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6.2 Simple Harmonic Motion

PHYSICS

AQA A Level Revision Notes

A Level Physics AQA

6.2 Simple Harmonic Motion

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6.2.1 Conditions for Simple Harmonic Motion

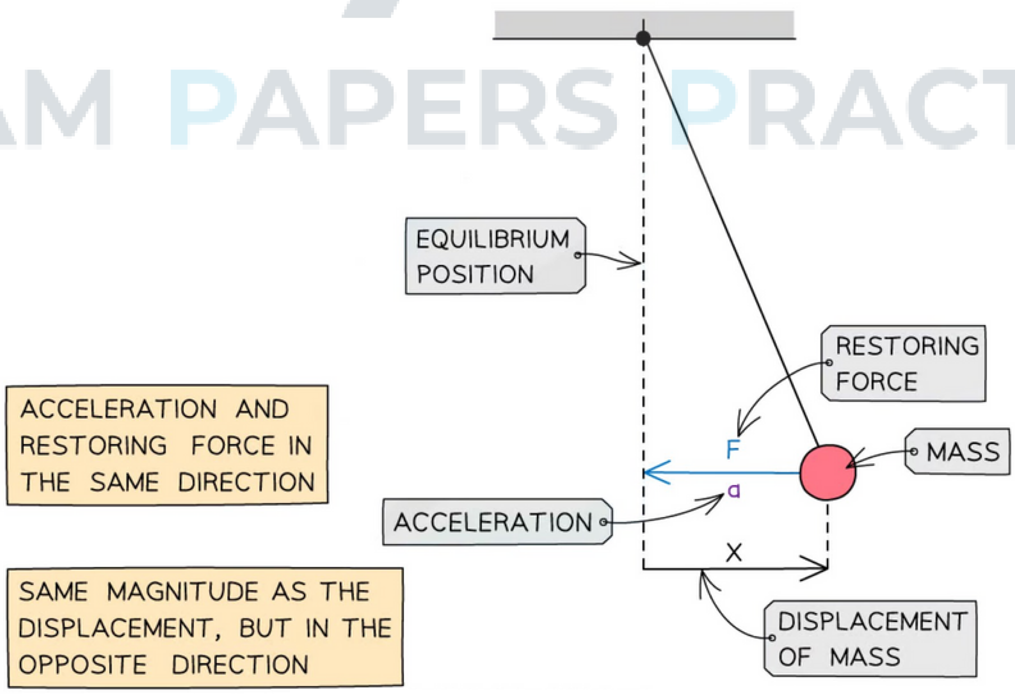
Conditions for Simple Harmonic Motion

- **Simple harmonic motion (SHM)** is a specific type of oscillation
- An oscillation is said to be SHM when:
  - **The acceleration is proportional to the displacement**
  - **The acceleration is in the opposite direction to the displacement**

- Examples of oscillators that undergo SHM are:
  - The pendulum of a clock
  - A mass on a spring
  - Guitar strings
  - The electrons in alternating current flowing through a wire
- These are always periodic, meaning they are repeated in regular intervals according to their frequency or time period
- Acceleration *a* and displacement *x* can be represented by the defining equation of SHM:

$$a \propto -x$$

- An object in SHM will also have a restoring force to return it to its equilibrium position
  - This restoring force will be directly proportional, but in the **opposite direction**, to the displacement of the object from the equilibrium position
- **Note:** the restoring force and acceleration act in the **same direction**



Force, acceleration and displacement of a pendulum in SHM

- This is why a person jumping on a trampoline is not an example of simple harmonic motion:
  - The restoring force on the person is **not** proportional to their distance from the equilibrium position
  - When the person is not in contact with the trampoline, the restoring force is equal to their weight, which is constant
  - This does not change, even if they jump higher

Acceleration

- The acceleration of an object oscillating in simple harmonic motion is:

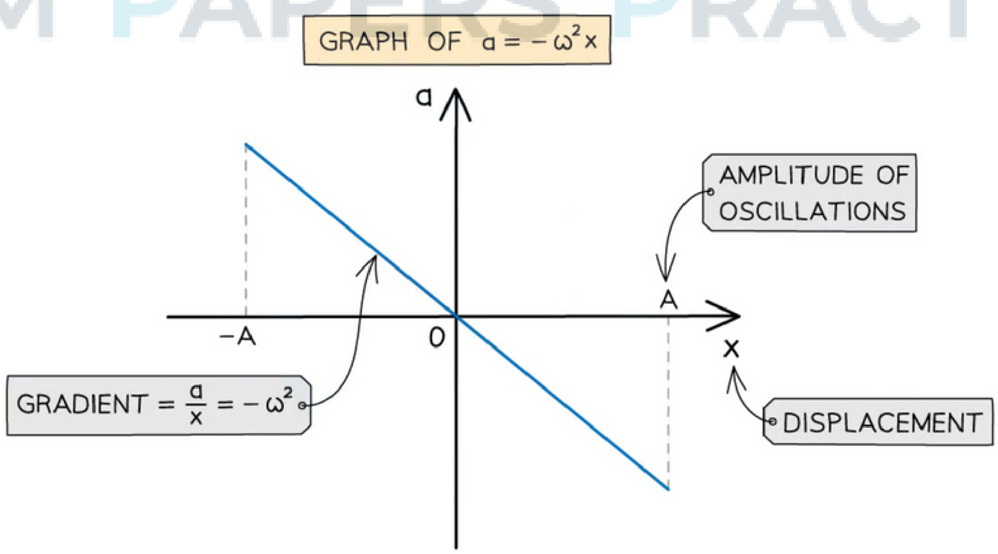
$$a = -\omega^2 x$$

- Where:
  - a = acceleration (m s^{-2})
  - ω = angular frequency (rad s^{-1})
  - x = displacement (m)

- This is used to find the acceleration of an object with a particular angular frequency ω at a specific displacement x
- The equation demonstrates:
  - The acceleration reaches its **maximum** value when the displacement is at a **maximum** ie. $x = A$ (amplitude)
  - The **minus** sign shows that when the object is displaced to the **right**, the direction of the acceleration is to the **left** and vice versa (a and x are always in opposite directions to each other)

Displacement

- The graph of acceleration against displacement is a straight line through the origin sloping downwards (similar to $y = -x$)



The acceleration of an object in SHM is directly proportional to the negative displacement

- The key features of the graph are:
  - The gradient is equal to $-\omega^2$
  - The maximum and minimum displacement x values are the amplitudes $-A$ and $+A$
- A solution to the SHM acceleration equation is the displacement equation:

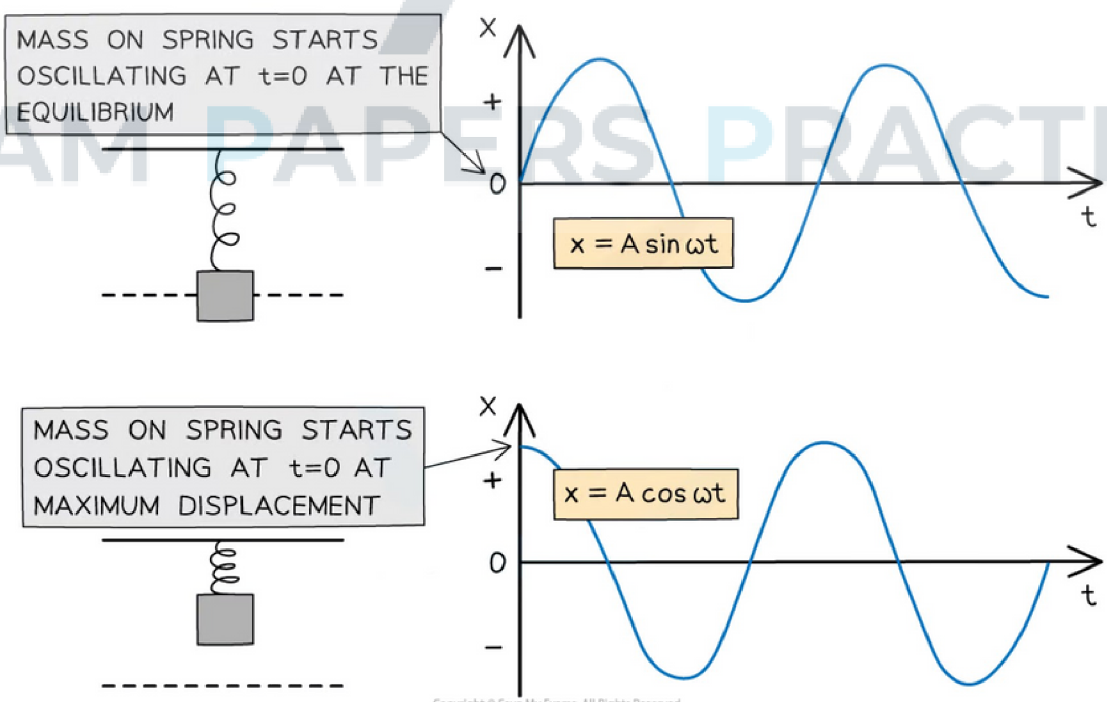
$$x = A\cos(\omega t)$$

- Where:
  - A = amplitude (m)
  - t = time (s)

