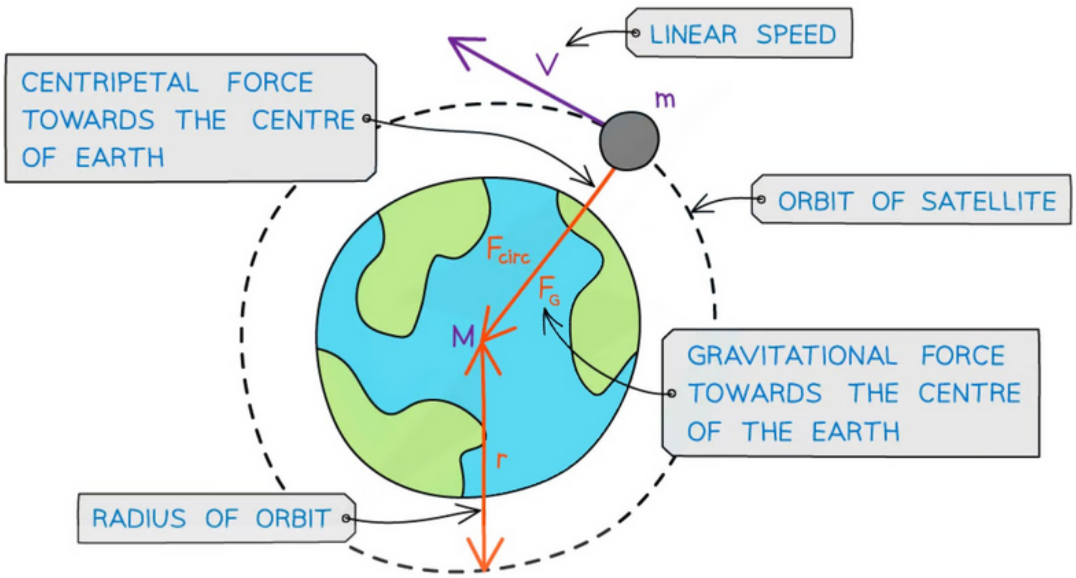
$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

- The mass of the satellite *m* will cancel out on both sides to give:

$$v^2 = \frac{GM}{r}$$

- Where:
  - v = linear speed of the mass in orbit (m s^{-1})
  - G = Newton's Gravitational Constant
  - M = mass of the object being orbited (kg)
  - r = orbital radius (m)

- This means that all satellites, whatever their mass, will travel at the same speed *v* in a particular orbit radius *r*
- Recall that since the direction of a planet orbiting in circular motion is constantly changing, it has **centripetal acceleration**



A satellite in orbit around the Earth travels in circular motion

Time Period & Orbital Radius Relation

- Since a planet or a satellite is travelling in circular motion when in orbit, its orbital time period *T* to travel the circumference of the orbit 2π*r*, the linear speed *v* is:

$$v = \frac{2\pi r}{T}$$

- This is a result of the well-known equation, speed = distance / time and first introduced in the circular motion topic
- Substituting the value of the linear speed *v* from equating the gravitational and centripetal force into the above equation gives:

$$v^2 = \left(\frac{2\pi r}{T}\right)^2 = \frac{GM}{r}$$

- Squaring out the brackets and rearranging for T^2 gives the equation relating the time period *T* and orbital radius *r*:

$$T^2 = \frac{4\pi^2 r^3}{GM}$$

- Where:
  - *T* = time period of the orbit (s)
  - *r* = orbital radius (m)
  - *G* = Newton's Gravitational Constant
  - *M* = mass of the object being orbited (kg)

- The equation shows that the orbital period *T* is related to the radius *r* of the orbit. This is also known as Kepler's third law:

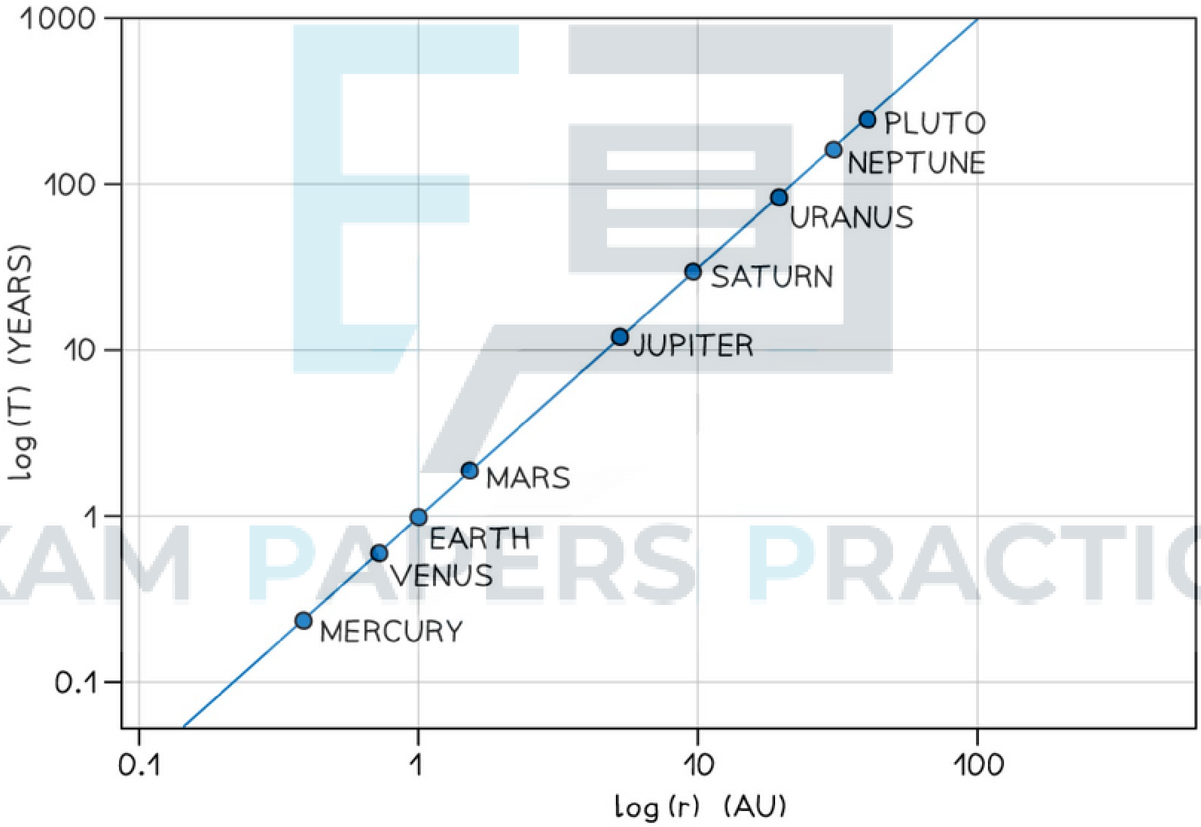
  For planets or satellites in a circular orbit about the same central body, the square of the time period is proportional to the cube of the radius of the orbit

- Kepler's third law can be summarised as:

$$T^2 \propto r^3$$

Graphical Representation of $T^2 \propto r^3$

- The relationship between *T* and *r* can be shown using a logarithmic plot
- The graph of log(*T*) in years against log(*r*) in AU (astronomical units) for the planets in our solar system is a straight line graph:



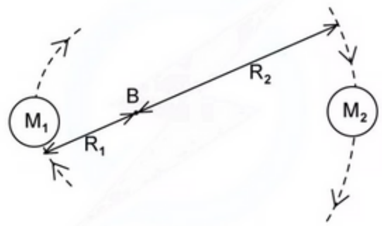
- The graph does not go through the origin since it has a negative y-intercept
- Only the log of both *T* and *r* will produce a straight line graph

Maths Tip

- The $\propto$ symbol means 'proportional to'
- Find out more about proportional relationships between two variables in the "proportional relationships " section of the A Level Maths revision notes

Worked Example

A binary star system constant of two stars orbiting about a fixed point **B**.The star of mass M_1 has a circular orbit of radius R_1 and mass M_2 has a radius of R_2. Both have linear speed v and an angular speed ω about **B**.



State the following formula, in terms of G, M_2, R_1 and R_2

(i) The angular speed ω of M_1

(ii) The time period T for each star in terms of angular speed ω

(i) The angular speed ω of M_1

Step 1: Equate the centripetal force to the gravitational force

$$M_1R_1\omega^2 = \frac{GM_1M_2}{(R_1+R_2)^2}$$

Step 2: M_1 cancels on both sides

$$R_1\omega^2 = \frac{GM_2}{(R_1+R_2)^2}$$

Step 3: Rearrange for angular velocity ω

$$\omega^2 = \frac{GM_2}{R_1(R_1+R_2)^2}$$

Step 4: Square root both sides

$$\omega = \sqrt{\frac{GM_2}{R_1(R_1+R_2)^2}}$$

(ii) The time period T for each star in terms of angular speed ω

Step 1: Write down the angular speed ω equation with time period T

$$\omega = \frac{2\pi}{T}$$

Step 2: Rearrange for T

$$T = \frac{2\pi}{\omega}$$

Step 3: Substitute in ω from part (i)

$$T = 2\pi \div \sqrt{\frac{GM_2}{R_1(R_1+R_2)^2}} = 2\pi\sqrt{\frac{R_1(R_1+R_2)^2}{GM_2}}$$

