6.1 Circular Motion

PHYSICS

AQA A Level Revision Notes

A Level Physics AQA

6.1 Circular Motion

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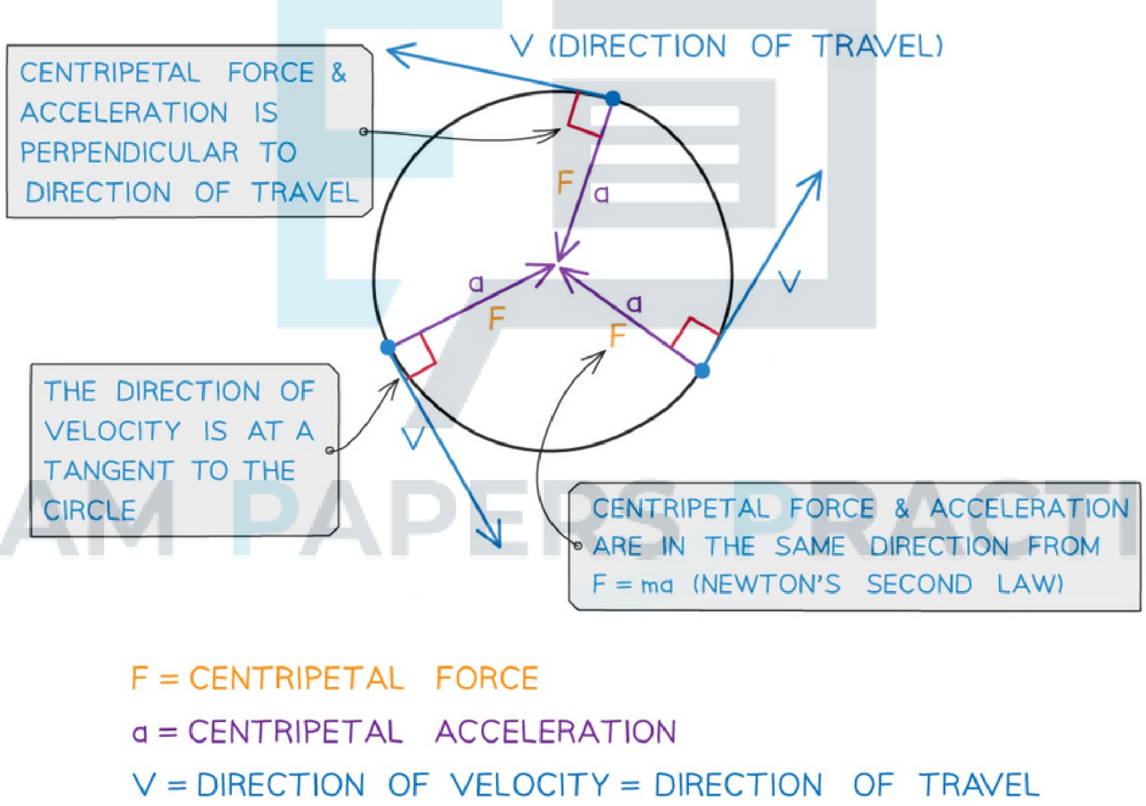
6.1.1 Circular Motion

Circular Motion

- Velocity and acceleration are both vector quantities
- An object in uniform circular motion has a constant **linear speed**
- However, it is **continuously changing direction.** Since velocity is the speed in a given direction, it, therefore, has a **constantly changing velocity**
    - The object therefore must be **accelerating**
    - This is because acceleration is defined as the rate of change of velocity

- This acceleration is called the **centripetal acceleration** and is **perpendicular** to the direction of the linear speed
    - Centripetal means it acts **towards the centre** of the circular path



Centripetal force and acceleration are always directed towards the centre of the circle

- The centripetal acceleration is caused by a **centripetal force** of constant magnitude that also acts **perpendicular** to the direction of motion (towards the centre)
    - This is a result of Newton's Second Law

- Therefore, the centripetal acceleration and force act in the **same direction**

Exam Tip

- The linear speed is sometimes referred to as the 'tangential' speed
- A tangent is a straight line which touches a circle or curve at exactly one point
- The key feature of a tangent of a circle is that it **always acts perpendicular** to its radius
- You can find out more in the A Level Maths revision notes on Tangents

6.1.2 Radians

Radians

- In circular motion, it is more convenient to measure angular displacement in units of **radians** rather than units of degrees
- The **angular displacement** (θ) of a body in circular motion is defined as:

  The change in angle, in radians, of a body as it rotates around a circle

- The **angular displacement** is the ratio of:

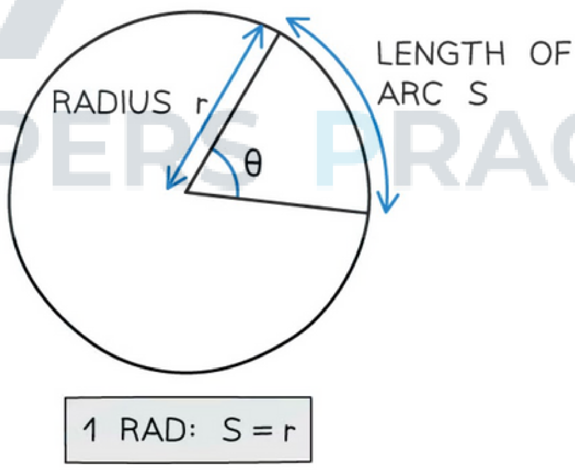
$$\Delta\theta = \frac{\text{distance travelled around the circle}}{\text{radius of the circle}}$$

- **Note**: both distances must be measured in the same units e.g. metres
- A **radian** (rad) is defined as:

  The angle subtended at the centre of a circle by an arc equal in length to the radius of the circle

- Angular displacement can be calculated using the equation:

$$\Delta\theta = \frac{\Delta S}{r}$$



When the angle is equal to one radian, the length of the arc (Δs) is equal to the radius (r) of the circle

- Where:
  - $\Delta\theta$ = angular displacement, or angle of rotation (radians)
  - s = length of the arc, or the distance travelled around the circle (m)
  - r = radius of the circle (m)

- Radians are commonly written in terms of π
- The angle in radians for a complete circle (360°) is equal to:

$$\frac{\text{circumference of circle}}{\text{radius}} = \frac{2\pi r}{r} = 2\pi$$

Radian Conversions

- If an angle of 360° = 2π radians, then 1 radian in degrees is equal to:

$$\frac{360}{2\pi} = \frac{180}{\pi} \approx 57.3^\circ$$

- Use the following equation to convert from degrees to radians:

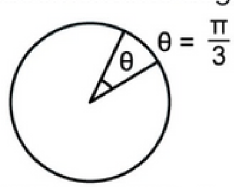
$$\theta^\circ \times \frac{\pi}{180} = \theta \text{ rad}$$

Table of common degrees to radians conversions

Degrees (°)	Radians (rads)
360	2π
270	$\frac{3\pi}{2}$
180	π
90	$\frac{\pi}{2}$

Worked Example

Convert the following angular displacement into degrees:



STEP 1

REARRANGE DEGREES TO RADIANS CONVERSION EQUATION

DEGREES → RADIANS $\theta^\circ \times \frac{\pi}{180} = \theta \text{ RAD}$

RADIANS → DEGREES $\theta \text{ RAD} \times \frac{180}{\pi} = \theta^\circ$

STEP 2

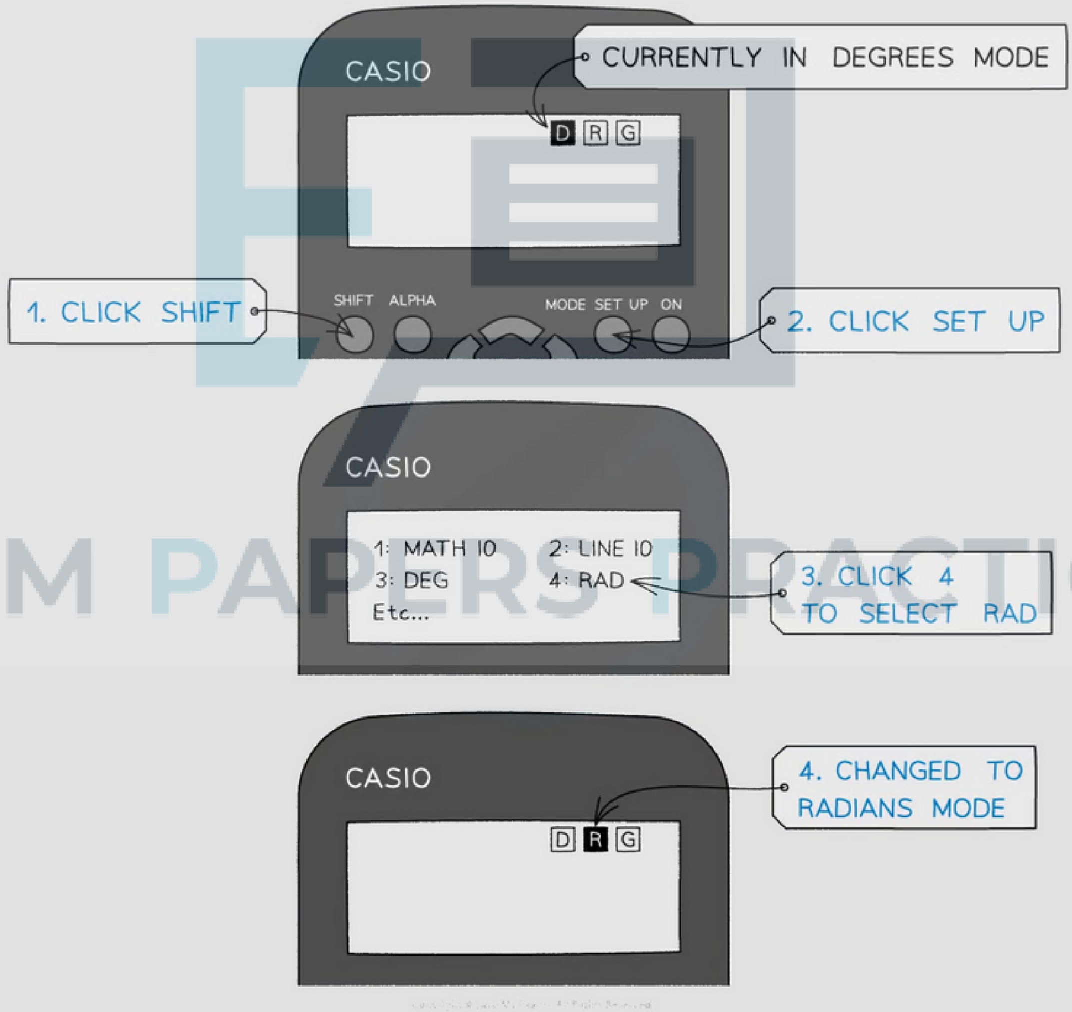
SUBSTITUTE VALUE

$\frac{\pi}{3} \text{ RAD} \times \frac{180}{\pi} = \frac{180^\circ}{3} = 60^\circ$

π's WILL CANCEL OUT

Exam Tip

- You will notice your calculator has a degree (Deg) and radians (Rad) mode
- This is shown by the "D" or "R" highlighted at the top of the screen
- Remember to make sure it's in the right mode when using **trigonometric** functions (sin, cos, tan) depending on whether the answer is required in **degrees** or **radians**
- It is extremely common for students to get the wrong answer (and lose marks) because their calculator is in the wrong mode - make sure this doesn't happen to you!
  - This mode only matters if you're using **sine, cos** or **tan**



6.1.3 Angular Speed

Angular Speed

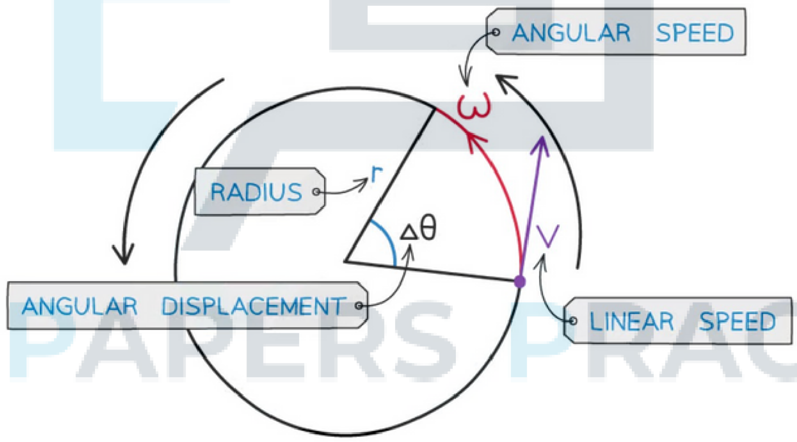
- Any object travelling in a uniform circular motion at the same speed travels with a **constantly changing velocity**
- This is because it is **constantly changing direction**, and is therefore accelerating
- The **angular speed** (ω) of a body in circular motion is defined as:

The rate of change in angular displacement with respect to time

- Angular speed is a **scalar quantity** and is measured in rad s^{-1}
- It can be calculated using:

$$\omega = \frac{\Delta\theta}{\Delta t}$$

- Where:
  - $\Delta\theta$ = change in angular displacement (radians)
  - Δt = time interval (s)



