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7.2 Gravitational Potential

PHYSICS

AQA A Level Revision Notes

A Level Physics AQA

7.2 Gravitational Potential

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7.2.1 Gravitational Potential

Gravitational Potential

- The gravitational potential energy (G.P.E) is the energy an object has when lifted off the ground given by the familiar equation:

$$G.P.E = mg\Delta h$$

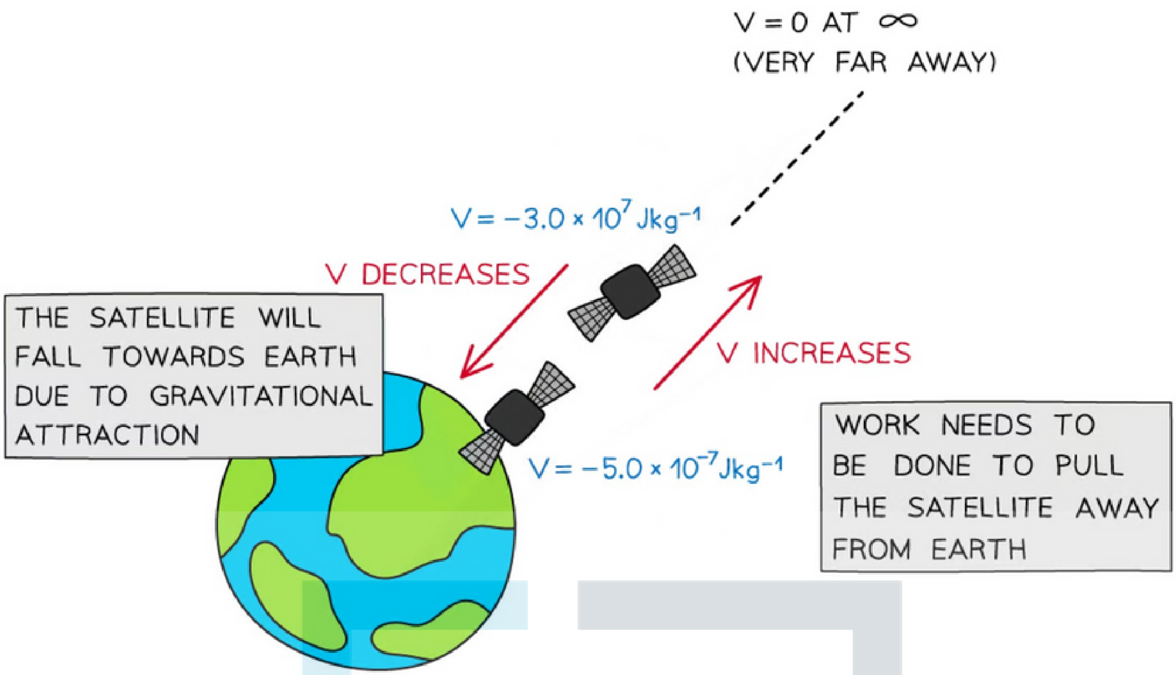
- The G.P.E on the surface of the Earth is taken to be zero
  - This means **work is done** to lift the object
- This equation is **only** used for objects that are within the Earth's surface
- However, **outside** the Earth's surface, G.P.E can be defined as:

The energy an object possess due to its position in a gravitational field

- The gravitational potential at a point is the **gravitational potential energy per unit mass at that point**
- Therefore, it is defined as:

The work done per unit mass in bringing a test mass from infinity to a defined point

- It is represented by the symbol, *V* and is measured in **J kg^{-1}**
- The gravitational potential is always a **negative** value. This is because:
  - It is defined as **zero at infinity**
  - Since the gravitational force is attractive, **work** must be done on a mass to reach infinity
- This means that the gravitational potential is negative on the surface of a mass (such as a planet), and **increases** with distance from that mass (becomes less negative)
- Work has to be done **against** the gravitational pull of the planet to take a unit mass **away** from the planet
- The gravitational potential at a point depends on the mass of the object producing the gravitational field and the distance the point is from that mass



Gravitational potential decreases as the satellite moves closer to the Earth

Exam Tip

Remember to memorise the gravitational potential definition and the reason why it is negative, as these are very common exam questions