Exam Tip

Many of the calculations in the Gravitation questions depend on the equations for circular motion. Be sure to revisit these and understand how to use them! You will be expected to remember the derivation for $T^2 \propto r^3$ relation, so make sure you understand each step

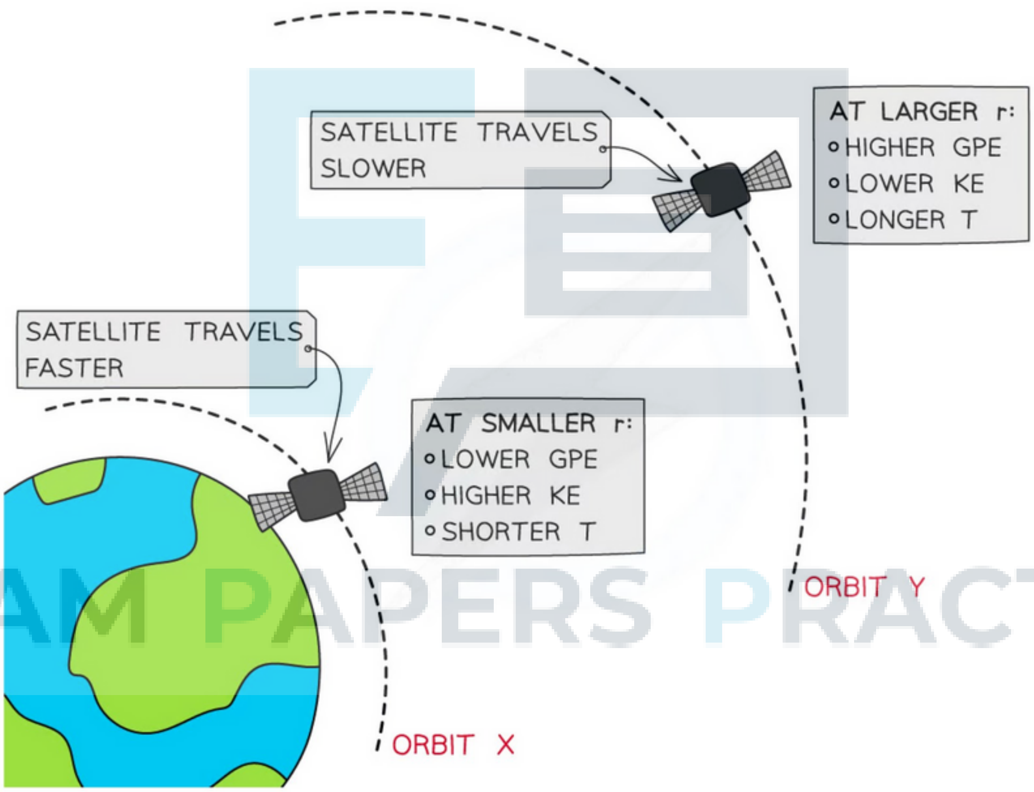
7.3.2 Energy of an Orbiting Satellite

Energy of an Orbiting Satellite

- An orbiting satellite follows a circular path around a planet
- Just like an object moving in circular motion, it has both kinetic energy (KE) **and** gravitational potential energy (GPE) and its **total** energy is always **constant**
- An orbiting satellite's total energy is calculated by:

Total energy = Kinetic energy + Gravitational potential energy

- This means that the satellite's KE and GPE are also both constant in a particular orbit
  - If the orbital radius of a satellite **decreases** its KE **increases** and its GPE **decreases**
  - If the orbital radius of a satellite **increases** its KE **decreases** and its GPE **increases**



At orbit Y, the satellite has greater GPE and less KE than at at orbit X

- A satellite is placed in two orbits, X and Y, around Earth
- At orbit X, where the radius of orbit *r* is smaller, the satellite has a:
  - Larger gravitational force on it
  - Higher speed
  - Higher KE
  - Lower GPE
  - Shorter orbital time period, *T*
- At orbit Y, where the radius of orbit *r* is larger, the satellite has a:

- Smaller gravitational force on it
- Smaller speed
- Lower KE
- Higher GPE
- Longer orbital time period, T

Worked Example

Two satellites A and B, of equal mass, orbit a planet at radii R and $3R$ respectively. Which one of the following statements is incorrect?