When an object is in uniform circular motion, velocity constantly changes direction, but the speed stays the same

- Taking the angular displacement of a complete cycle as 2π, the angular speed ω can be calculated using the equation:

$$\omega = \frac{v}{r} = 2\pi f = \frac{2\pi}{T}$$

- Where:
  - v = linear speed (m s^{-1})
  - r = radius of orbit (m)
  - T = the time period (s)
  - f = frequency (Hz)

- Angular velocity is the same as angular speed, but it is a **vector quantity**
- This equation shows that:
  - The greater the rotation angle θ in a given amount of time, the greater the angular velocity ω
  - An object rotating further from the centre of the circle (larger *r*) moves with a smaller angular velocity (smaller ω)

Worked Example

A bird flies in a horizontal circle with an angular speed of 5.25 rad s^{-1} of radius 650 m.

Calculate:

a) The linear speed of the bird

b) The frequency of the bird flying in a complete circle

a)

STEP 1 — LINEAR SPEED EQUATION

$v = r\omega$

STEP 2 — SUBSTITUTE IN VALUES

$V = 650 \times 5.25 = 3412.5 = 3410\ ms^{-1}$ (3 s.f.)

b)

STEP 1 — ANGULAR SPEED WITH FREQUENCY EQUATION

$\omega = 2\pi f$

STEP 2 — REARRANGE FOR FREQUENCY

$f = \frac{\omega}{2\pi}$

STEP 3 — SUBSTITUTE IN VALUES

$f = \frac{5.25}{2\pi} = 0.83556... = 0.836\ Hz$ (3 s.f.)

6.1.4 Centripetal Acceleration

Centripetal Acceleration Formula

- Centripetal acceleration is defined as:

  The acceleration of an object towards the centre of a circle when an object is in motion (rotating) around a circle at a constant speed

- It can be defined using the radius *r* and linear speed *v*:

$$a = \frac{v^2}{r}$$

- Where:
  - a = centripetal acceleration (m s^{-2})
  - v = linear speed (m s^{-1})
  - r = radius of the circular orbit (m)
- Using the equation relating angular speed ω and linear speed v:

$$v = r\omega$$

- Where:
  - ω = angular speed (rad s^{-1})

- These equations can be combined to give another form of the centripetal acceleration equation:

$$a = \frac{(r\omega)^2}{r}$$

$$a = r\omega^2$$

- This equation shows that centripetal acceleration is equal to the radius times the square of the angular speed
- Alternatively, rearrange for *r:*

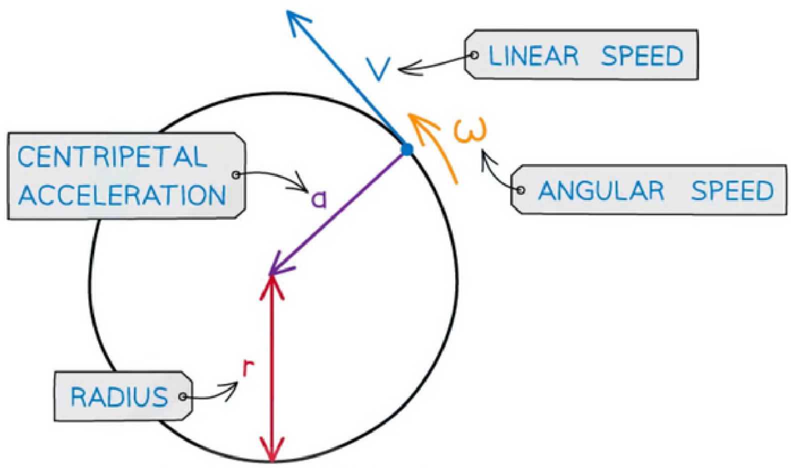
$$r = \frac{v}{\omega}$$

- This equation can be combined with the first one to give us another form of the centripetal acceleration equation:

$$a = \frac{v^2}{\left(\frac{v}{\omega}\right)}$$

$$a = v\omega$$

- This equation shows how the centripetal acceleration relates to the linear speed and the angular speed



Centripetal acceleration is always directed toward the centre of the circle, and is perpendicular to the object's velocity

- Where:
  - a = centripetal acceleration ($m\ s^{-2}$)
  - v = linear speed ($m\ s^{-1}$)
  - ω = angular speed ($rad\ s^{-1}$)
  - r = radius of the orbit (m)

Worked Example

A ball tied to a string is rotating in a horizontal circle with a radius of 1.5 m and an angular speed of 3.5 rad s^{-1}. Calculate its centripetal acceleration if the radius was twice as large and angular speed was twice as fast.

