| 6.5 Boolean algebra | Name: |  |
| :---: | :---: | :---: |
|  | Class: |  |
|  | Date: |  |

Time:
126 minutes

Marks:
121 marks

Comments:

## Q1.

The diagram shows a logic circuit.

(a) Complete the part of the truth table for the circuit in the diagram that is shown below.

| Inputs |  |  |  |  |  |  |  |  |  |  |  | Outputs |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| A | B | C | D | E | F | G | H |  |  |  |  |  |  |
| 0 | 0 | 0 |  |  |  |  |  |  |  |  |  |  |  |
| 0 | 0 | 1 |  |  |  |  |  |  |  |  |  |  |  |
| 0 | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  |
| 1 | 1 | 1 |  |  |  |  |  |  |  |  |  |  |  |

(b) Using the diagram above, write a Boolean expression to show how the output $\mathbf{G}$ is calculated from the inputs $\mathbf{A}, \mathbf{B}$ and $\mathbf{C}$.

$\qquad$
$G=$ $\qquad$
(c) Explain the purpose of the circuit.
$\qquad$
$\qquad$

Q2.
(a) Complete the table below and draw the symbol for an AND gate in the box.

Truth table for an AND gate

| Input A | Input B | Output |
| :--- | :--- | :--- |
|  |  |  |
|  |  |  |
|  |  |  |
|  |  |  |

(b) Using the laws of Boolean algebra, simplify the following Boolean expression.

$$
\text { A.B. }(A+B)
$$

$\qquad$
$\qquad$

(c) Using the laws of Boolean algebra, simplify the following Boolean expression.

$$
(X+Y) \cdot(X+\bar{Y})
$$

$\qquad$
$\qquad$
$\qquad$
$\qquad$
Answer $\qquad$

Q3.
A burglar alarm system is to be implemented that has the following sensors:

- a door sensor D that outputs TRUE when the door is open and FALSE when the door is shut
- a pressure mat sensor $\mathbf{M}$ that outputs TRUE while a weight is detected on it and FALSE when no weight is detected on it.

The alarm also has a key $\mathbf{K}$ that turns the alarm on and off. $\mathbf{K}$ outputs a TRUE signal when the alarm is switched on and FALSE when the alarm is off.

The alarm output A sounds a bell. It should be TRUE if:

- the alarm is on AND
- either of the sensors $\mathbf{D}$ or $\mathbf{M}$ are set to the value TRUE.
(a) Draw a logic circuit that will behave as described above for the inputs $\mathbf{D}, \mathbf{M}$ and $\mathbf{K}$ and the output $\mathbf{A}$.

(b) Write a Boolean expression to represent the logic of this alarm system. $A=$ $\qquad$
(c) In this alarm system, the alarm bell will sound only while the door is open or a
weight is placed on the pressure mat. If someone who has stepped on to the mat
moves off it, or an open door is closed, the alarm bell will stop ringing.
A D-type flip-flop could be incorporated into the logic circuit so that the alarm bell would continue to sound after a person closed the door or moved off the pressure mat.

Explain how this could be achieved. In your answer refer to:

- why a D-type flip-flop would be suitable for this task
- where the D-type flip-flop would need to be inserted into the circuit
- what additional input the D-type flip-flop would need.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Q4.
(a) Complete the truth table below for a NAND gate.

| NAND gate |  |  |
| :---: | :---: | :---: |
| Input A | Input B | Output |
| 0 | 0 |  |
| 0 | 1 |  |
| 1 | 0 |  |
| 1 | 1 |  |

(b) Multiplexors are used in electronic switching.

A 2-to-1 multiplexor has a Boolean equation where $A$ and $B$ are two inputs, $S$ is the selector input, and $Q$ is the output.

$$
Q=(A \cdot \bar{S})+(B . S)
$$

(i) Complete the truth table for the above Boolean equation.

(ii) Draw a circuit for the Boolean equation in the rectangle below.

(iii) By considering its inputs and outputs, describe what the 2-to-1 multiplexor circuit does.


## Q5.

(a) Complete the truth tables for the following logic gates.


| Input A | Input B | Output |
| :---: | :---: | :---: |
| 0 | 0 |  |
| 0 | 1 |  |
| 1 | 0 |  |
| 1 | 1 |  |

NAND gate

| Input A | Input B | Output |
| :---: | :---: | :---: |
| 0 | 0 |  |
| 0 | 1 |  |


| 1 | 0 |  |
| :--- | :--- | :--- |
| 1 | 1 |  |

(b) Represent the following Boolean equation as a logic circuit by completing the diagram below.

$$
\mathrm{Q}=\overline{\overline{\mathrm{A} \cdot \mathrm{~B}}+\mathrm{B} \cdot \mathrm{C}}
$$


(c) Simplify the following expression.
Show each stage of your working.