Gravitational Potential Difference

- Two points at different distances from a mass will have different gravitational potentials
  - This is because the gravitational potential increases with distance from a mass
- Therefore, there will be a **gravitational potential difference** between the two points
  - This is represented by the symbol ΔV
- ΔV is normally given as the equation

$$\Delta V = V_f - V_i$$

- Where:
  - V_f = final gravitational potential (J kg^{-1})
  - V_i = initial gravitational potential (J kg^{-1})
- A difference in gravitational potential will give a difference in gravitational potential energy, that can also be calculated

Exam Tip

When exam questions ask for the 'difference' or 'change in' a value (denoted by Δ), they are asking for the magnitude. Therefore, don't worry too much about negative or positive signs. As long as you consistently calculate the difference in two values as '**final value – initial value**', a negative difference will mean that the value is decreasing and vice versa

7.2.2 Calculating Gravitational Potential

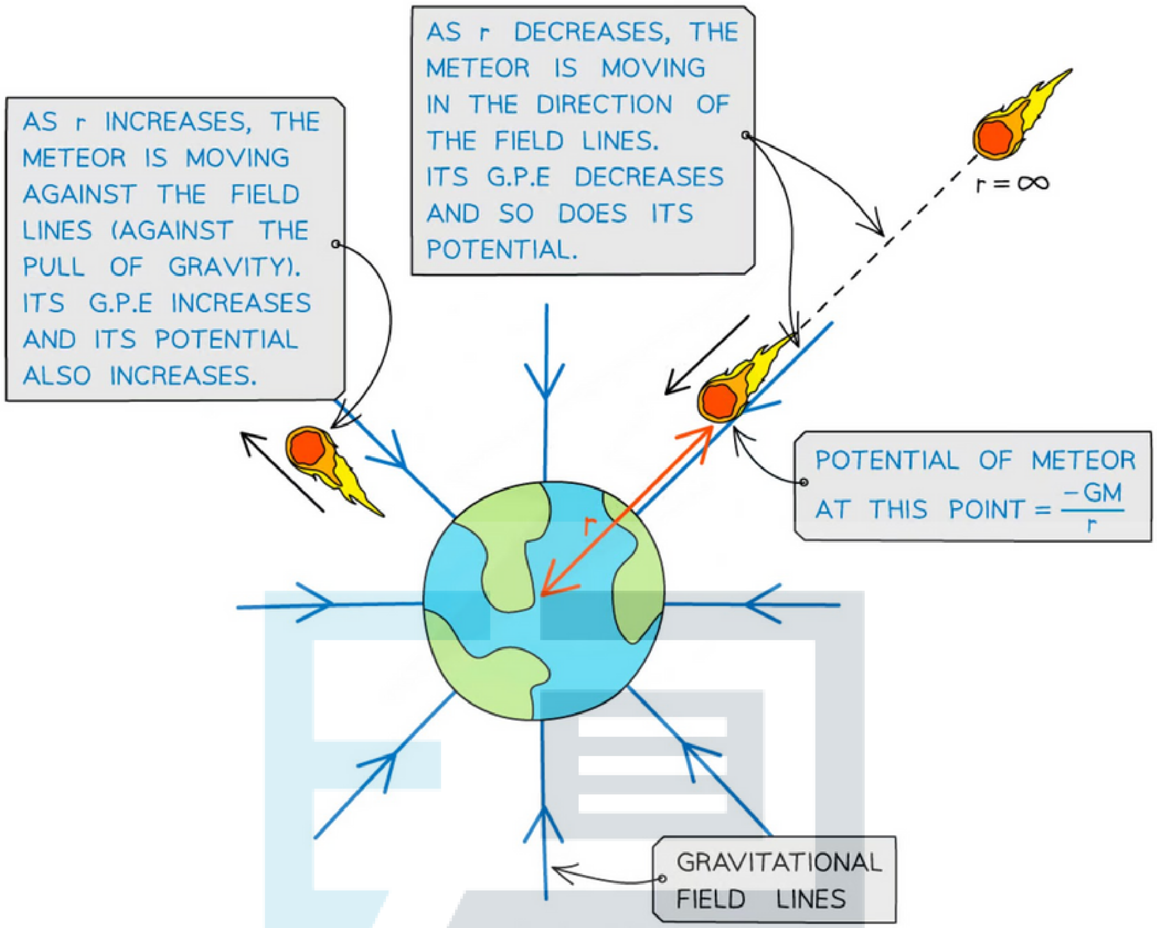
Calculating Gravitational Potential

- The equation for gravitational potential *V* is defined by the mass *M* and distance *r*:

$$V = -\frac{GM}{r}$$

- Where:
  - *V* = gravitational potential (J kg^{-1})
  - *G* = Newton's gravitational constant
  - *M* = mass of the body producing the gravitational field (kg)
  - *r* = distance from the centre of the mass to the point mass (m)
- The gravitational potential always is negative near an isolated mass, such as a planet, because:
  - The potential when *r* is at infinity (∞) is defined as 0
  - Work must be done to move a mass away from a planet (V becomes less negative)
- It is also a **scalar** quantity, unlike the gravitational field strength which is a **vector** quantity

