Exam Papers Practice

6.2 Newton's Law of Gravitation

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Course	DP IB Physics
Section	6. Circular Motion & Gravitation
Topic	6.2 Newton's Law of Gravitation
Difficulty	Medium

To be used by all students preparing for DP IB Physics HL
Students of other boards may also find this useful

1

The correct answer is **D** because:

- The question is asking about the weight of the module on Earth and **not** on Jupiter
- Weight = mass × gravitational field strength on Earth
- Weight = 4000 × 10 = 40 000 N

A is incorrect as	this is the mass of the module and **not** the weight of the module on Earth
B is incorrect as	The question is asking about the weight of the object on Earth and **not** on Jupiter, so the mass should be multiplied by the gravitational field strength on Earth and not by that on Jupiter
C is incorrect as	the weight of the module on Earth is its mass × gravitational field strength on Earth. It is not mass × the increase in gravitational field strength on Jupiter

This question is easy, but you need to read it carefully. It is asking about the weight of the object on Earth and **not** on Jupiter. Remember that you can assume *g* on earth is 10 N kg^{-1} for paper 1.

2

The correct answer is **B** because:

- The gravitational field strength on Earth is defined by the equation:
  - $g_{earth} = \frac{Gm}{r^2}$
- For the new planet:
  - mass = 3*m* and radius = $\frac{1}{3}r$

- Substituting these values into the equation gives:
  - $g_{planet} = \frac{G \times 3m}{\left(\frac{1}{3}r\right)^2} = \frac{3Gm}{\frac{1}{9}r^2} = \frac{3Gm \times 9}{r^2} = \frac{27Gm}{r^2}$
- Therefore:
  - $g_{planet} = 27 \times g_{earth} = 27 \times 10\ \text{N kg}^{-1}$
  - $g_{planet} = 270\ \text{N kg}^{-1}$

A is incorrect as	this is the gravitational field strength on Earth and not on the new planet
C is incorrect as	the radius of the new planet is $\frac{1}{3}$ of the radius of Earth. When substituted into the formula $g = \frac{Gm}{r^2}$ then $\frac{1}{3}$r must be **squared**, so *g* is 9 × 3 times bigger on the new planet than on Earth and not just 3 × 3 times bigger, as $3^2 = 9$
D is incorrect as	the radius of the new planet is $\frac{1}{3}$ of the radius of Earth. $3^2 = 9$ and not 6, so *g* is 9 × 3 times bigger on the new planet than on Earth and not just 6 × 3 times bigger

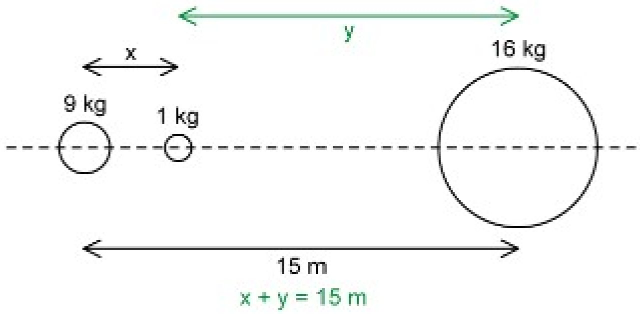
This question requires you to use the known equation for gravitational field strength and substitute the correct values in carefully. Show your workings clearly so that you do not make a mistake. For non-calculator questions, it is valid to assume that $g = 10\ \text{N kg}^{-1}$.

3

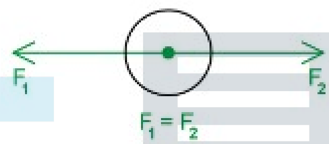
The correct answer is **A** because:

- The equation for the gravitational force *F* between two masses m_1 and m_2 is:
  - $F = \frac{Gm_1m_2}{r^2}$
- Therefore, the force between $m_1 = 1$ kg and $m_2 = 9$ kg if the distance between them is *x* is given by $F_1 = \frac{9G}{x^2}$

- Additionally, the force between $m_1 = 1$ kg and $m_2 = 16$ kg if the distance between them is is given by $F_2 = \frac{16G}{y^2}$
- Therefore $x + y = 15$ as shown below:



- If the 1 kg mass experiences zero resultant force, then $F_1 = F_2$:



- Therefore, equating the force equations gives:
  - $\frac{9G}{x^2} = \frac{16G}{y^2}$
  - $\frac{16}{9} = \frac{y^2}{x^2}$
  - $\frac{y}{x} = \frac{4}{3}$ and so $y = \frac{4}{3}x$
- Using the fact that x + y = 15, then substituting in $y = \frac{4}{3}x$ gives:
  - $x + \frac{4}{3}x = 15$
  - $\frac{7}{3}x = 15$
  - $7x = 45$
  - $x = \frac{45}{7} = 6.4$ m

B is incorrect as	$x + y = 15$ and not $15 + x = y$

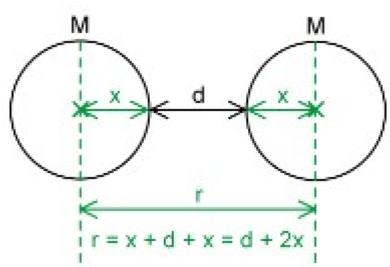
C is incorrect as	$\frac{16}{9} = \frac{y^2}{x^2}$ and not $\frac{16}{9} = \frac{y}{x}$
D is incorrect as	$\frac{7}{3}x = 15$ when rearranged becomes $x = \frac{15 \times 3}{7} = \frac{45}{7}$ and not $x = \frac{15 \times 7}{3}$

Always annotate the diagrams to help visualise any missing distances so you can make the correct calculation. Simultaneous equations are a common occurrence in IB physics that you would have learnt from GCSE, an annotated diagram will also show you other equations such as the sum of distances.

4

The correct answer is **D** because:

- The equation for the gravitational force F between two masses m_1 and m_2 is:
  - $F = \frac{Gm_1m_2}{r^2}$
- r is the distance between the centre of masses, which means you need to consider the additional distance due to the radius of the spheres:



- Therefore:
  - $r = d + x + x = d + 2x$ where x is the radius of the sphere
- Substituting this equation for r into the force F equation gives:
  - $F = \frac{GM \times M}{r^2} = \frac{GM^2}{(d+2x)^2}$
- Rearranging the equation to obtain and expression for M is:
  - $M^2 = \frac{F(d+2x)^2}{G}$
  - So, $M = \sqrt{\frac{F(d+2x)^2}{G}}$

A is incorrect as	$F = \frac{GM^2}{r^2}$ and not $F = \frac{GM}{r^2}$ (which is the equation for g) so when rearranged the final equation must be square rooted to obtain the correct expression for M
B is incorrect as	the gravitational force acts from the centre of each mass and not just from the surface of each mass, so the radii of both masses need to be considered. F must be multiplied by d the distance between the masses + $2x$ the distance from the surface of each mass to the centre. This gives $d + 2x$
C is incorrect as	in the equation for gravitational force $F = \frac{Gm_1m_2}{r^2}$ where r is squared and not just r this means the top line of the fraction should be $F \times (d + 2x)^2$ and not $F \times (d + 2x)$

You should remember that the distance between bodies in gravitational fields needs to be between the **centre of mass** of each body. This is an important consideration for the distance r.

Especially in questions with additional context, like a satellite orbiting the Earth, you are sometimes given only the distance between the Earth's **surface** and the body. This distance is **not** the distance you should substitute into equations for gravitational force and / or field strength: you would need to include the **radius** of the entire planet (i.e., from the surface to its centre) as well! Only then would the distance between the two bodies (the Earth and the satellite) be between their centres of mass.

5

The correct answer is **D** because:

- The gravitational force on the satellite from the Earth is a centripetal force
- Centripetal force is defined by the equation:
  - $F = m\omega^2 r$
- The gravitational is defined by Newton's law of gravitation:
  - $F = \frac{GMm}{r^2}$
- Therefore, equating both forces gives:
  - $m\omega^2 r = \frac{GMm}{r^2}$
  - Where m is the mass of the satellite, M is the mass of the body being orbited (the Earth) and r is the orbital radius
- Rearranging for r gives:
  - $\omega^2 r = \frac{GM}{r^2}$
  - $r^3 = \frac{GM}{\omega^2}$
  - $r = \sqrt[3]{\frac{GM}{\omega^2}} = \left(\frac{GM}{\omega^2}\right)^{\frac{1}{3}}$
- Therefore, the circumference of the satellite's orbit is:
  - $2\pi \times r = 2\pi\left(\frac{GM}{\omega^2}\right)^{\frac{1}{3}}$

A is incorrect as	this is the expression for the radius of the satellites orbit and not the circumference. The answer for r must be multiplied by 2π as circumference is $2\pi r$
B is incorrect as	ω^2 is not equal to $\frac{GMm}{r^2}$ the centripetal force $F = m\omega^2 r = \frac{GMm}{r^2}$

C is incorrect as	the radius of the Earth's orbit is $\left(\frac{GM}{\omega^2}\right)^{\frac{1}{3}}$ and not just $\left(\frac{GM}{\omega^2}\right)$ because the equation is r^3 so needs to be cube-rooted.