- This occurs when:
  - An object is oscillating from its amplitude position ($x = A$ or $x = -A$ at $t = 0$)
  - The displacement will be at its maximum when $\cos(\omega t)$ equals 1 or −1, when $x = A$
- This equation can be used to find the position of an object in SHM with a particular angular frequency and amplitude at a moment in time
- If an object is oscillating from its equilibrium position ($x = 0$ at $t = 0$) then the displacement equation will be:

$$x = A\sin(\omega t)$$

- The displacement will be at its maximum when $\sin(\omega t)$ equals 1 or −1, when $x = A$
- This is because the sine graph starts at 0, whereas the cosine graph starts at a maximum



These two graphs represent the same SHM. The difference is the starting position

Speed

- The speed of an object in simple harmonic motion varies as it oscillates back and forth
  - Its speed is the magnitude of its velocity
- The greatest speed of an oscillator is at the equilibrium position ie. when its displacement $x = 0$
- How the speed v changes with the oscillator's displacement x in SHM is defined by:

$$v = \pm\omega\sqrt{(A^2 - x^2)}$$

- Where:
  - v = speed (m s^{-1})
  - A = amplitude (m)
  - ± = 'plus or minus'. The value can be negative or positive
  - ω = angular frequency (rad s^{-1})
  - x = displacement (m)
- This equation shows that when an oscillator has a greater amplitude A, it has to travel a greater distance in the same time and hence has greater speed v
- Although the symbol v is commonly used to represent velocity, not speed, exam questions focus more on the magnitude of the velocity than its direction in SHM

Worked Example

A mass of 55 g is suspended from a fixed point by means of a spring. The stationary mass is pulled vertically downwards through a distance of 4.3 cm and then released at t = 0. The mass is observed to perform simple harmonic motion with a period of 0.8 s. Calculate the displacement x, in cm, of the mass at time t = 0.3 s.

Step 1: Write down the SHM displacement equation

Since the mass is released at t = 0 at its maximum displacement, the displacement equation will be with the cosine function:

$$x = A\cos(\omega t)$$

Step 2: Calculate angular frequency

$$\omega = \frac{2\pi}{T} = \frac{2\pi}{0.8} = 7.85 \text{ rads}^{-1}$$

Remember to use the value of the time period given, not the time where you are calculating the displacement from

Step 3: Substitute values into the displacement equation

$$x = 4.3\cos(7.85 \times 0.3) = -3.0369... = -3.0 \text{ cm (2 s.f)}$$

Make sure the calculator is in **radians mode**

The negative value means the mass is 3.0 cm on the opposite side of the equilibrium position to where it started (3.0 cm above it)

Worked Example

A simple pendulum oscillates with simple harmonic motion with an amplitude of 15 cm. The frequency of the oscillations is 6.7 Hz. Calculate the speed of the pendulum at a position of 12 cm from the equilibrium position.

Step 1: Write out the known quantities

- Amplitude of oscillations, ***A* = 15 cm = 0.15 m**
- Displacement at which the speed is to be found, ***x* = 12 cm = 0.12 m**
- Frequency, **f = 6.7 Hz**

Step 2: Oscillator speed with displacement equation

$$v = \pm\omega\sqrt{(A^2 - x^2)}$$

- Since the speed is being calculated, the ± sign can be removed as direction does not matter in this case

Step 3: Write an expression for the angular frequency

- Equation relating angular frequency and normal frequency:

$$\omega = 2\pi f = 2\pi \times 6.7 = 42.097...$$

Step 4: Substitute in values and calculate

$$v = (2\pi \times 6.7) \times \sqrt{(0.15)^2 - (0.12)^2}$$

$$\boldsymbol{v = 3.789 = 3.8\ m\,s^{-1}\ (2\ s.f)}$$

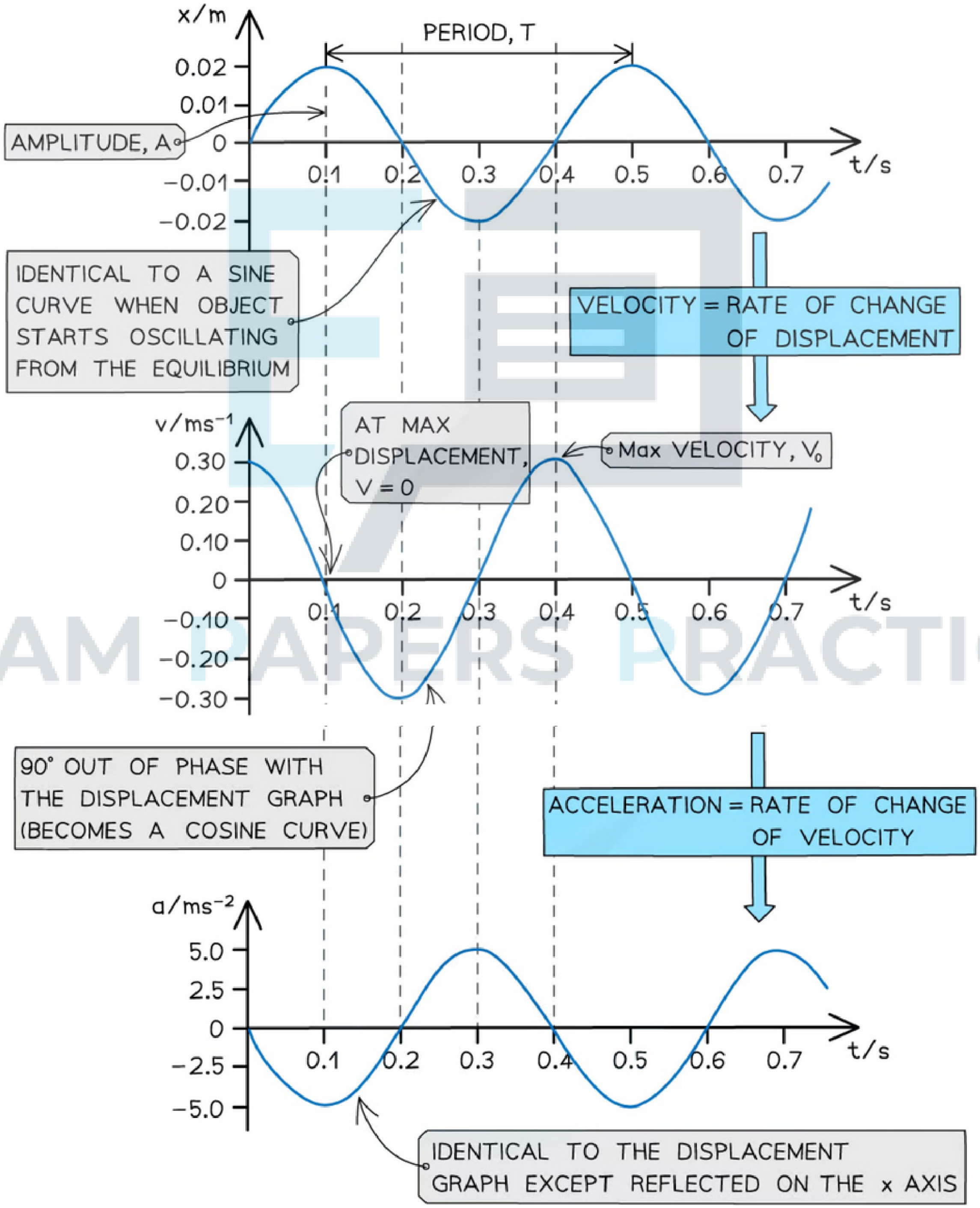
Exam Tip

Since displacement is a vector quantity, remember to keep the minus sign in your solutions if they are negative, you could lose a mark if not! Also, remember that your calculator must be in **radians** mode when using the cosine and sine functions. This is because the angular frequency ω is calculated in rad s^{-1}, **not** degrees. You often have to convert between time period *T*, frequency *f* and angular frequency ω for many exam questions – so make sure you revise the equations relating to these.

