Exam Papers Practice

6.1 Circular Motion

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Course	DP IB Physics
Section	6. Circular Motion & Gravitation
Topic	6.1 Circular Motion
Difficulty	Medium

To be used by all students preparing for DP IB Physics HL
Students of other boards may also find this useful

1

The correct answer is **C** because:

- The kinetic energy is given by the equation
  - $\frac{1}{2}mv^2$
- The linear velocity, *v* is given by the equation
  - $v = \omega r = 2\pi f r$
- Substituting the equation for *v* into the kinetic energy equation gives
  - $\frac{1}{2}m(2\pi f r)^2 = \frac{1}{2}m4\pi^2 f^2 r^2$
- The 4 and the 2 cancel out to give
  - $2mr^2\pi^2 f^2$ which is option **C**

When variables are multiplied together in algebraic equations, your final answer might appear in a slightly different order. So although it may look like your answer is not one of the options, always double check whether the variables are just given in a different order. For example $r^2 m\pi^2 2f^2$ would still be option **C**.

2

The correct answer is **D** because:

- The centripetal force, *F* is calculated using the equation:
  - $F = \frac{mv^2}{r}$
- The radius of the circular orbit of the ball is the length of the string, r = 0.1 m
- The velocity, *v* is defined by:
  - $v = r\omega = \frac{2\pi r}{r}$
- Substituting in the values gives:
  - $v = \frac{2\pi \times 0.1}{\frac{\pi}{10}} = 0.2 \times 10 = 2\text{ m s}^{-1}$

- Substituting in the values gives us
  - $F = \frac{0.05 \times (2)^2}{0.1} = \frac{0.05 \times 4}{0.1} = 2.0$ N

A is incorrect as	the velocity v has not been squared. The formula says v^2 and not just v
B is incorrect as	both the mass and velocity have been squared. The formula says v^2 and not $(mv)^2$
C is incorrect as	mv^2 has not been divided by radius r. The formula states: $F = \frac{mv^2}{r}$

To make the fraction easier to calculate with decimals $\frac{0.05 \times 4}{0.1}$, try and remove the decimals by multiplying the numerator and denominator 100. This would give $\frac{5 \times 4}{10} = \frac{20}{10} = 2$. Only to multiply the 0.05 **or** the 4 by 100, and not both, since they are multiplied together anyway.

3

The correct answer is **C** because:

- Linear velocity is defined by the equation:
  - $v = r\omega$
- Comparing v for both X and Y:
  - X: $v_X = r\omega = 3L\omega$
  - Y: $v_Y = r\omega = L\omega$
- So, $v_X = 3v_Y$
- Centripetal acceleration is defined by the equation:
  - $a = \frac{mv^2}{r}$
- Comparing a for both X and Y:
  - X: $a_X = \frac{mv^2}{r} = \frac{m(3L\omega)^2}{3L} = \frac{9mL^2\omega^2}{3L} = 3mL\omega^2$
  - Y: $a_Y = \frac{mv^2}{r} = \frac{m(L\omega)^2}{L} = \frac{mL^2\omega}{L} = mL\omega^2$
- So, $a_X = 3a_Y$

A is incorrect as	the linear velocity of 3v has not been squared the formula for acceleration is $a = \frac{mv^2}{r}$ and not $a = \frac{mv}{r}$
B is incorrect as	linear velocity and acceleration are not the same for both X and Y because points X and Y sit at different distances from the centre of the circle
D is incorrect as	the radius of X is 3 times the radius of Y and not 2 times

There are quite a few formulas in this topic, but make sure to check your data booklet for the ones that are given for you!

4

The correct answer is **D** because:

- The centripetal force, F is given by the equation:
  - $F = \frac{mv^2}{r}$
- This is equal to the frictional force
  - Since this is the force that keeps the girl moving in a circle
- The linear velocity, v is given by the equation
  - $v = \omega r$
  - Where, angular velocity $\omega = \frac{2\pi}{T}$
- So, $v = \frac{2\pi r}{T}$ where T is the time period
- Substituting the equation for v into the centripetal force equation gives
  - $F = \frac{m\left(\frac{2\pi r}{T}\right)^2}{r} = \frac{m\frac{4\pi^2 r^2}{T^2}}{r} = \frac{m4\pi^2 r}{T^2}$

- Rearranging for the time period, T:
  - $T = \sqrt{\frac{m4\pi^2 r}{F}}$

- Substituting in the values to calculate T gives:
  - $T = \sqrt{\frac{50 \times 4 \times \pi^2 \times 1}{600}} = \sqrt{\frac{200\ \pi^2}{600}} = \sqrt{\frac{1}{3}}\ \pi$