This question requires very careful substitution and rearranging of the equations for gravitational and centripetal forces. Remembering also that the satellite remains in orbit because of the centripetal force caused by gravity is key to this question, and is very common in this topic! You will see many circular motion equations appearing in this topic too, so make sure to brush up on them.

6

The correct answer is **C** because:

- The gravitational field strength on Earth is defined by the equation:
  - $g_{earth} = \frac{Gm}{r^2}$
- For the new planet:
  - Mass = $2m$ and Radius = $3r$
- Substituting this into the gravitational field strength equation gives:
  - $g_{planet} = \frac{G \times 2m}{(3r)^2} = \frac{2Gm}{9r^2}$
- Therefore:
  - $g_{planet} = \frac{2}{9} g_{earth}$

A is incorrect as	the mass of the planet is 2 × mass of earth and not 3 × m. The radius of the planet is 3 × radius of earth and not 2 × r.
B is incorrect as	the 3 and r both need to be squared, so the final fraction should be and not as the formula is g = and not g =

D is incorrect as	the fraction has been manipulated incorrectly to produce 2 × 9 as the numerator

This question requires you to use the known equation for gravitational field strength and substitute the correct values in carefully. Show your workings clearly so that you do not make a mistake and make sure that **all** values are squared that need to be!

7

The correct answer is **A** because:

- Newton's law of gravitational states that the force between two planets is:
  - $F = \frac{GMm}{r^2}$ where r is the separation between two masses M and m
- During separation R the force is:
  - $F = \frac{GMm}{R^2}$
- During separation $4R$ the force is:
  - $F = \frac{GMm}{(4R)^2} = \frac{GMm}{16R^2}$
- This means F for a separation of $4R$ is:
  - $\frac{F}{16}$ (i.e. 16 times less)

B is incorrect as	the radius is 4 × R, so it is now 4 times further away. According to the inverse square law the force will be $\frac{1}{4^2}$ smaller and not 4 times larger

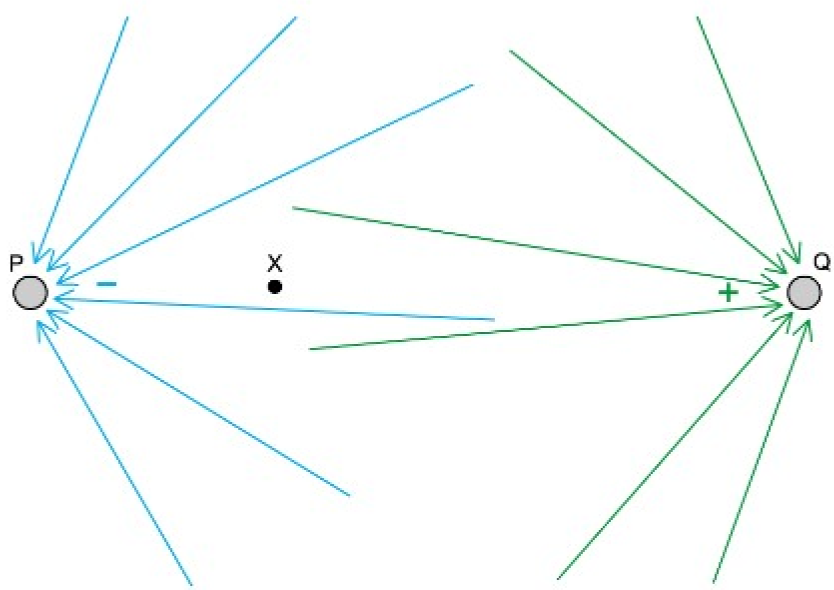
C is incorrect as	the inverse square law states that the force will be reduced by $\frac{1}{4^2}$ and not just $\frac{1}{4}$
D is incorrect as	the distance between the planets has changed, so the force between them will also change

The inverse square law relationship appears many times in this topic, remember to always consider it's impact.