6.2.2 SHM Graphs

SHM Graphs

- The displacement, velocity and acceleration of an object in simple harmonic motion can be represented by graphs against time
- All undamped SHM graphs are represented by **periodic functions**
  - This means they can all be described by sine and cosine curves



The displacement, velocity and acceleration graphs in SHM are all 90° out of phase with each other

- **Key features of the displacement-time graph:**
  - The amplitude of oscillations *A* can be found from the maximum value of *x*
  - The time period of oscillations *T* can be found from reading the time taken for one full cycle
  - The graph might not always start at 0
  - If the oscillations starts at the positive or negative amplitude, the displacement will be at its maximum

- **Key features of the velocity-time graph:**
  - It is 90° out of phase with the displacement-time graph
  - Velocity is equal to the rate of change of displacement
  - So, the velocity of an oscillator at any time can be determined from the **gradient of the displacement-time graph:**

$$v = \frac{\Delta x}{\Delta t}$$

  - An oscillator moves the fastest at its equilibrium position
  - Therefore, the velocity is at its **maximum** when the **displacement is zero**

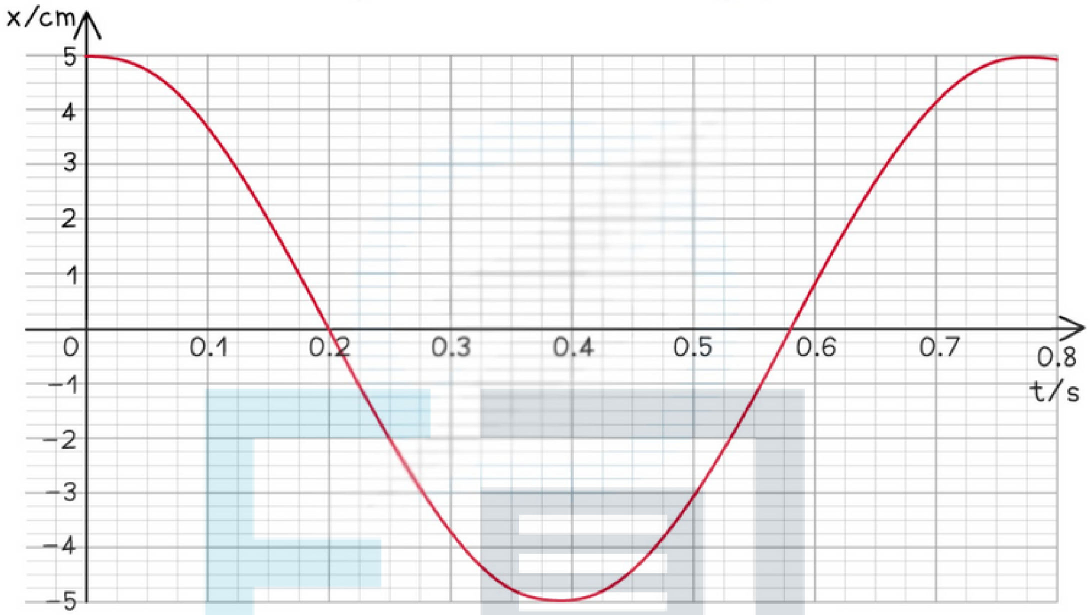
- **Key features of the acceleration-time graph:**
  - The acceleration graph is a reflection of the displacement graph on the x axis
  - This means when a mass has positive displacement (to the right) the acceleration is in the opposite direction (to the left) and vice versa
  - It is 90° out of phase with the velocity-time graph
  - Acceleration is equal to the rate of change of velocity
  - So, the acceleration of an oscillator at any time can be determined from the **gradient of the velocity-time graph:**

$$a = \frac{\Delta v}{\Delta t}$$

  - The maximum value of the acceleration is when the oscillator is at its **maximum displacement**

Worked Example

A swing is pulled 5 cm and then released. The variation of the horizontal displacement *x* of the swing with time t is shown on the graph below.



The swing exhibits simple harmonic motion. Use data from the graph to determine at what time the velocity of the swing is first at its maximum.

Step 1: The velocity is at its maximum when the displacement x = 0

Step 2: Reading value of time when x = 0

From the graph this is equal to 0.2 s

Exam Tip

These graphs might not look identical to what is in your textbook, depending on where the object starts oscillating from at t = 0 (on either side of the equilibrium, or at the equilibrium). However, if there is no damping, they will all always be a general sine or cosine curves.

6.2.3 Calculating Maximum Speed & Acceleration

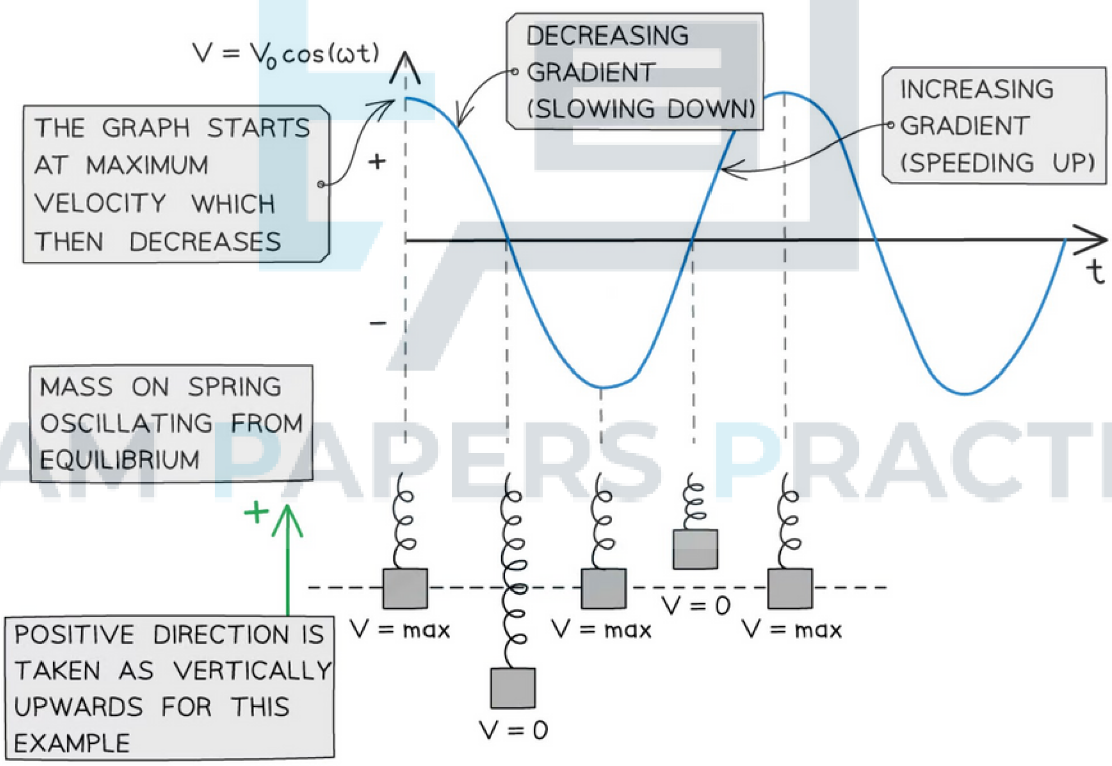
Maximum Speed

- The speed v of an oscillator will vary in SHM. It is:
  - **Maximum** at the **equilibrium** position ($x = 0$)
  - **Zero** at the **amplitude** ($x = A$)

- The maximum speed, v_{max}, is given by the equation:

$$v_{max} = \omega A$$

- Where:
  - v_{max} = maximum speed (m s^{-1})
  - ω = angular frequency (rad s^{-1})
  - A = amplitude (m)



The maximum speed of a mass on a spring is at the equilibrium position. Its speed is 0 at its positive and negative amplitude

> **?** **Worked Example**
>
> Calculate the frequency of an oscillator with a maximum speed of 12 m s^{-1} and amplitude of 1.4 m.