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Final answer $\qquad$
(Total 10 marks)

Q6.
(a) State the names of the logic gates represented by each of the three truth tables below.

| Input A | Input B | Output |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 0 | 1 | 0 |

Logic gate name $\qquad$

| 1 | 0 | 0 |
| :--- | :--- | :--- |
| 1 | 1 | 1 |


| Input A | Input B | Output |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 0 | 1 | 0 |
| 1 | 0 | 0 |
| 1 | 1 | 0 |

Logic gate name $\qquad$

| Input A | Input B | Output |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| 0 | 1 | 1 |
| 1 | 0 | 1 |
| 1 | 1 | 0 |

(b) Simplify the following Boolean expressions.
(i) $\mathrm{B} \cdot(\mathrm{A}+\overline{\mathrm{A}})$
$\qquad$

(ii) $\mathrm{A} \cdot \mathrm{B}+\mathrm{B}$
$\qquad$
$\qquad$
(iii) $\overline{\mathrm{B}} \cdot(\overline{\overline{\mathrm{A}}+\overline{\mathrm{B}}})$
$\qquad$
$\qquad$
(c) Draw a logic circuit for the following Boolean expression:

$$
Q=(A \oplus B) \cdot B
$$

You will need to make use of the symbols below when drawing your logic circuit.

(a) Represent the Boolean equation $Q=\bar{A} \cdot \bar{B}$ as a logic circuit by drawing a diagram using only the following symbols:
EXAM
AND

NOT

(2)

## Q7.


(b) Use the following truth tables to demonstrate that $A+B=\overline{\bar{A}} \cdot \bar{B}$

| $\mathbf{A}$ | $\mathbf{B}$ | $\mathbf{A}+\mathbf{B}$ |
| :--- | :--- | :--- |
| 0 | 0 |  |
| 0 | 1 |  |
| 1 | 0 |  |
| $\mathbf{A}$ | $\mathbf{B}$ |  |
| 0 |  |  |
| 0 | 1 |  |
| 0 | 1 |  |$|$| 1 | 0 |  |  |  |
| :--- | :--- | :--- | :--- | :--- |
| $\mathbf{A} \cdot \overline{\mathbf{B}}$ |  |  |  |  |
| 1 | 1 |  |  |  |

(c) What is the name commonly associated with the statement $A+B=\overline{\bar{A}} \cdot \overline{\bar{B}}$ ?

(d) Simplify the Boolean expression below.

$$
\text { A.B. } \bar{C}+A \cdot \bar{C}
$$

Show each stage of your workin.

Final answer $\qquad$

Q8.
(a) Complete the truth tables for the following logic gates.

| AND Gate |  |  |
| :---: | :---: | :---: |
| Input $\mathbf{X}$ | Input $\mathbf{Y}$ | Input $\mathbf{Q}$ |
| 0 | 0 |  |
| 0 | 1 |  |
| 1 | 0 |  |
| 1 | 1 |  |


| XOR Gate |  |  |
| :---: | :---: | :---: |
| Output $\mathbf{X}$ | Input $\mathbf{Y}$ | Output $\mathbf{Q}$ |
| 0 | 0 |  |
| 0 | 1 |  |
| 1 | 0 |  |
| 1 | 1 |  |

(b) A line-following robot has three sensors. It moves along a black line on a white background whilst the following conditions are met:

- the ultrasonic sensor $U$ does not detect any obstacle
- either, but not both, of the infrared sensors $L$ and $R$ are on the black line.

Sensor U returns 1 if it detects an obstacle and 0 if the path is clear.
Sensors $L$ and $R$ each return 1 if they detect black and 0 if they detect white.
A logic circuit will process the input from the sensors and produce an output $M$.
$M$ should be 1 if the robot is to move and 0 if the robot should stop.
(i) Represent the output $M$ as a Boolean expression.
$M=$ $\qquad$
$E M \frac{\Delta}{\text { (ii) }}$ The following symbols are used to represent logic gates:


NOT


XOR


OR


NOR


AND


NAND

Using a combination of any of the above logic gates draw a logic circuit for this system in the box below. You will not need to use all of the different types of logic gates.

(c) Apply De Morgan's Law(s) to the following expression and simplify the result.
$\mathrm{Q}=\overline{\overline{\mathrm{A}}+(\overline{\mathrm{B} \cdot \mathrm{A}})}$
Show the stages of your working.


Final answer

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Q9.
The diagram below shows a logic circuit.

(a) Write a Boolean expression for D.
$\qquad$
(b) Write a Boolean expression for $\mathbf{B}$.
$\qquad$
(c) The diagram below shows a different logic circuit.