A is incorrect as	the 2π hasn't been squared correctly (to give $4\pi^2$)
B is incorrect as	4 is not included in the question, as it should be, so the answer is half the size it should be
C is incorrect as	The square root has not been taken (which would give T^2 instead of T)

The easiest way to divide a fraction by a fraction is to rewrite it as follows:

$\frac{\frac{r^2}{T^2}}{r} = \frac{r^2}{T^2} \div r$ where r can also be written as $\frac{r}{1}$ (same for any whole number)

When dividing by a fraction (or whole number), multiply them together and **flip** the numerator and denominator of the second fraction

$$\frac{r^2}{T^2} \div r = \frac{r^2}{T^2} \times \frac{1}{r} = \frac{r^2}{T^2 r} = \frac{r}{T^2}$$

This makes it easier, since multiplying by two fractions is done by multiplying the numerators and multiplying the denominators.

Remember that not all the equations will be in your data sheet. You will be expected to remember $\omega = \frac{2\pi}{T}$ (although it is technically in the HL part of the data sheet, if you carry onto this course).

5

The correct answer is **A** because:

- First calculate the frequency of the spinning top:
  - Frequency is defined as the number of revolutions in 1 second
- Convert five minutes into seconds:
  - $5 \times 60 = 300$ s
- Calculate the frequency:
  - Frequency = $\frac{20}{300} = \frac{1}{15}$ Hz
- The angular speed, ω is defined as:
  - $\omega = 2\pi f$
- Substitute in the values into the angular speed equation:
  - $\omega = 2\pi \times \frac{1}{15} = \frac{2\pi}{15}$ rad s^{-1}

B is incorrect as	the frequency is not just $\frac{1}{300}$, it is $\frac{20}{300}$ because there are 20 revolutions in five minutes
C is incorrect as	the time must be measured in seconds and not minutes. There should be 5×60 seconds in five minutes and not just 5. This means the frequency has been calculated incorrectly
D is incorrect as	this is the frequency of the spinning top and not the angular velocity

Understanding the definition of frequency is important – it is the number of revolutions per **second**. This question can also be done by:

- Finding the time period, T (time taken for 1 revolution)
  - $\frac{300}{20} = 15$
- Substitute this into the equation $\omega = \frac{2\pi}{T}$
  - $\omega = \frac{2\pi}{15} = \frac{2\pi}{15}$

Remember back in GCSE how to simplify fractions. This is really important when working with a non-calculator paper, since it makes the numbers a lot easier to calculate with.

E.g. To simplify $\frac{20}{300}$

1. Determine the highest common factor for both 20 and 300. This is 20 (i.e. 20 is the largest number that goes into 20 and 300)
2. Divided the numerator and denominator by this factor
3. This gives $\frac{1}{15}$

6

The correct answer is **A** because:

- Angular acceleration, a is defined by the equation:
  - $a = \frac{\Delta\omega}{\Delta t}$
- From t = 5.0 s to t = 15 s
  - $a = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{2.4 - 0.4}{15 - 5.0}$ = 0.2 rad s^{-2}
- From t = 15 s to t = 25 s
  - $a = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{4.4 - 2.4}{25 - 15}$ = 0.2 rad s^{-2}
- From t = 25 s to t = 35 s
  - $a = \frac{\omega_f - \omega_i}{t_f - t_i} = \frac{6.4 - 4.4}{35 - 25}$ = 0.2 rad s^{-2}
- Therefore, the angular acceleration a is constant at = 0.2 rad s^{-2}

B is incorrect as	$a = \frac{\Delta\omega}{\Delta t}$ so, $a = \frac{\text{change in angular velocity}}{\text{change in time}}$ and not just the angular velocity at one point
C is incorrect as	$a = \frac{\Delta\omega}{\Delta t}$ so, $a = \frac{\text{change in angular velocity}}{\text{change in time}}$ and is not equal to the initial angular velocity ω

D is incorrect as	*a* is constant during the whole motion it does not increase during the motion but remains at 0.2 rad s^{-2}

The key to this question is recognising that a = change in angular velocity ÷ by the change in time. This is the angular version to the equation a = change in linear velocity / change in time.