Step 1: State the known values

- Maximum speed, $v_{max} = 12 \text{ m s}^{-1}$
- Amplitude, $A = 1.4 \text{ m}$

Step 2: Write down the equation

$$v_{max} = \omega A$$

Step 3: Rewrite angular velocity in terms of frequency *f*

$$\omega = 2\pi f$$

$$v_{max} = 2\pi f A$$

Step 4: Rearrange for frequency, *f*

$$f = \frac{v_{max}}{2\pi A}$$

Step 5: Substitute in the values

$$f = \frac{12}{2\pi \times 1.4} = 1.364 = 1.4 \text{ Hz}$$

Maximum Acceleration

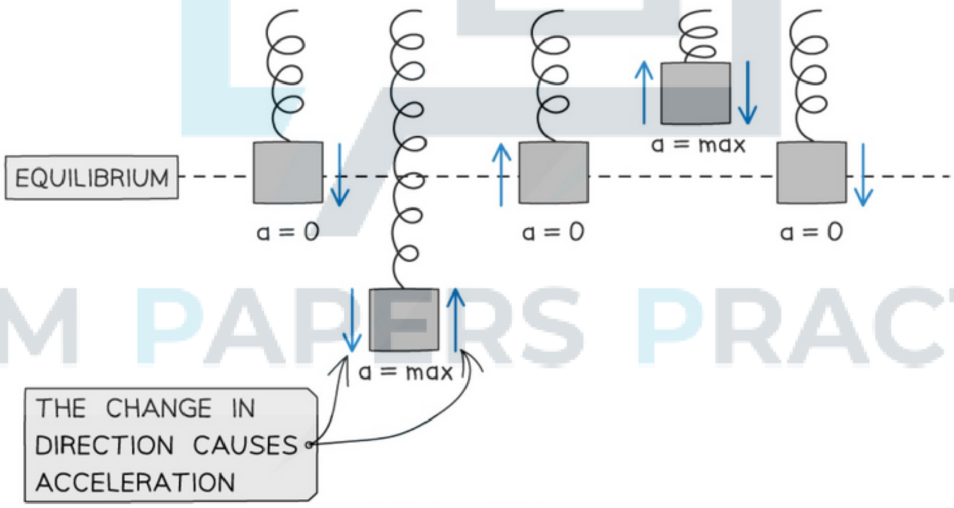
- The acceleration *a* of an oscillator will also vary in SHM. It is:
  - **Maximum** at the **amplitude** ($x = A$)
  - **Zero** at the **equilibrium** position ($x = 0$)

- This is because the acceleration is directly proportional to the displacement of an oscillator
- The maximum acceleration is given by the equation:

$$a_{max} = \omega^2 A$$

- Where:
  - a_{max} = maximum acceleration ($m\ s^2$)
  - ω = angular frequency ($rad\ s^{-1}$)
  - A = amplitude (m)

- Although at the amplitude, the speed is zero, the oscillator has **changed direction**
- This means that it has a non-zero velocity, and since acceleration is the rate of change of velocity, the oscillator has an acceleration at the amplitude too



The maximum acceleration of a mass on a spring is at its positive and negative amplitude. Its acceleration is 0 at the equilibrium position

> **?**
> ### Worked Example
> Calculate the maximum acceleration of an oscillator with a time period of 0.4 s and amplitude of 2.8 m.

Step 1: State the known values

- Time period, $T = 0.4$ s

- Amplitude, $A = 2.8$ m

Step 2: Write down the equation

$$a_{max} = \omega^2 A$$

Step 3: Rewrite maximum acceleration with time period T

$$a_{max} = \left(\frac{2\pi}{T}\right)^2 A$$

Step 4: Substitute in the values

$$a_{max} = \left(\frac{2\pi}{0.4}\right)^2 \times 2.8 = 690.97 = \mathbf{690\ m\ s^{-2}\ (2\ s.f)}$$

Exam Tip

Make sure not to get mixed up with lower case *a* (acceleration) and upper case *A* (amplitude)

6.2.4 Period of Mass-Spring System

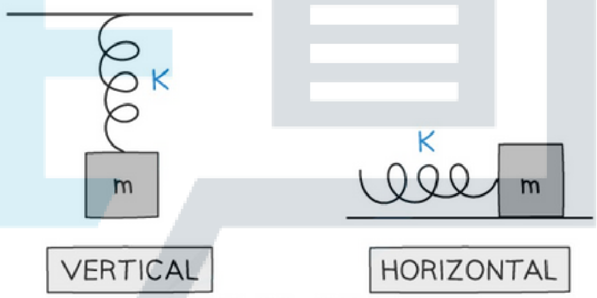
Period of Mass-Spring System

- A mass and spring system is a type of simple harmonic oscillator
- The time period of a mass-spring system is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- Where:
  - T = time period (s)
  - m = mass on the end of the spring (kg)
  - k = spring constant (N m^{-1})

- This equation applies for both a horizontal or vertical mass-spring system



A mass-spring system can be either vertical or horizontal. The time period equation applies to both

- The equation shows that the time period and frequency, of a mass-spring system, does **not** depend on the force of gravity
  - Therefore, the oscillations would have the same time period on Earth and the Moon

- The higher the spring constant k, the stiffer the spring and the shorter the time period

Worked Example

Calculate the frequency of a mass of 2.0 kg attached to a spring of spring constant 0.9 N m^{-1} oscillating in simple harmonic motion.

Step 1: Write down the known quantities

- Mass, $m = 2.0$ kg
- Spring constant, $k = 0.9$ N m^{-1}

Step 2: Calculate the time period

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$T = 2\pi\sqrt{\frac{2.0}{0.9}} = 9.3664$$

Step 3: Calculate the frequency

$$\text{Frequency, } f = \frac{1}{T}$$

$$f = \frac{1}{9.3664} = \mathbf{0.11\ Hz}$$

Exam Tip

Another area of physics where you may have seen the spring constant *k* is from Hooke's Law. Exam questions commonly merge these two topics together, so make sure you're familiar with the Hooke's Law equation too.

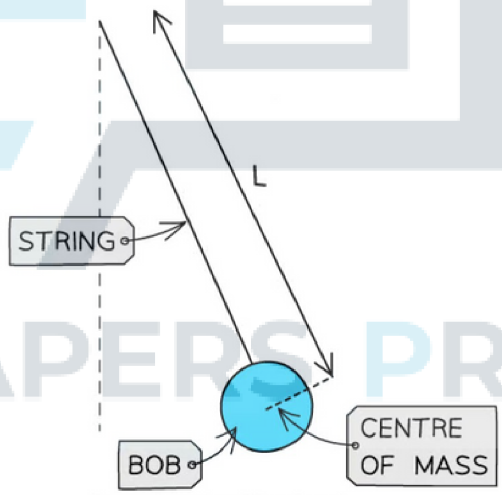
6.2.5 Period of Simple Pendulum

Period of Simple Pendulum

- A simple pendulum is another type of simple harmonic oscillator
  - The pendulum consists of a string and a bob (a weight, generally spherical) at the end
- The **time period** of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- Where:
  - T = time period (s)
  - L = length of string (m)
  - g = gravitational field strength (N kg^{-1})
- Length L is taken from the pivot to the centre of mass of the bob
- Normally, the bob is taken as a point mass, so l is just the length of the string



A simple pendulum

- The time period of a pendulum does depend on the gravitational field strength, meaning its period would be different on the Earth and the Moon
- The limitation with this formula, and the conditions for which it applies one for **small angles** from the equilibrium point (~10°) (and therefore small **amplitudes**)

Worked Example

Calculate the time period of a simple pendulum on the Moon, if on Earth it has a time period of 7 s. g on the moon is 1/6 of that on Earth.