- Gravitational forces are always **attractive**, this means as *r* decreases, positive work is done by the mass when moving from infinity to that point
  - When a mass is closer to a planet, its gravitational potential becomes smaller (more negative)
  - As a mass moves away from a planet, its gravitational potential becomes larger (less negative) until it reaches 0 at infinity
- This means when the distance (*r*) becomes very large, the gravitational force tends rapidly towards 0 at a point further away from a planet



Gravitational potential increases and decreases depending on whether the object is travelling towards or against the field lines from infinity

Worked Example

A planet has a diameter of 7600 km and a mass of 3.5×10^{23} kg. A rock of mass 528 kg accelerates towards the planet from infinity. At a distance of 400 km above the planet's surface, calculate the gravitational potential of the rock.

Step 1: Write the gravitational potential equation

$$V = -\frac{GM}{r}$$

Step 2: Determine the value of r

- r is the distance from the centre of the planet

Radius of the planet = planet diameter ÷ 2 = 7600 ÷ 2 = 3800 km

$r = 3800 + 400 = 4200 \text{ km} = 4.2 \times 10^6 \text{ m}$

Step 3: Substitute in values

$$V = -\frac{(6.67 \times 10^{-11}) \times (3.5 \times 10^{23})}{4.2 \times 10^{6}} = -5.6 \times 10^{6}\ \text{J kg}^{-1}$$

Exam Tip

Remember to keep the negative sign in your solution for the gravitational potential at a point. However, if you're asked for the 'change in' gravitational potential, no negative sign needs to be included since you are finding a difference in values and just the magnitude is normally required. Remember to also calculate *r* as the distance from the **centre** of the planet, and not just the distance above the planet's surface

7.2.3 Graphical Representation of Gravitational Potential

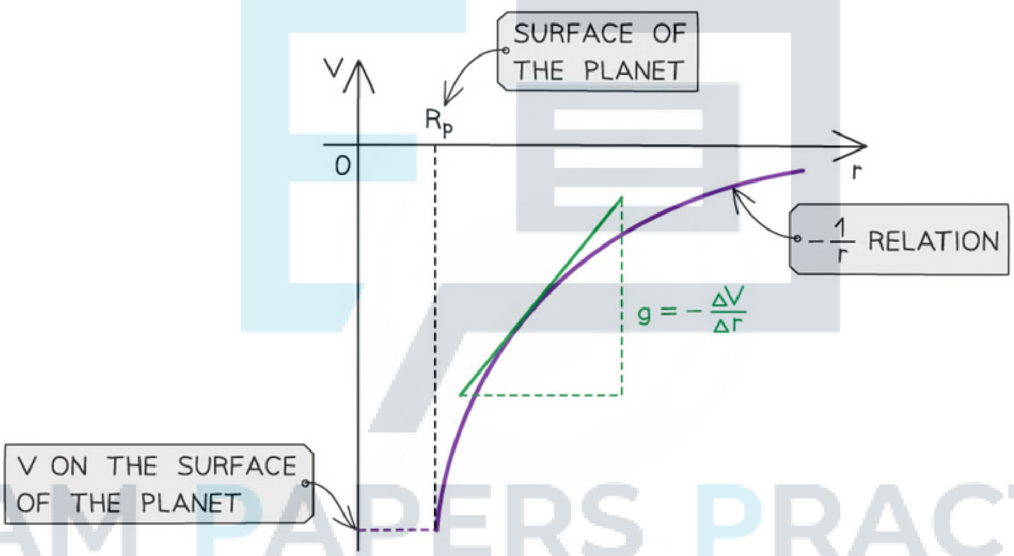
Graphical Representation of Gravitational Potential