8

The correct answer is **B** because:

- Gravitational field strength is a **vector** quantity
- In the second situation, *P* is now between two gravitational fields acting in **opposite** directions (towards P and towards Q)
- The overall magnitude of the gravitational field strength at *x* will be less than when *x* is in just one gravitational field
- Therefore, the correct answer is between *g* and zero
- This can be seen from the diagram below:



A is incorrect as	the distances between *PX* and *QX* are not the same, so the gravitational field strengths from the two masses do not cancel each other out to give zero. PX is a lot smaller than QX
C & **D** are incorrect as	gravitational field strength is a vector, so the gravitational field strength of *P* acts in one direction and in *Q* acts in the other, so the overall magnitude of the gravitational field strength at *X* is less and not more than between *P* and *X*.

This question requires you to recognise that gravitational field strength is a vector, so acting in one direction, it is positive and in the other it is negative. This means that it has a **direction** and this must be taken into consideration.

9

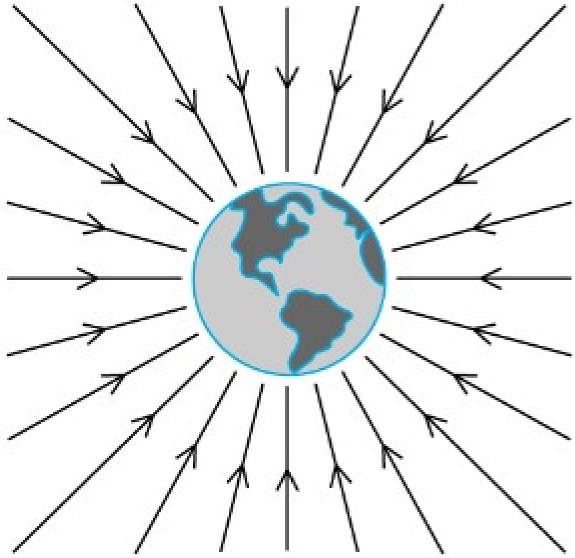
The correct answer is **C** because:

- Since the spacecraft is travelling with its motors shut down, this means it is decelerating due to the gravitational field strength of Earth
- Newton's second law states: *F* = *ma* = *mg*
- So, the acceleration of the rocket can be calculated using the equation:
  - *a* = change in velocity / change in time = $\frac{v-u}{t}$ where *v* is the final velocity and *u* is the initial velocity
- To use this formula km s^{-1} must be converted to m s^{-1}
  - There are 1000 m in 1 km
    - 2 km s^{-1} = 5200 m s^{-1}
    - 7 km s^{-1} = 5700 m s^{-1}
- Therefore:
  - Gravitational field strength, $g = \frac{v-u}{t} = \frac{5200-5700}{1000}$ = -0.5 N kg^{-1}

- In standard form, this is -5×10^{-1} N kg^{-1}
- The acceleration is negative because the space shuttle is moving away from the Earth and the gravitational field strength is acting towards the Earth, therefore it is decelerating due to the gravitational field pulling it back

A is incorrect as	the velocities of the spacecraft must be converted into m s^{-1} to use the equations of motion to calculate acceleration and not leave the velocities in km s^{-1}
B is incorrect as	this is the initial velocity 5.7 minus final velocity 5.2. The equation states that $a = \frac{v-u}{t} = \frac{\text{final} - \text{initial}}{t}$, not the other way around
D is incorrect as	this is the strength of the Earth's gravitational field on Earth's surface. Whilst the spacecraft is affected by the Earth's gravitational field, it is only affected by a **fraction** of it since it is moving away where the field strength is getting weaker

Remembering to use the equation $a = \frac{v-u}{t}$ is key to answering this question correctly. Also recognising that the gravitational field strength will be negative because the object is moving away from the Earth is important to success in this question. The Earth's gravitational field strength acts towards the centre of the Earth. So, it acts in the opposite direction to a rocket moving away from the Earth.



10

The correct answer is **B** because:

- The acceleration *a* of a mass *m* is given by $a = \frac{F}{m}$ where *F* is the resultant force on the mass
- The resultant force on a mass in a gravitational field is given by $F = mg$ (because the gravitational field strength $g = \frac{F}{m}$)
- Therefore, $a = \frac{F}{m} = \frac{mg}{m} = g$
- Therefore, statement **B** is incorrect

This question is vital to help you understand that the acceleration of any freefalling body in a gravitational field is independent of its mass: it only depends on the strength of the field. This is one of the major discoveries of physics, famously investigated by Galileo Galilei and verified by astronauts on the moon, who observed a hammer and a feather falling to the surface of the moon at exactly the same rate (in the absence of air resistance!)