Step 1: Write down the known quantities

$g_E = 9.81 \text{ N kg}^{-1}$

$g_M = \dfrac{9.81}{6} \text{ N kg}^{-1}$

$T_E = 7 \text{ s}$

L will be the same for both since it is the same pendulum

Step 2: Write down the time period equation for Earth & Moon

$$T_E = 2\pi\sqrt{\frac{L}{g_E}} = 7 \text{ s}$$

$$T_M = 2\pi\sqrt{\frac{L}{g_m}} = 2\pi\sqrt{\frac{L}{\frac{g_E}{6}}}$$

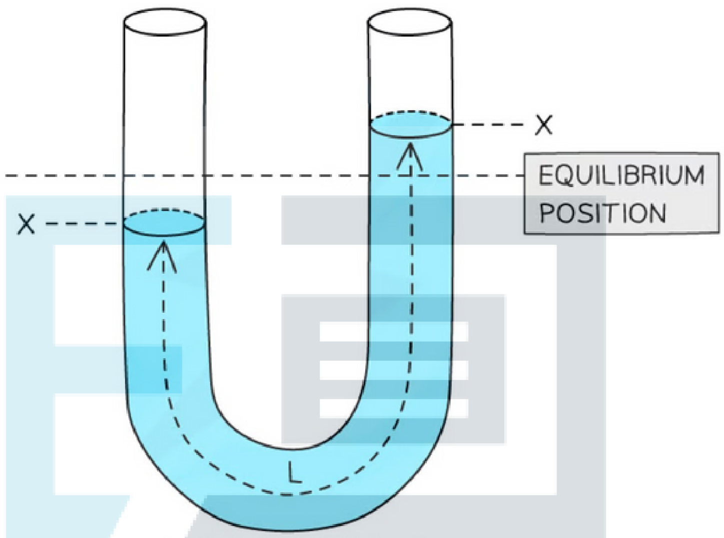
Step 3: Calculate the time period

$$T_M = T_E \times \sqrt{\frac{1}{\frac{1}{6}}} = 7 \times \sqrt{\frac{1}{\frac{1}{6}}} = 17 \text{ s}$$

6.2.6 Harmonic Oscillators in Context

Harmonic Oscillators in Context

- Any oscillations about an equilibrium where its restoring force is proportional to its displacement will be simple harmonic
- This doesn't apply just to pendulums and mass on springs
- An example of another harmonic oscillator is the oscillation of a liquid in a U-tube



A U-tube is a type of simple harmonic oscillator with a displacement x from the equilibrium position and length of liquid column L

- The time period for these oscillations are found from the same equation as the time period of a simple pendulum, where *l* is known the length of the liquid column
- An exam question will likely state whether a particular oscillation should be treated as simple harmonic. Some other examples include:
  - A bungee jumper
  - An acrobat on a trapeze
  - A swing
  - A ball in a concave dish
  - Oscillating platforms

6.2.7 Energy in SHM

Energy in SHM

- During **simple harmonic motion**, energy is constantly exchanged between two forms: kinetic and potential
- The potential energy could be in the form of:
  - Gravitational potential energy (for a pendulum)
  - Elastic potential energy (for a horizontal mass on a spring)
  - Or **both** (for a vertical mass on a spring)
- Speed *v* is at a maximum when displacement $x = 0$, so:

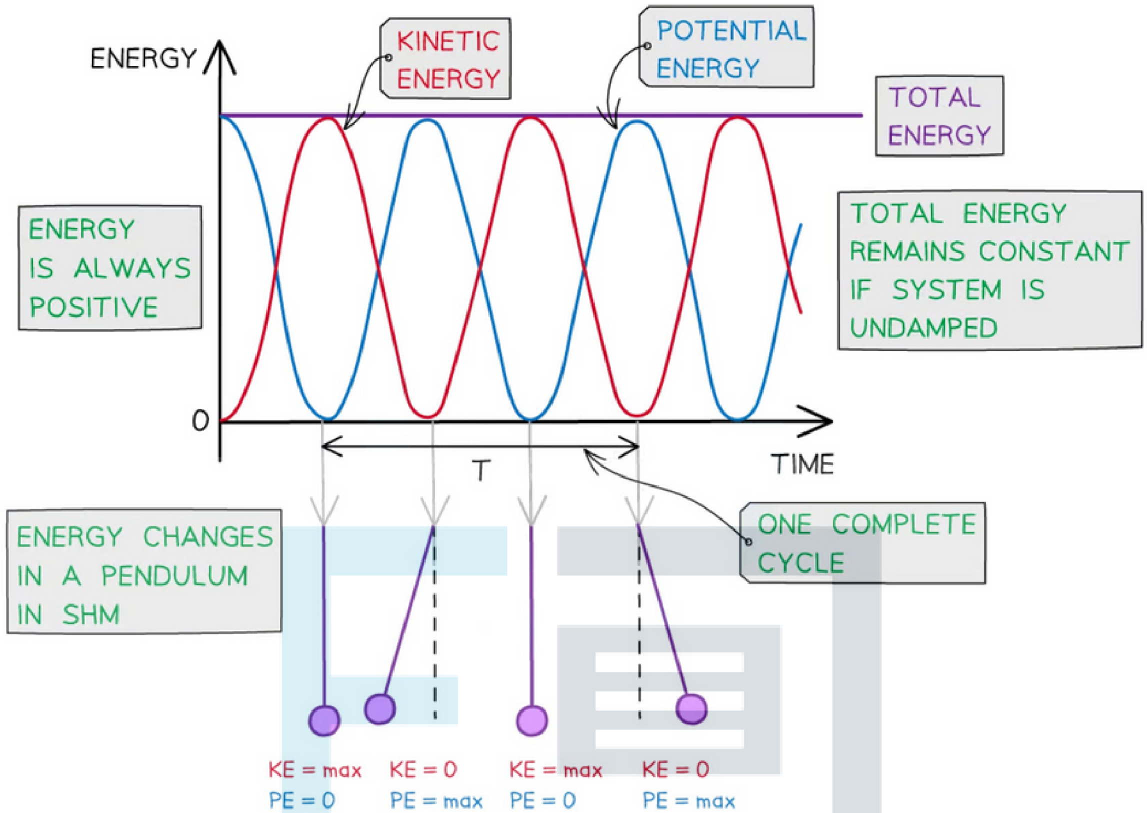
  The kinetic energy is at a maximum when the displacement x = 0 (equilibrium position)

- Speed *v* is 0 (and kinetic energy is 0) at maximum displacement $x = A$, so:

  The potential energy is at a maximum when the displacement (both positive and negative) is at a maximum x = A (amplitude)

- A simple harmonic system is therefore constantly converting between kinetic and potential energy
- When one increases, the other decreases and vice versa, therefore:

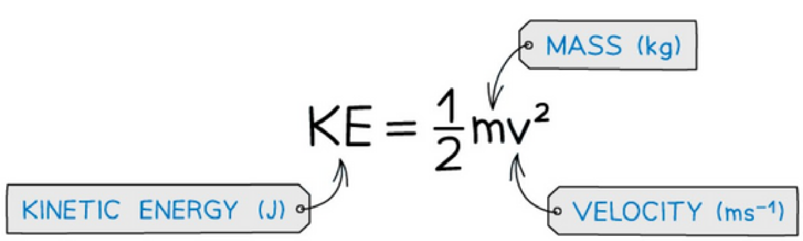
  The total energy of a simple harmonic system always remains constant and is equal to the sum of the kinetic and potential energies



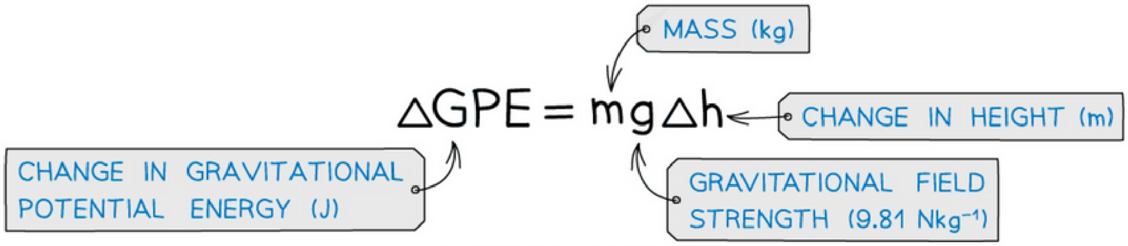
The kinetic and potential energy of an oscillator in SHM vary periodically

- **The key features of the energy-time graph are**:
  - Both the kinetic and potential energies are represented by periodic functions (sine or cosine) which are varying in opposite directions to one another
  - When the potential energy is 0, the kinetic energy is at its maximum point and vice versa
  - The **total energy** is represented by a **horizontal straight line** directly above the curves at the maximum value of both the kinetic or potential energy
  - Energy is **always positive** so there are no negative values on the y axis