7

The correct answer is **C** because:

- When the mass is released it travels off at a **tangent** to the circle
- Gravity will affect the vertical velocity of the mass and accelerate it downwards
- However, it still continues at a constant velocity in the horizontal direction
- Therefore, it still follows a parabolic path when viewed in the vertical plane

8

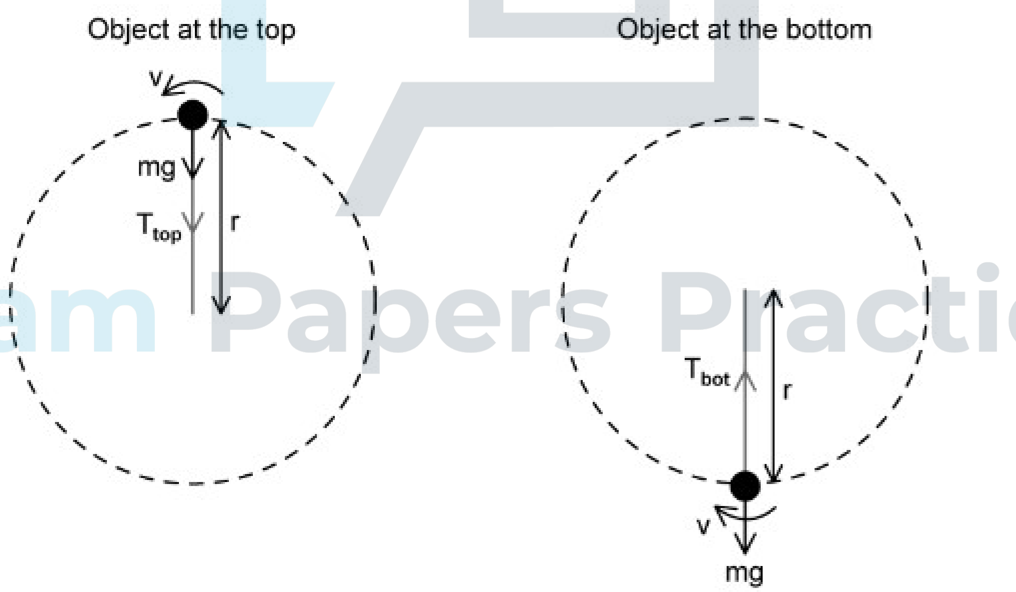
The correct answer is **B** because:

- The centripetal force on the object is the tension in the steel rod
    - This is the force that keeps the object moving in a circle
- The tension in the rod will be greatest when it is directly opposing the weight of the object
- This occurs when the rod is hanging vertically below the centre of the circle.
    - It is the rod that is keeping the object in circular motion

A is incorrect as	the tension in the rod will be greatest when opposing the total weight of the object and not just a component of the weight downwards at that point. The maximum values of the weight are when the object is directly at the bottom of the circle

C is incorrect as	the tension in the rod changes continually as the centripetal force in the circle. This is because of the changing components of weight acting on the object at different angles to the vertical
D is incorrect as	the tension in the rod is least at the top of the circle because it is acting in the same direction as the weight of the object. At this point, it is the weight that keeps the object moving in a circle, so the tension can be at a minimum.

This diagram explains the principles behind the maximum and minimum values of tension, *T* in vertical circular motion:



For object at the top

$\downarrow$ Centripetal force $= \frac{mv^2}{r} = mg + T_{top}$

$\Rightarrow T_{top} = m\left(\frac{v^2}{r} - g\right)$

For object at the botom

$\uparrow$ Centripetal force $= \frac{mv^2}{r} = T_{bot} - mg$

$\Rightarrow T_{bot} = m\left(\frac{v^2}{r} + g\right)$

9

The correct answer is **C** because:

- Momentum, p is defined by the equation:
  - $p = mv$
- Since velocity is a vector, so is momentum
- Velocity is defined as the **rate of change of displacement**
  - Since the direction of an object moving in a circle is always changing, therefore its velocity is also constantly changing
- If the velocity is constant changing, then so much its momentum and therefore its momentum is not constant
- Therefore **C** is incorrect

10

The correct answer is **D** because:

- The time period is defined as:
  - The time taken for an object to complete one full orbit
- The equation for the time period for X in circular motion is given by:
  - $T_X = \frac{2\pi}{\omega} = \frac{1}{f} = \frac{2\pi r}{v}$
- Therefore, the time period for Y is:
  - $T_Y = \frac{\frac{2\pi r}{2}}{2v} = \frac{\pi r}{2v}$
- T_Y is therefore 4 times less than T_X
  - This is $\frac{T}{4}$
- Objects X and Y follow the same orbital path, so the time period of their orbits will be the same

This question requires you to consider the formulae for time period and radius and identify the variables involved. The time period equations come from:

- $v = \omega r$ where $\omega = 2\pi f = \frac{2\pi}{T}$
- Therefore, $\omega = \frac{v}{r} = \frac{2\pi}{T}$
- Rearranging for T gives $T = \frac{2\pi r}{v}$

This can also be seen from the

$$speed = \frac{distance}{time}$$

Rearranging, we could see here

$$time = \frac{distance}{speed}$$