A A has more kinetic energy and less potential energy than B

B A has a shorter time period and travels faster than B

C B has less kinetic energy and more potential energy than A

D B has a longer time period and travels faster than A

ANSWER: D

- Since B is at a larger orbital radius ($3R$ instead of R) it has a longer time period since $T^2 \propto R^3$ for an orbiting satellite
- However, satellite B will travel much slower than A
- Its larger orbital radius means the force of gravity will be much lower for B than for A

Exam Tip

If you can't remember which way around the kinetic and potential energy increases and decreases, think about the velocity of a satellite at different orbits. When it is orbiting close to a planet, it experiences a larger gravitational pull and therefore orbits faster. Since the kinetic energy is proportional to v^2, it, therefore, has higher kinetic energy closer to the planet. To keep the total energy constant, the potential energy must decrease too.

7.3.3 Escape Velocity

Escape Velocity

- To escape a gravitational field, a mass must travel at the **escape velocity**
- This is dependent on the mass and radius of the object creating the gravitational field, such as a planet, a moon or a black hole
- Escape velocity is defined as:

The minimum speed that will allow an object to escape a gravitational field with no further energy input

- It is the same for all masses in the same gravitational field ie. the escape velocity of a rocket is the same as a tennis ball on Earth

- An object reaches escape velocity when all its kinetic energy has been transferred to gravitational potential energy
- This is calculated by equating the equations:

$$\frac{1}{2} \times m \times v^2 = \frac{G \times M \times m}{r}$$

- Where:
  - m = mass of the object in the gravitational field (kg)
  - v = escape velocity of the object (m s^{-1})
  - G = Newton's Gravitational Constant
  - M = mass of the object to be escaped from (ie. a planet) (kg)
  - r = distance from the centre of mass M (m)

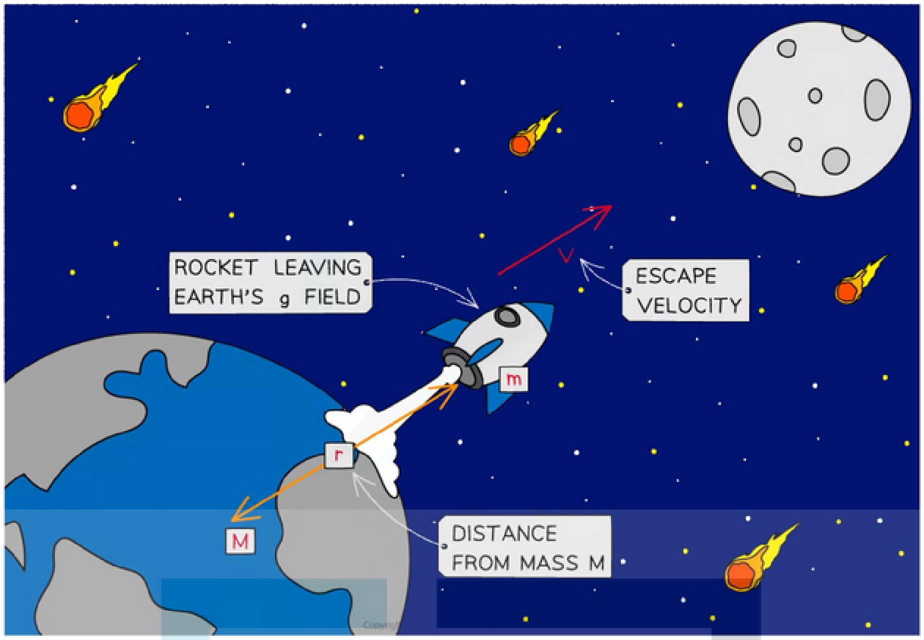
- Since mass m is the same on both sides of the equations, it can cancel on both sides of the equation:

$$\frac{1}{2} \times v^2 = \frac{G \times M}{r}$$

- Multiplying both sides by 2 and taking the square root gives the equation for escape velocity, v:

$$v = \sqrt{\frac{2 \times G \times M}{r}}$$

- This equation is **not** given on the datasheet. Be sure to memorise how to derive it