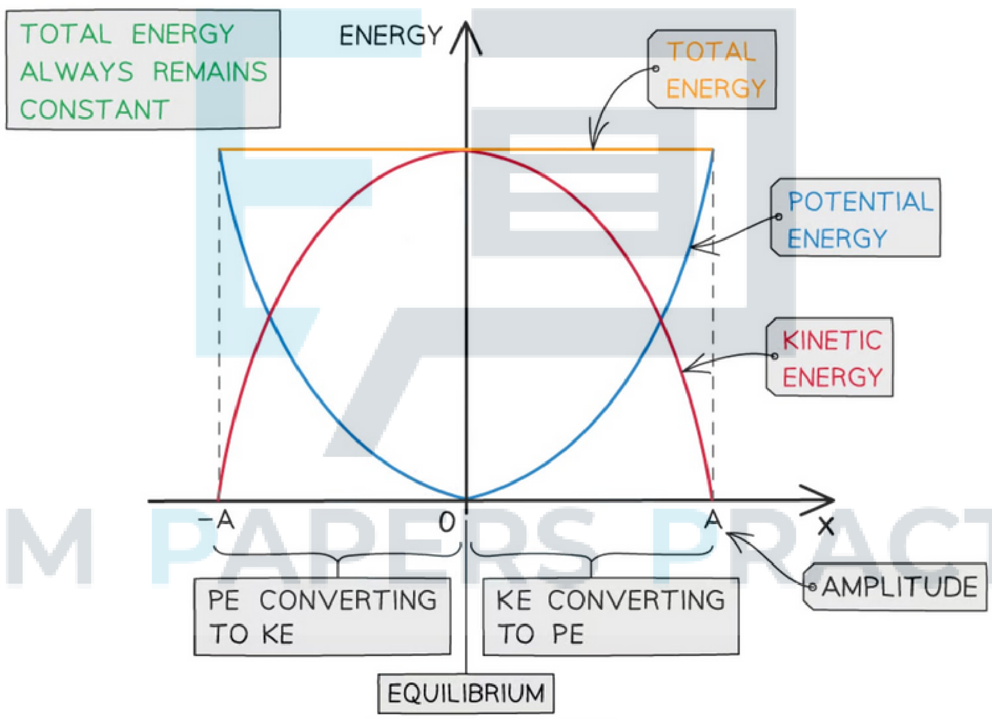
- Recall that the kinetic energy is defined by the equation



- Gravitational potential energy is defined by the equation



- **Note:** kinetic and potential energy go through **two** complete cycles during one **period** of oscillation
  - This is because one complete oscillation reaches the maximum displacement **twice** (positive and negative)
- The energy-displacement graph for **half** a cycle looks like:



Graph showing the potential and kinetic energy against displacement in half a period of an SHM oscillation

- **The key features of the energy-displacement graph are:**
  - Displacement is a vector, so, the graph has both **positive** and **negative** *x* values
  - The potential energy is always at a maximum at the amplitude positions $x = A$, and 0 at the equilibrium position $x = 0$
    - This is represented by a **'U' shaped curve**
  - The kinetic energy is the opposite: it is 0 at the amplitude positions $x = A$, and maximum at the equilibrium position $x = 0$
    - This is represented by an **'n' shaped curve**

- The total energy is represented by a **horizontal straight line** above the curves

Exam Tip

You may be expected to draw as well as interpret energy graphs against time or displacement in exam questions. Make sure the sketches of the curves are as even as possible and **use a ruler** to draw straight lines, for example, to represent the total energy.

6.2.8 Required Practical: Investigating SHM

Required Practical: Investigating SHM

Equipment List

Apparatus	Purpose
Clamp stand	Used to hold the spring, masses, and pendulum string vertically
Spring	Used to calculate the spring constant and provide the oscillations
Fiducial marker (e.g. Needle)	To mark the equilibrium position
Mass hanger + 50 g masses	To hang from the spring and vary the mass
Stopwatch	To measure the time for a certain number of oscillations
Metre ruler	To check all oscillations have the same amplitude (mass on spring) To measure the length of the pendulum (simple pendulum)
Pendulum bob on 2 m long string	To provide the oscillations for simple pendulum
Wooden Block	To clamp the string in place to create the pivot for the pendulum

- Resolution of measuring equipment:
  - Stopwatch = 0.01 s
  - Metre Ruler = 1 mm

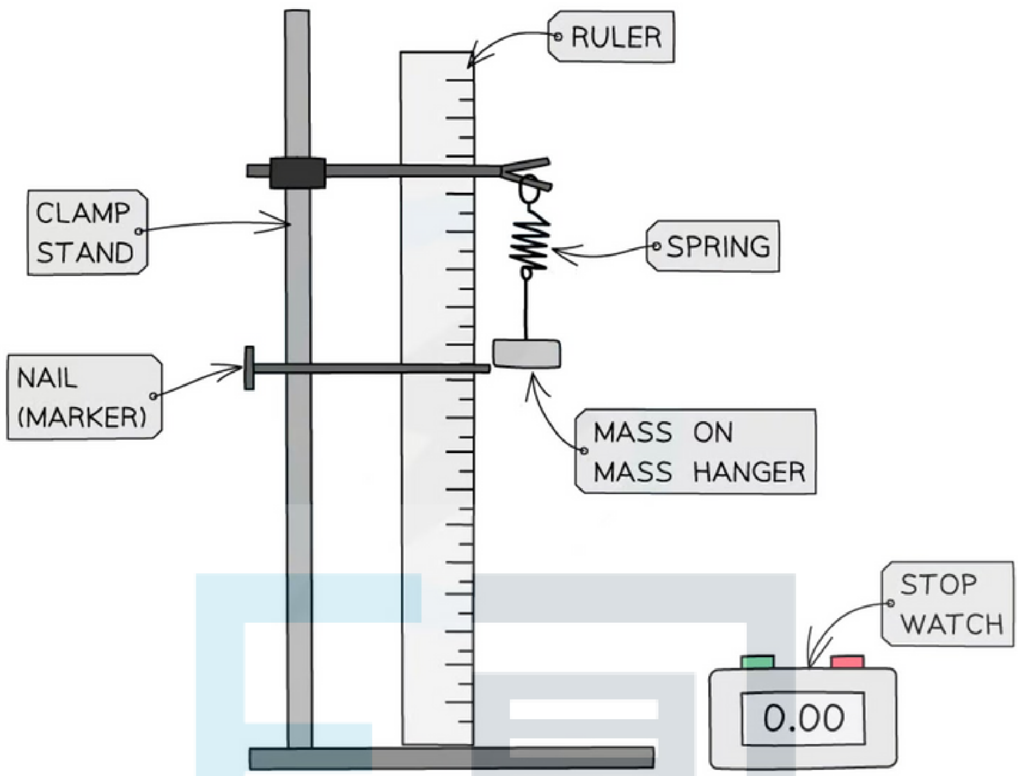
SHM in a Mass-Spring System

- The overall aim of this experiment is to calculate the spring constant of a mass-spring system
- This is done by investigating how the time period of the oscillations varies with the mass
  - This is just one example of how this required practical might be carried out

Variables

- Independent variable = mass, *m*
- Dependent variable = time period, *T*
- Control variables:
  - Spring constant, *k*
  - Number of oscillations

Method



The setup of apparatus to detect oscillations of a mass-spring system

1. Set up the apparatus, with no masses hanging on the holder to begin with (just the 100 g mass attached to it)
2. Pull the mass hanger vertically downwards between 2–5 cm as measured from the ruler and let go. The mass hanger will begin to oscillate
3. Start the stopwatch when it passes the nail marker
4. Stop the stopwatch after 10 complete oscillations and record this time. Divide the time by 10 for the time period (which is the mean)
5. Add a 50 g mass to the holder and repeat the above between 8–10 readings. Make sure the mass is pulled down by the same length before letting go

- An example table might look like this:

TIME TAKEN FOR 10 OSCILLATIONS → T_{10}/s

MEAN TIME $\frac{T_{10}}{10}$ → T/s

Mass/kg	T_{10}/s	T/s	T^2/s^2
0.05			
0.10			
0.15			
0.20			
0.25			
0.30			
0.35			
0.40			

Analysing the Results

- The time period of a mass-spring system is given by:

$$T = 2\pi\sqrt{\frac{m}{k}}$$

- Where:
  - T = time period (s)
  - m = mass (kg)
  - k = spring constant (N m^{-1})

- Squaring both sides of the equation gives:

$$T^2 = 4\pi^2\frac{m}{k}$$

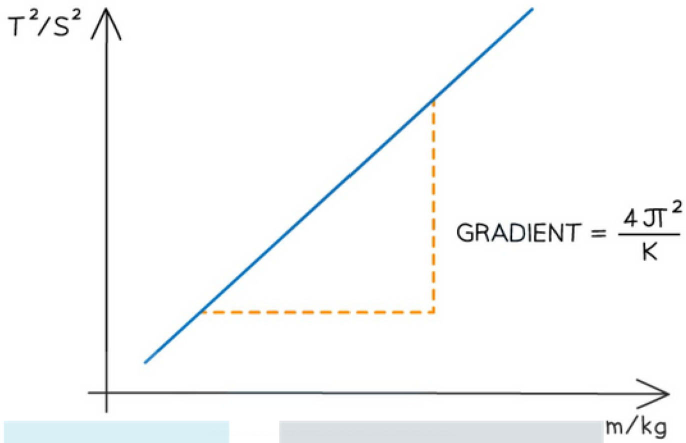
- Comparing this to the equation of a straight line: y = mx
  - $y = T^2$
  - $x = m$
  - Gradient = $4\pi^2 / k$