STEP 1

ANGULAR ACCELERATION EQUATION WITH ANGULAR SPEED

$$a = r\omega^2$$

STEP 2

CHANGE IN ANGULAR ACCELERATION WITH TWICE THE RADIUS AND ANGULAR SPEED

$$a = (2r) \times (2\omega)^2 = 2r \times 4\omega^2 = 8r\omega^2$$

THE CENTRIPETAL ACCELERATION WILL BE 8x BIGGER

STEP 3

SUBSTITUTE IN VALUES OF RADIUS AND ANGULAR SPEED

$$a = 8r\omega^2 = 8 \times 1.5 \times 3.5^2 = 147\ ms^{-2}$$

6.1.5 Centripetal Force

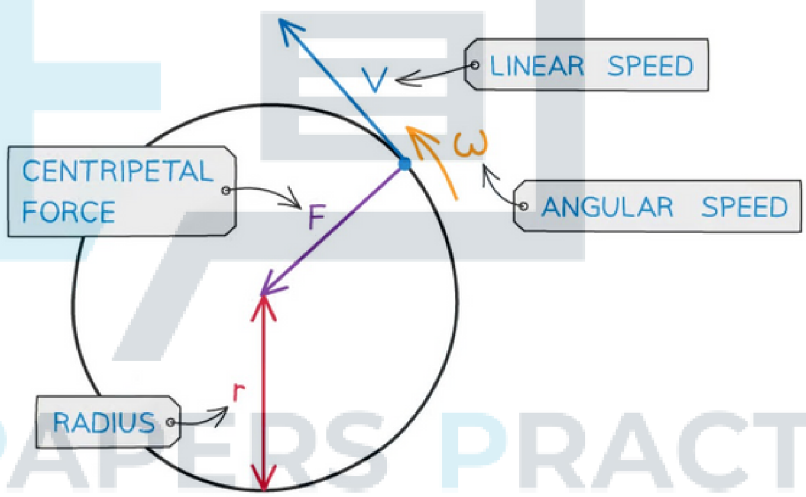
Calculating Centripetal Force

- An object moving in a circle is not in equilibrium, it has a resultant force acting upon it
  - This is known as the **centripetal force** and is what keeps the object moving in a circle
- The centripetal force (*F*) is defined as:

  The resultant force towards the centre of the circle required to keep a body in uniform circular motion. It is always directed towards the centre of the body's rotation.

- Centripetal force can be calculated using:

$$F = \frac{mv^2}{r} = mr\omega^2 = mv\omega$$



Centripetal force is always perpendicular to the direction of travel

- Where:
  - F = centripetal force (N)
  - v = linear velocity (m s^{-1})
  - ω = angular speed (rad s^{-1})
  - r = radius of the orbit (m)

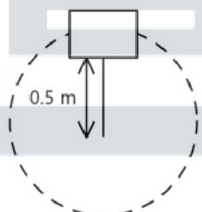
- **Note:** centripetal force and centripetal acceleration act in the **same direction**
  - This is due to Newton's Second Law
- The centripetal force is **not** a separate force of its own
  - It can be any type of force, depending on the situation, which keeps an object moving in a circular path

Examples of centripetal force

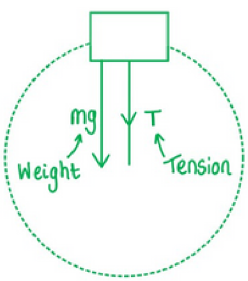
Situation	Centripetal force
Car travelling around a roundabout	Friction between car tyres and the road
Ball attached to a rope moving in a circle	Tension in the rope
Earth orbiting the Sun	Gravitational force

Worked Example

A bucket of mass 8.0 kg is filled with water is attached to a string of length 0.5 m. What is the minimum speed the bucket must have at the top of the circle so no water spills out?



Step 1: Draw the forces on the bucket at the top



Step 2: Calculate the centripetal force

- The weight of the bucket = mg
- This is equal to the centripetal force since it is directed towards the centre of the circle

$$mg = \frac{mv^2}{r}$$

Step 3: Rearrange for velocity *v*

- *m* cancels from both sides

$$v = \sqrt{gr}$$

Step 4: Substitute in values

$$v = \sqrt{9.81 \times 0.5} = 2.21 \text{ m s}^{-1}$$