- Gravitational field strength, *g* and the gravitational potential, *V* can be graphically represented against the distance from the centre of a planet, *r*
- *g*, *V* and *r* are related by the equation:

$$g = -\frac{\Delta V}{\Delta r}$$

- Where:
  - g = gravitational field strength ($N\ kg^{-1}$)
  - ΔV = change in gravitational potential ($J\ kg^{-1}$)
  - Δr = distance from the centre of a point mass (m)

- The graph of *V* against *r* for a planet is:



The gravitational potential and distance graphs follows a -1/r relation

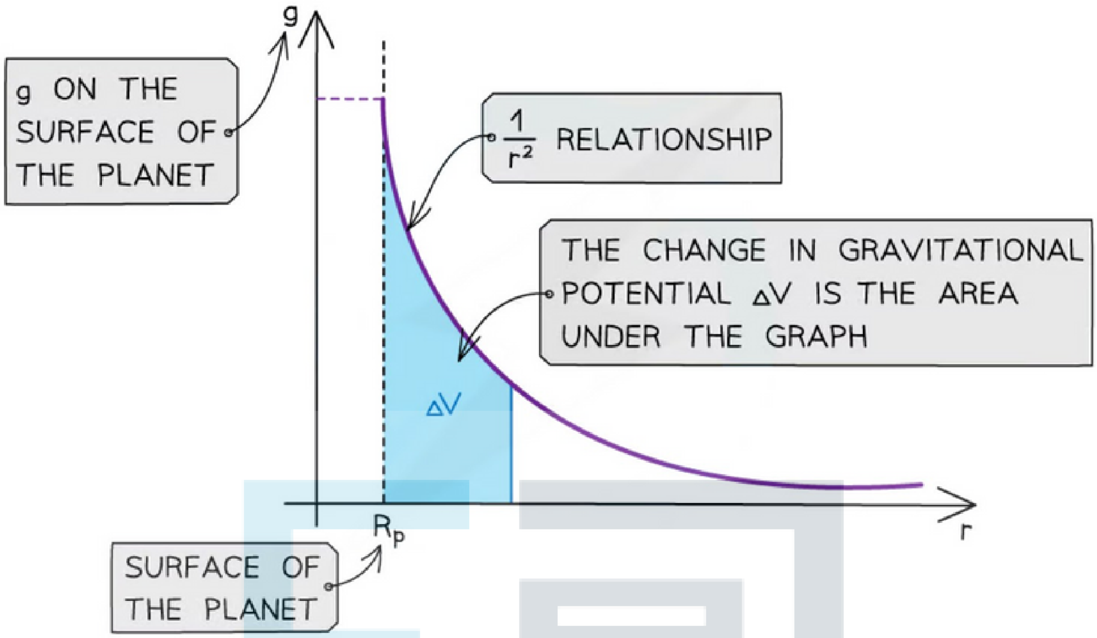
- **The key features of this graph are:**
  - The values for *V* are all negative
  - As *r* increases, *V* against *r* follows a -1/r relation
  - The **gradient** of the graph at any particular point is the value of *g* at that point
  - The graph has a shallow increase as *r* increases

- To calculate *g*, draw a tangent to the graph at that point and calculate the gradient of the tangent
- This is a graphical representation of the equation:

$$V = -\frac{GM}{r}$$

where *G* and *M* are constant

- The graph of g against *r* for a planet is:



The gravitational field strength and distance graph follows a 1/r2 relation

- **The key features of this graph are**:
  - The values for *g* are all positive
  - As *r* increases, g against *r* follows a $1/r^2$ relation (inverse square law)
  - The **area** under this graph is the change in gravitational potential ΔV
  - The graph has a steep decline as *r* increases