1. Plot a graph of T^2 against m and draw a line of best fit
2. Calculate the gradient
3. The spring constant, k, is therefore equal to:

$$k = \frac{4\pi^2}{\text{gradient}}$$

- The spring constant can also be found using the Hooke's Law equation ($F = -kx$)

- An experiment can be carried out and *k* found from the gradient of a plot of the force *F* against the extension *x*
  - The two values could then be compared



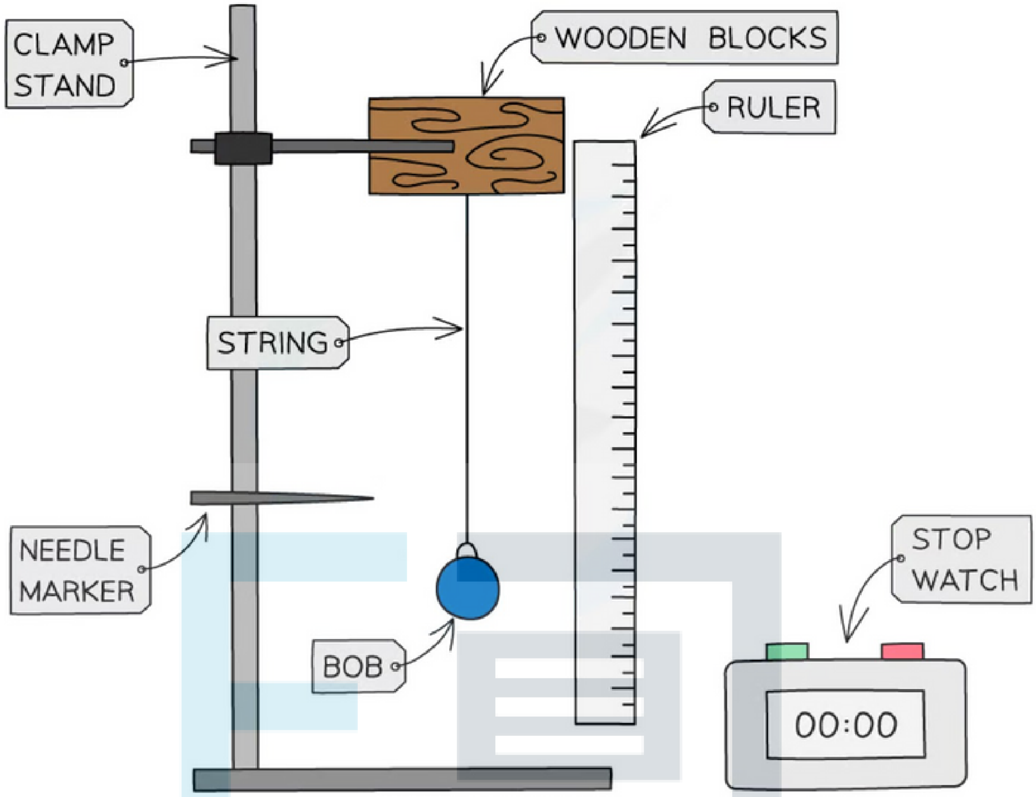
SHM in a Simple Pendulum

- The overall aim of this experiment is to calculate the acceleration due to gravity from a simple pendulum
- This is done by investigating how the time period of its oscillations varies with its length
  - This is just one example of how this required practical might be carried out

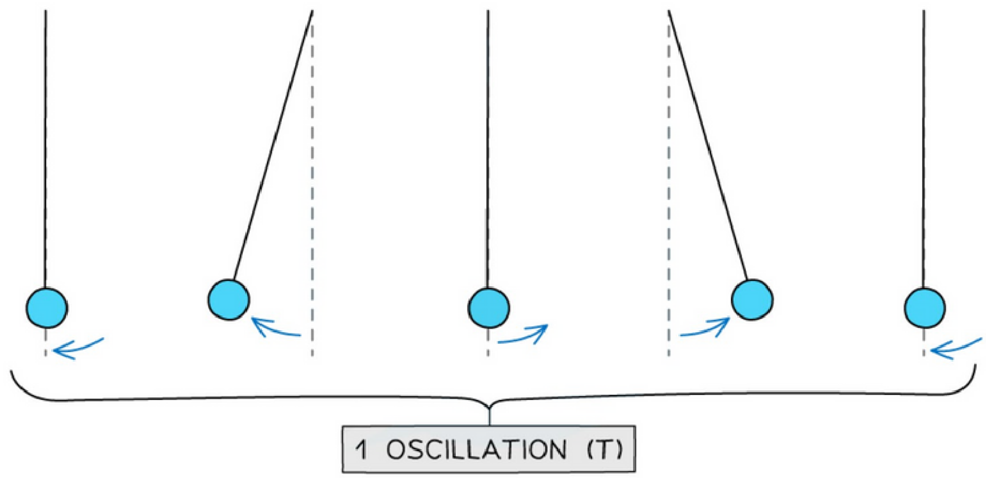
Variables

- Independent variable = length, *L*
- Dependent variable = time period, *T*
- Control variables:
  - Mass of pendulum bob, *m*
  - Number of oscillations

Method



1. Set up the apparatus, with the length of the pendulum at 0.2 m
2. Make sure the pendulum hangs vertically downwards at equilibrium and inline directly in front of the needle marker
3. Pull the pendulum to the side at a very small angle then let go. The pendulum will begin to oscillate
4. Start the stopwatch when the pendulum passes the needle marker. One complete oscillation is when the pendulum passes through the equilibrium, then to one amplitude and the other and then back to the equilibrium again (not just from side to side)
5. Stop the stopwatch after 10 complete oscillations and record this time. Divide the time by 10 for the time period (which is the mean)
6. Increase the length of the pendulum by adjusting the string and the wooden block and repeat the above for 8–10 readings. The ruler is used to measure the string and ensure it is measured from the wooden blocks to the centre of mass of the bob. Also, make sure the mass is pulled to the side by the same angle before letting go for the oscillations



- An example table might look like this:

TIME TAKEN FOR 10 OSCILLATIONS → T_{10}

MEAN TIME $\frac{T_{10}}{10}$ → T

LENGTH / m	T_{10} / s	T / s	T^2 / s^2
0.2			
0.4			
0.6			
0.8			
1.0			
1.2			
1.4			
1.6			
1.8			
2.0			

Analysing the Results

- The time period of a simple pendulum is given by:

$$T = 2\pi\sqrt{\frac{L}{g}}$$

- Where:
  - T = time period (s)
  - L = length of the pendulum (m)
  - g = acceleration due to gravity ($m\ s^{-2}$)

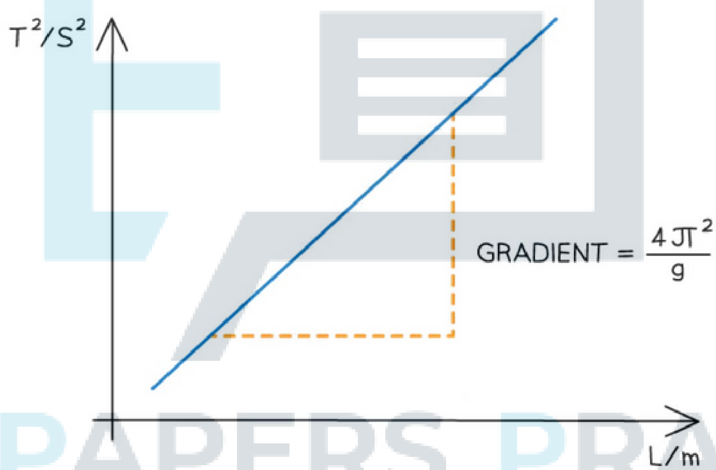
- Squaring both sides of the equation gives

$$T^2 = 4\pi^2 \frac{L}{g}$$

- Comparing this to the equation of a straight line: y = mx + c
  - $y = T^2$
  - x = L
  - gradient $m = 4\pi^2 / g$
  - c = 0

1. Plot a graph of T^2 against L and draw a line of best fit
2. Calculate the gradient
3. The acceleration due to gravity is equal to:

$$g = \frac{4\pi^2}{\text{gradient}}$$



Evaluating the Experiments

Systematic Errors:

- Reduce parallax error by being at eye level with the marker

Random Errors:

- Record the time taken for 10 full oscillations, then divide by 10 for one period, to reduce random errors
- The oscillations may not completely go from side to side, and end up in a circle. Therefore, keep the amplitudes relatively small (only a few cm) and repeat the readings if they do take a different trajectory
- A motion tracker and data logger could provide a more accurate value for the time period and reduce the random errors in starting and stopping the stopwatch (due to reflex times)
- The equation for the time period of a pendulum bob only works for small angles, so make sure the pendulum is not pulled too far out to the side for the oscillations

- For the mass-spring system, the oscillations may not stay completely vertical. Therefore, keep the amplitudes relatively small (only a few cm) and repeat the reading making sure they are vertical

Safety Considerations

- The suspended masses or pendulum bob could damage the surface if they were to fall. Make sure to keep a soft surface directly below the equipment
- Only pull down the mass and spring system a few centimetres for the oscillations, as larger oscillations could cause the masses to fall off and damage the equipment
- The wooden blocks must be tightly clamped together to hold the string for the pendulum in place, otherwise the pendulum may dislodge during oscillations and fall off

Worked Example

A student investigates the relationship between the time period and the mass on a mass-spring system that oscillates with simple harmonic motion. They obtain the following results

Mass/kg	T_{10} /s	T /s
0.05	6.3	0.63
0.10	6.9	0.69
0.15	7.4	0.74
0.20	7.9	0.79
0.25	8.2	0.82
0.30	8.5	0.85
0.35	8.8	0.88
0.40	9.1	0.91

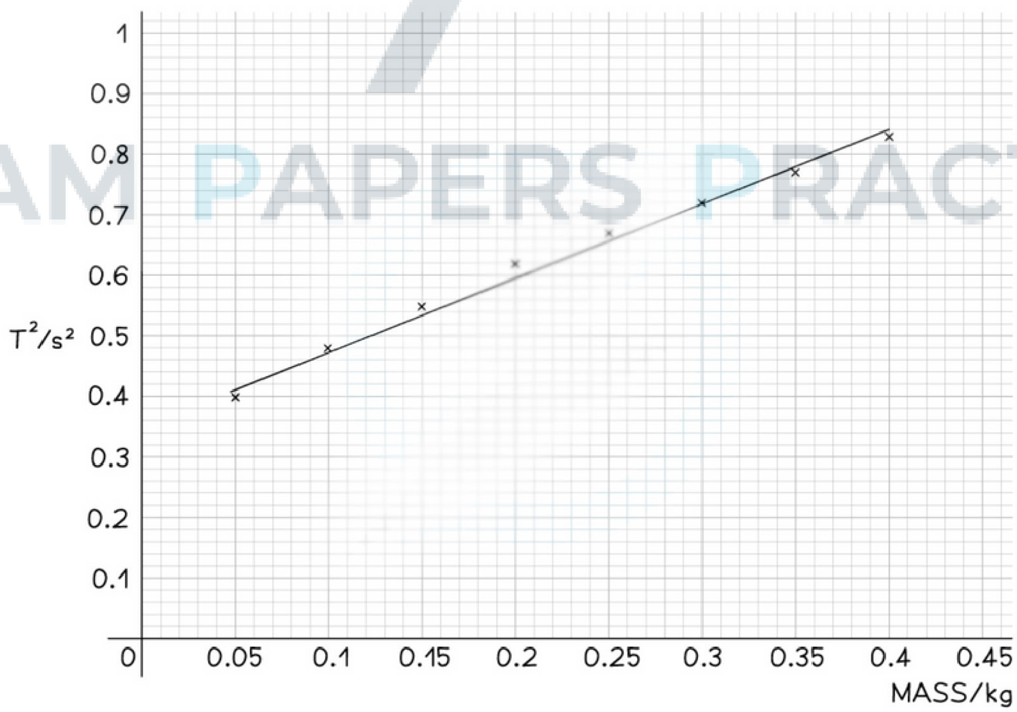
Calculate the value of the spring constant of the spring used in this experiment

Step 1: Complete the table

Add the extra column T^2 and calculate the values

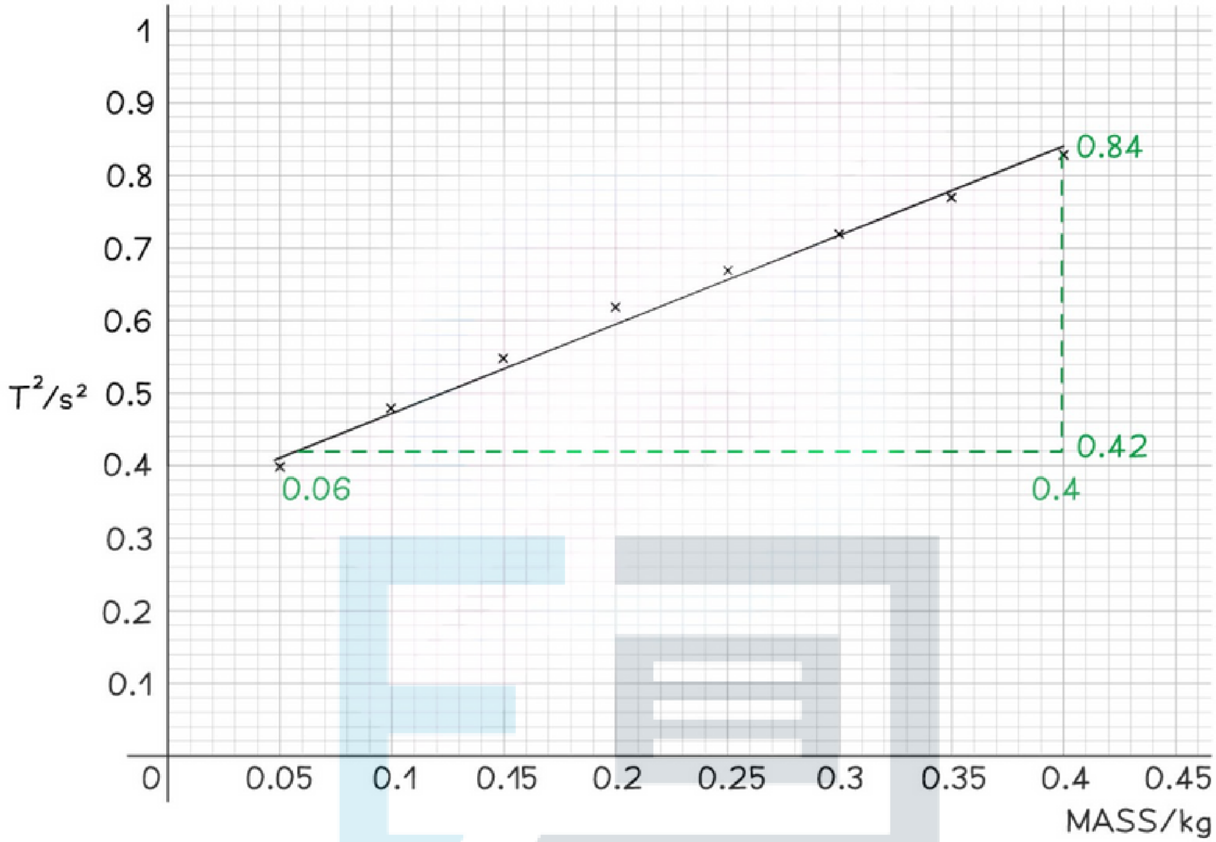
Mass/kg	T_{10} /s	T /s	T^2/s^2
0.05	6.3	0.63	0.40
0.10	6.9	0.69	0.48
0.15	7.4	0.74	0.55
0.20	7.9	0.79	0.62
0.25	8.2	0.82	0.67
0.30	8.5	0.85	0.72
0.35	8.8	0.88	0.77
0.40	9.1	0.91	0.83

Step 2: Plot the graph of T^2 against the mass *m*



Make sure the axes are properly labelled and the line of best fit is drawn with a ruler

Step 3: Calculate the gradient of the graph



The gradient is calculated by:

$$\text{gradient} = \frac{0.84-0.42}{0.4-0.06} = 1.23529$$

Step 4: Calculate the spring constant, *k*

$$k = \frac{4\pi^2}{\text{gradient}} = \frac{4\pi^2}{1.23529} = \mathbf{32.0\ N\ m^{-1}}$$