For an object to leave the Earth's gravitational field, it will have to travel at a speed greater than the Earth's escape velocity, v

- Rockets launched from the Earth's surface do **not** need to achieve escape velocity to reach their orbit around the Earth
- This is because:
  - They are continuously given energy through fuel and thrust to help them move
  - Less energy is needed to achieve orbit than to escape from Earth's gravitational field
- The escape velocity is **not** the velocity needed to escape the planet but to escape the planet's **gravitational field** altogether
  - This could be quite a large distance away from the planet

Worked Example

Calculate the escape velocity at the surface of the Moon given that its density is 3340 kg m^{-3} and has a mass of 7.35 × 10^{22} kg.Newton's Gravitational Constant = 6.67 × 10^{-11} N m^2 kg^{-2}

Step 1: Rearrange the density equation for radius

$$\rho = \frac{M}{V} \;\&\; V = \frac{4}{3}\pi r^3$$

$$\rho = \frac{M}{\frac{4}{3}\pi r^3} = \frac{3M}{4\pi r^3}$$

$$r = \sqrt[3]{\frac{3M}{4\pi\rho}}$$

Step 2: Calculate the radius by substituting in the values

$M = 7.35 \times 10^{22}$ kg

$\rho = 3340$ kg m^{-3}

$$r = \sqrt[3]{\frac{3 \times (7.35 \times 10^{22})}{4\pi \times 3340}} = 1.7384 \times 10^6 \text{ m}$$

Step 3: Substitute *r* into escape velocity equation

$$v = \sqrt{\frac{2GM}{r}} = \sqrt{\frac{2 \times (6.67 \times 10^{-11}) \times (7.35 \times 10^{22})}{1.7384 \times 10^6}}$$

$$v = 2.37 \text{ km s}^{-1}$$

Exam Tip

When writing the definition of **escape velocity**, avoid terms such as 'gravity' or the 'gravitational pull / attraction' of the planet. It is best to refer to its gravitational field.

7.3.4 Geostationary Orbits

Synchronous Orbits

- A synchronous orbit is:

  When an orbiting body has a time period equal to that of the body being orbited and in the same direction of rotation as that body

- These usually refer to **satellites** (the orbiting body) around **planets** (the body being orbited)
- The orbit of a synchronous satellite can be above any point on the planet's surface and in any plane
  - When the plane of the orbit is directly above the equator, it is known as a **geosynchronous** orbit