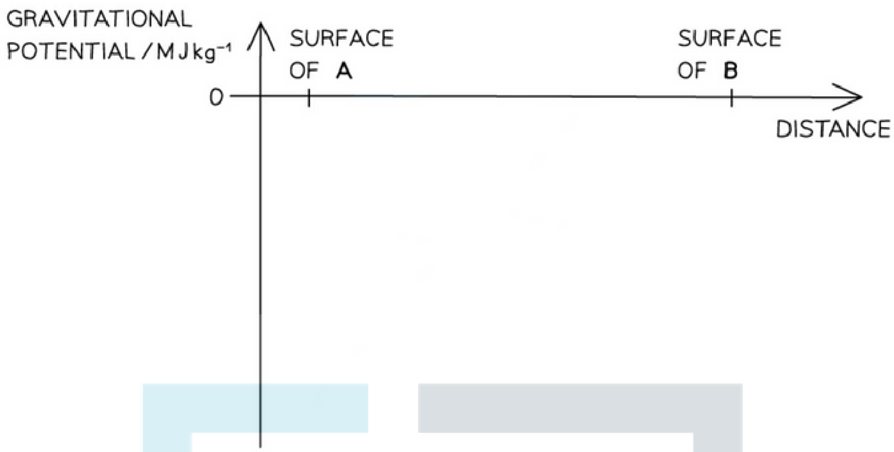
- The area under the graph can be estimated by counting squares, if it is plotted on squared paper, or by splitting it into trapeziums and summing the area of each trapezium
- The inverse square law relation means that as the distance *r* doubles, *g* decreases by a factor of **4**
- This is a graphical representation of the equation:

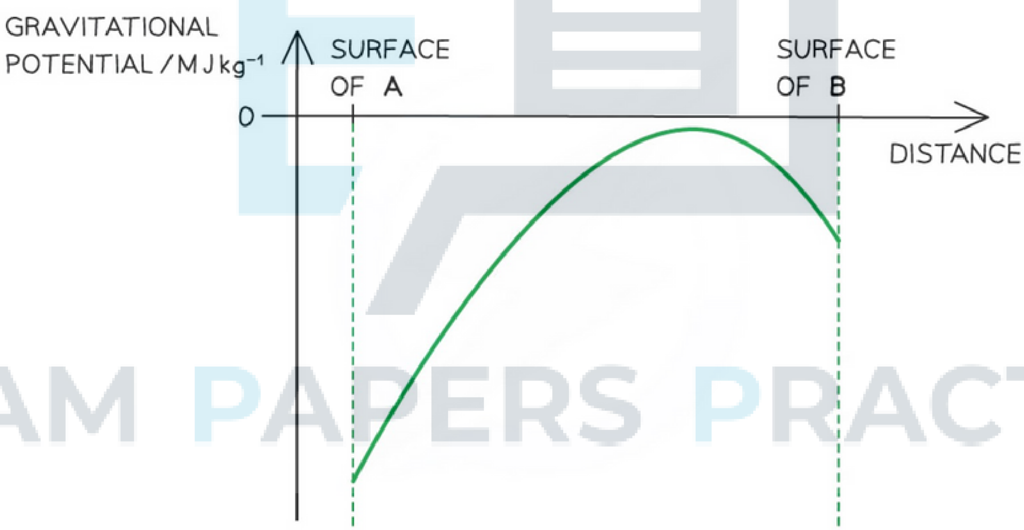
$$g = \frac{GM}{r^2}$$

where *G* and *M* are constant

Worked Example

Sketch a graph on the axes below to indicate how the gravitational potential varies with distance along a line outwards from the surface of planet **A** which is 80 times the mass of planet **B**.





- Graph increases from a large negative value for *V* at the surface of **A** as distance increases
- Up to a value close to, but below 0 near the surface of planet **B**
- The graph then falls near the surface of planet **B**
- From a point much closer to planet **B** than **A**

Exam Tip

Drawing, interpreting or calculating from either of these graphs are common exam questions. The graph of *g* against *r* should start off **steeper** and decrease **rapidly** compared to that of *V* against *r*, to distinguish it as an inverse square law ($1/r^2$) relation instead of just $1/r$.

7.2.4 Work Done on a Mass

Work Done on a Mass

- When a mass is moved against the force of gravity, work is done
- The work done in moving a mass *m* is given by:

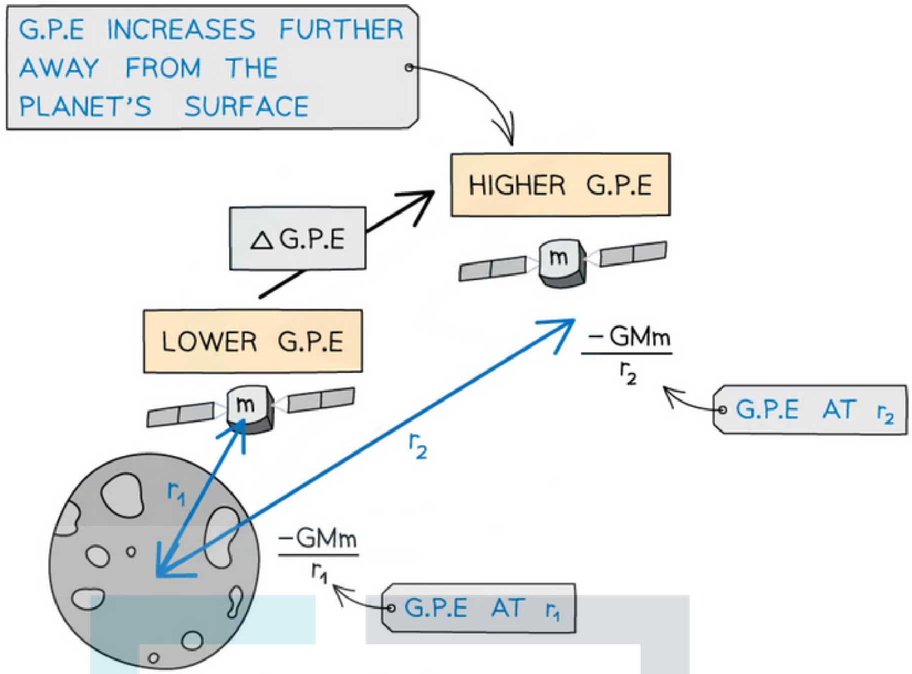
$$\Delta W = m\Delta V$$

- Where:
  - ΔW = change in work done (J)
  - m = mass (kg)
  - ΔV = change in gravitational potential (J kg^{-1})

- This change in work done is equal to the change in **gravitational potential energy** (G.P.E)
  - When $V = 0$, then the G.P.E = 0
- The change in G.P.E, or work done, for an object of mass m at a distance r_1 from the centre of a larger mass M, to a distance of r_2 further away can be written as:

$$\Delta G.P.E = -\frac{GMm}{r_2} - \left(-\frac{GMm}{r_1}\right) = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

- Where:
  - M = mass that is producing the gravitational field (eg. a planet) (kg)
  - m = mass that is moving in the gravitational field (eg. a satellite) (kg)
  - r_1 = first distance of m from the centre of M (m)
  - r_2 = second distance of m from the centre of M (m)
- Work is done when an object in a planet's gravitational field moves **against** the gravitational field lines ie. away from the planet



Gravitational potential energy increases as a satellite leaves the surface of the Moon

Maths Tip

- Multiplying two negative numbers equals a positive number, for example:

$$-\left(-\frac{GM}{r}\right) = +\frac{GM}{r}$$

Worked Example

A spacecraft of mass 300 kg leaves the surface of Mars to an altitude of 700 km.Calculate the work done by the spacecraft.Radius of Mars = 3400 km

Mass of Mars = 6.40×10^{23} kg

Step 1: **Write down the work done (or change in G.P.E) equation**

$$\Delta G.P.E = GMm\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Step 2: **Determine values for r_1 and r_2**

r_1 is the radius of Mars = 3400 km = 3400×10^3 m

r_2 is the radius + altitude = 3400 + 700 = 4100 km = 4100×10^3 m

Step 3: **Substitute in values**

$$\Delta G.P.E = (6.67 \times 10^{-11}) \times (6.40 \times 10^{23}) \times 300 \times \left(\frac{1}{3400 \times 10^3} - \frac{1}{4100 \times 10^3}\right)$$

$$\Delta G.P.E = 643.076 \times 10^6 = \mathbf{640\ MJ\ (2\ s.f.)}$$

Exam Tip

Make sure to not confuse the ΔG.P.E equation with

$$\mathbf{\Delta G.P.E = mg\Delta h}$$

The above equation is only relevant for an object lifted in a uniform gravitational field (close to the Earth's surface). The new equation for G.P.E will not include *g*, because this varies for different planets and is no longer a constant (decreases by $1/r^2$) outside the surface of a planet.