(i) Complete the truth table below for the logic circuit in the diagram above.

| Inputs |  | Outputs |  |
| :---: | :---: | :---: | :---: |
| $\mathbf{X}$ | $\mathbf{Y}$ | $\mathbf{C}$ | $\mathbf{s}$ |
| 0 | 0 |  |  |
| 0 | 1 |  |  |
| 1 | 0 |  |  |
| 1 | 1 |  |  |

=2. (ii) What arithmetic function does the logic circuit in the diagram above perform?
$\qquad$
(d) Without using a truth table, simplify the Boolean expression below.
$(X+Y) \cdot(X+\bar{Y})$
Show the stages of your working.
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Final answer $\qquad$

Q10.
(a) Complete the truth tables for the following logic gates.

| NAND Gate |  |  |
| :---: | :---: | :---: |
| Input <br> $\mathbf{X}$ | Input <br> $\mathbf{Y}$ | Output <br> $\mathbf{Q}$ |
| 0 | 0 |  |
| 0 | 1 |  |
| 1 | 0 |  |
| 1 | 1 |  |


(b) Represent the Boolean equation $Z=\bar{A} \cdot \bar{B}+C$ in the form of a logic circuit by drawing a diagram using the following symbols.



(c) Simplify the Boolean expression below.


## 

Final answer $\qquad$

## Q11.

Write the following Boolean expressions in their simplest forms.
(a) $(\overline{\bar{A}} \cdot \overline{\bar{B}})$
$\qquad$
(b) $B+B \cdot \bar{C}$
$\qquad$
(c) $\mathrm{A} \cdot \mathrm{B}+\mathrm{A} \cdot \overline{\mathrm{B}}$
(d) $\mathrm{A} \cdot(\mathrm{B}+1)$
$\qquad$

Q12.
(a) Complete the truth tables for the following logic gates.

| OR Gate |  |  | XOR Gate |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Input A | Input B | Output Q | Input A | Input B | Output Q |
| 0 | 0 |  | 0 | 0 |  |
| 0 | 1 |  | 0 | 1 |  |
| 1 | 0 |  | 1 | 0 |  |
| 1 | 1 |  | 1 | 1 |  |

(b) Represent the Boolean equation $Q=A+B \cdot C$. as a logic circuit by drawing a diagram of it.

(c) Simplify the Boolean expression:

$$
B \cdot(A+\bar{B})
$$

Show your working.
$\qquad$
$\qquad$
$\qquad$
$\qquad$

## Q13.

Simplify the Boolean expression:

$$
\overline{\mathrm{A} \cdot \mathrm{~B}}+\mathrm{A}
$$

Show your working.
$\qquad$
$\qquad$
$\qquad$

(Total 3 marks)

## Q14.

(a) Look at the truth table below


What logic gate does the table represent?
$\qquad$
(b) An interior light in a two-door car is controlled by two switches that the driver can turn on or off and two sensors, one per door.

The switches are named $A$ and $B$.
The door sensors are named $C$ and $D$.
The interior light is named L .
If a door is open the output of its sensor is on.
If a door is closed the output of its sensor is off.

- If both switches $A$ and $B$ are off then the light $L$ is always off.
- If switch $A$ is on the light $L$ is always on.
- If switch $B$ is on and switch $A$ is off then:
- the light $L$ turns on if one or more of the car doors is opened
- the light L turns off if both of the doors are closed.

The following symbols are used to represent logic gates:

AND

OR

NOT
(i) Using only AND, OR and NOT gates draw a logic circuit for this system in the box below. You may not need to use all three types of gate.

(3)
(ii) Write a Boolean expression to represent the logic of the interior light system.
$\qquad$
(c) Simplify the Boolean expression below, showing your working.

$$
\overline{\bar{A}+\bar{B}}+B \cdot \bar{A}
$$

$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$
$\qquad$

Q15.
(a) Complete the truth tables for the following logic gates.

OR gate AND gate

| Input <br> $\mathbf{A}$ | Input <br> $\mathbf{B}$ | Output |  | Input <br> $\mathbf{A}$ | Input <br> $\mathbf{B}$ | Output |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 |  |  | 0 | 0 |  |
| 0 | 1 |  |  | 0 | 1 |  |
| 1 | 0 |  |  | 1 | 0 |  |
| 1 | 1 |  | 1 | 1 |  |  |

(b) (i) $A$ single output $Q$ is produced from three inputs $A, B$ and $C$. Output $Q$ is required to be 1 only if inputs $A$ and $B$ are 1 , or input $C$ is 1 and input $B$ is 0 .

EDAA Express this as a Boolean equation. P
(ii) Represent this Boolean equation diagrammatically by completing the logic gate diagram below.

(4)
(Total 8 marks)


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