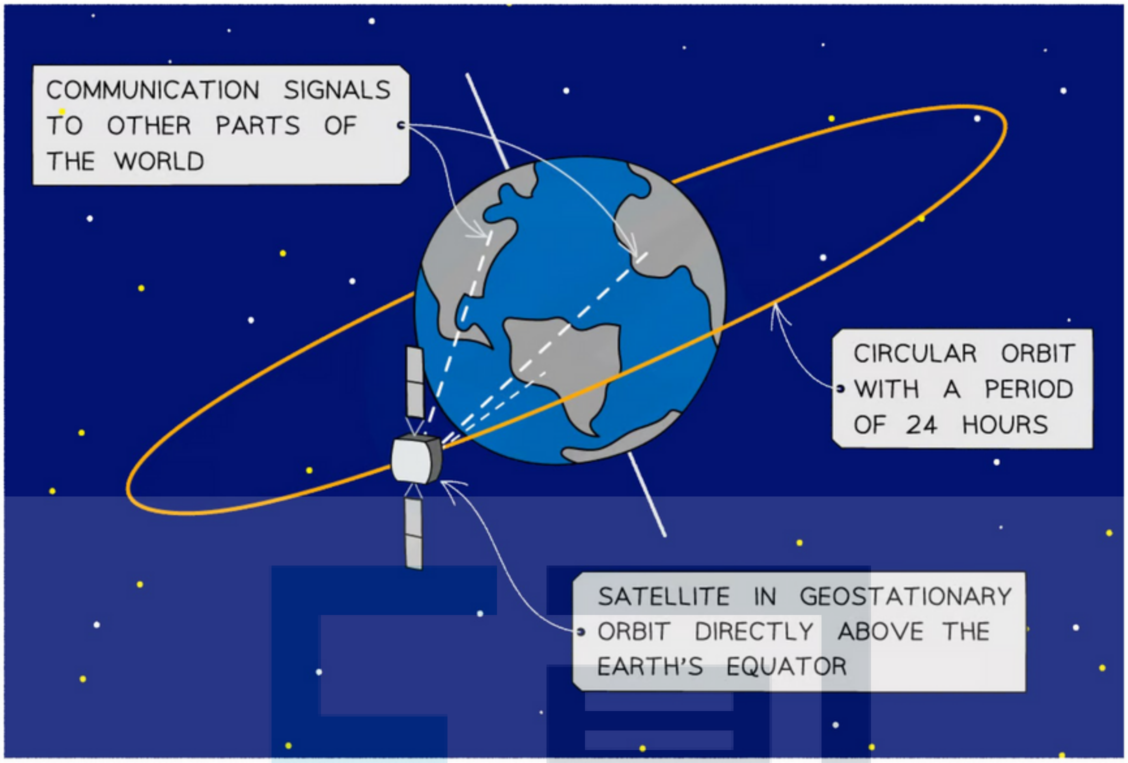
Geostationary Orbits

Geostationary Orbit

- Many communication satellites around Earth follow a **geostationary orbit**
  - This is sometimes referred to as a **geosynchronous** orbit
- This is a specific type of orbit in which the satellite:
  - Remains directly **above the equator**
  - Is in the **plane of the equator**
  - Always orbits at the **same point** above the Earth's surface
  - Moves from **west to east** (same direction as the Earth spins)
  - Has an orbital time period equal to Earth's rotational period of **24 hours**
- Geostationary satellites are used for **telecommunication** transmissions (e.g. radio) and television broadcast
- A base station on Earth sends the TV signal up to the satellite where it is amplified and broadcast back to the ground to the desired locations
- The satellite receiver dishes on the surface must point towards the same point in the sky
  - Since the geostationary orbits of the satellites are fixed, the receiver dishes can be fixed too

Low Orbits

- Some satellites are in low orbits, which means their altitude is closer to the Earth's surface
- One example of this is a **polar** orbit, where the satellite orbits around the north and south pole of the Earth
- Low orbits are useful for taking high-quality photographs of the Earth's surface. This could be used for:
  - Weather
  - Military applications



Geostationary satellite in orbit

Worked Example

The table gives data for two types of satellite, a low-Earth orbit (LEO) and a geostationary orbit

Orbit type	*T* / min	*h* / km
LEO	89	250
Geostationary	**X**	**Y**

For the geostationary orbit, calculate

(i)
the orbital period **X** in minutes.
(ii)
the height **Y** above the Earth's surface that a geostationary satellite will orbit in km.

(i)

Step 1: Convert the time period from seconds to minutes

- The period of a geostationary orbit is **X** = 24 hrs
- The period of a geostationary orbit is **X** = 24 × 60 = 1440 minutes

(ii)

Step 1: List the known quantities

- Period of the LEO, T_L = 89 min
- Period of a geostationary orbit, T_G = 1440 min
- Height above Earth of the LEO, h_L = 250 km
- Radius of the Earth, $R = 6.37 \times 10^6$ m (from the data sheet)

Step 2: Recall the relationship between orbital period and radius

- Orbital period T is related to the radius r of the orbit by $T^2 \propto r^3$

Step 3: Convert the proportional relationship into an equation

- $$\frac{T_G^{\ 2}}{T_L^{\ 2}} = \frac{r_G^{\ 3}}{r_L^{\ 3}} \quad \Rightarrow \quad r_G^{\ 3} = r_L^{\ 3}\left(\frac{T_G}{T_L}\right)^2$$
- $$r_G = \sqrt[3]{r_L^{\ 3}\left(\frac{T_G}{T_L}\right)^2} \quad \Rightarrow \quad r_G = r_L\left(\frac{T_G}{T_L}\right)^{2/3}$$

Step 4: Evaluate a final value for Y

- Orbital radius of LEO: $r_L = R + h_L = (6.37 \times 10^6) + (250 \times 10^3) = 6.62 \times 10^6 \text{ m}$
- Orbital radius of geostationary: $r_G = (6.62 \times 10^6)\left(\frac{1440}{89}\right)^{2/3} = 4.235 \times 10^7 \text{ m}$
- Height above the Earth's surface: **Y** = $(4.235 \times 10^7) - (6.37 \times 10^6) = 3.6 \times 10^7 \text{ m}$
- Height above the Earth's surface: **Y** = 36 000 km

Exam Tip

Make sure to memorise the key features of a geostationary orbit, since this is a common exam question. Remember:

- Equatorial orbit
- Moves west to east
- Period of 24 hours

You will also be expected to remember that the time period of the orbit is 24 hours for calculations on a geostationary satellite.