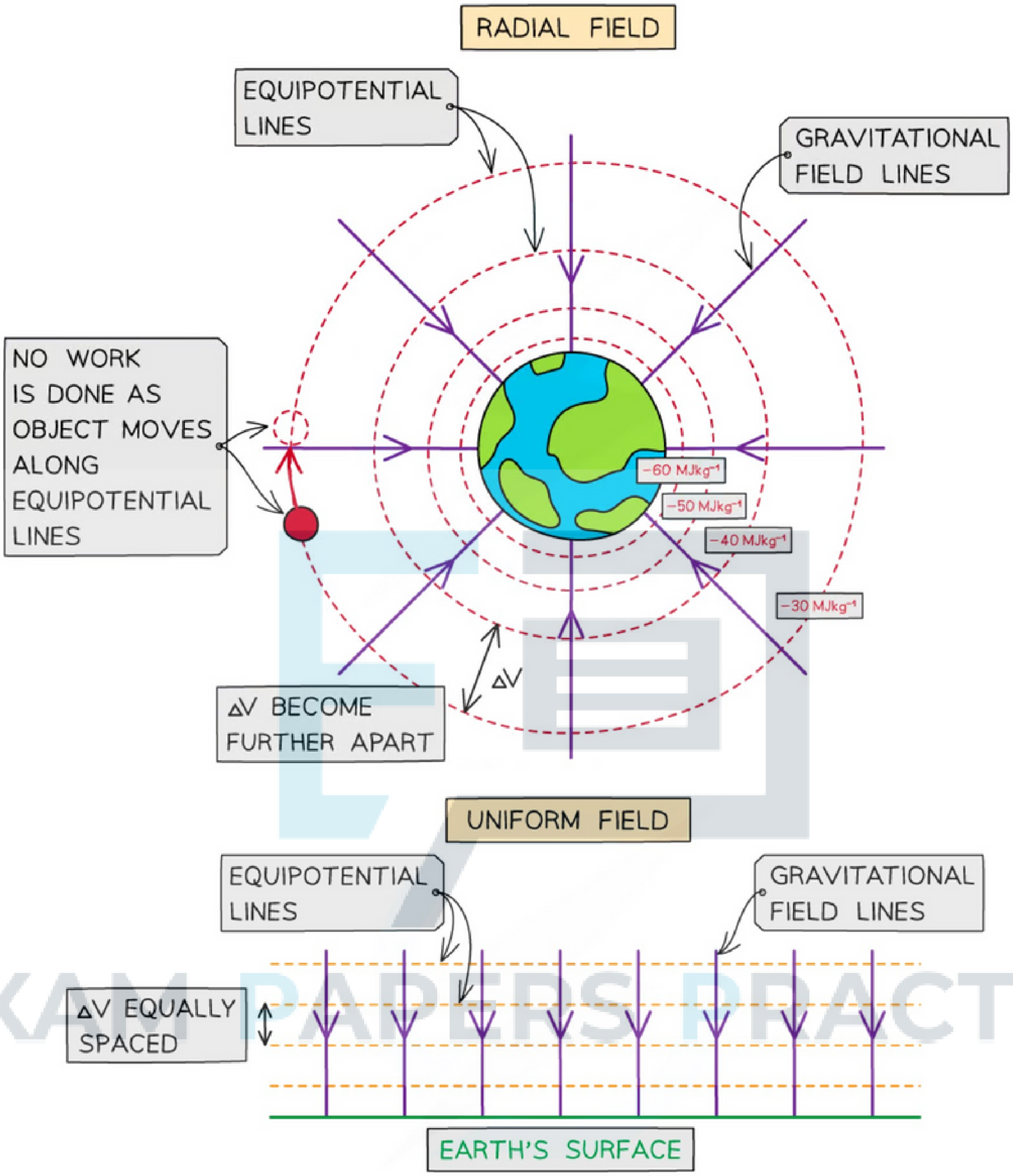
Gravitational Equipotential Surfaces

- Equipotential lines (2D) and surfaces (3D) join together points that have the **same gravitational potential**
- These are always:
  - **Perpendicular** to the gravitational field lines in both radial and uniform fields
  - Represented by **dotted** lines (unlike field lines, which are solid lines with arrows)

- In a radial field (eg. a planet), the equipotential lines:
  - Are concentric circles around the planet
  - Become further apart further away from the planet

- In a uniform field (eg. near the Earth's surface), the equipotential lines are:
  - Horizontal straight lines
  - Parallel
  - Equally spaced

- **No work is done** when moving along an equipotential line or surface, only **between** equipotential lines or surfaces
  - This means that an object travelling along an equipotential doesn't lose or gain energy and $\Delta V = 0$



Gravitational equipotential lines in a non-uniform and uniform gravitational field

Exam Tip

Remember equipotential lines **should not** have arrows on them like gravitational field lines do, since they have no particular direction and are not vectors. Make sure to draw any straight lines with a ruler or a straight edge.