Exam Papers Practice

6.1 Extended Questions

Mark Schemes

Course	DP IB Maths
Section	6. Extended Questions
Topic	6.1 Extended Questions
Difficulty	Medium

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful

Question 1

a) $u_n = u_1 + (n-1)d$ } n^{th} term of an arithmetic sequence

(i) $u_4 = u_1 + (4-1)d = u_1 + 3d = 25$

$$u_1 + 3d = 25$$

(ii) $u_{16} = u_1 + (16-1)d = u_1 + 15d = 49$

$$u_1 + 15d = 49$$

b) Subtract ① from ②

$$u_1 + 15d = 49$$
$$-(u_1 + 3d = 25)$$

$$12d = 24 \Rightarrow d = 2$$

$u_1 = 25 - 3d = 25 - 3(2) = 25 - 6$ (from ①)

$$u_1 = 19$$

You could also solve simultaneous equations with your GDC.

c) $S_n = \frac{n}{2}(2u_1 + (n-1)d)$ and $S_n = \frac{n}{2}(u_1 + u_n)$ } Sum of n terms of an arithmetic sequence

$$S_{18} = \frac{18}{2}(2(19) + (18-1)(2))$$

$$= 9(38 + 34) = 9(72) = 648$$

There are 648 seats.

d) (i) $u_n = u_1 r^{n-1}$ } n^{th} term of a geometric sequence

$$22(0.95)^{n-1} = 10 \Rightarrow (0.95)^{n-1} = \frac{10}{22}$$

$$\Rightarrow n - 1 = \log_{0.95}\left(\frac{10}{22}\right) \quad a^x = b \Longleftrightarrow x = \log_a b$$

$$\Rightarrow n = 1 + \log_{0.95}\left(\frac{10}{22}\right) = 16.371548...$$

round to next higher integer

$n = 17$ (17^{th} row)

(ii) $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}$ } Sum of n terms of a geometric sequence $(r \neq 1)$

$$\text{Total} = 22 \times \underbrace{\left(\frac{22(1 - 0.95^{18})}{1 - 0.95}\right)}_{= S_{18}\ (\text{sum of prices for one ticket from each row})} = 5834.96539...$$

$ 5835

Question 2

a) (i) $\bar{x} = 17.3666666... = 17.4\%$ (3 s.f.)

(ii) $\bar{y} = 61.8333333... = 61.8$ bpm (3 s.f.)

(iii) $r = 0.74981007 = 0.750$ (3 s.f.)

b) (i) $m = 0.90106629 = 0.901$ (3 s.f.)

$c = 46.1848154 = 46.2$ (3 s.f.)

$y = 0.901x + 46.2$

(ii) Find y when $x = \bar{x}$, using exact values

When $x = \bar{x}$, $= \frac{521}{30}$

$y = 0.90106629(17.36) + 46.1848154$

$= 61.8333333 = \bar{y}$

So $(\bar{x}, \bar{y})$ is on the regression line.

c) (i) $y = 0.901x + 46.2 \Rightarrow x = \dfrac{y - 46.2}{0.901}$

When $y = 60$, $x = \dfrac{60 - 46.2}{0.901} = 15.316315\ldots$

15.3 % (3 s.f.)

(ii) **In general, the regression line of y on x should not be used to predict a value of x from a value of y. So the estimate of 15.3 % may not be reliable.**

Percentage error

$$\varepsilon = \left| \frac{V_A - V_E}{V_E} \right| \times 100\ \%$$

V_A is the approximate value

V_E is the exact value

d) $\varepsilon = \left| \dfrac{15.3 - 13.5}{13.5} \right| \times 100$

$= \dfrac{40}{3} = 13.333333\ldots$

13.3 % (3 s.f.)

Question 3

a) $c^2 = a^2 + b^2 - 2ab\cos C$ } Cosine rule

$(BC)^2 = 184^2 + 222^2 - 2(184)(222)\cos 115$

$= 83140 - 81696\cos 115$

$BC = \sqrt{83140 - 81696\cos 115}$

$= 343.025103...$

343 m (3 s.f.)

b) $89.99 \times (184 + 222 + \underbrace{\sqrt{83140 - 81696\cos 115}}_{\text{exact value of BC}})$

$= 67404.7690...$

$67404.77 (2 d.p.)

c) $\text{Area} = \frac{1}{2}ab\sin C$ } area of a triangle

$\text{Area} = \frac{1}{2}(184)(222)\sin 115$

$= 18510.4302...$

18510 m^2 (to nearest m^2)

d) $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ } Sine rule

$$\frac{BC}{\sin 115} = \frac{184}{\sin A\hat{B}C} \Rightarrow \sin A\hat{B}C = \frac{184}{BC}\sin 115$$

$$A\hat{B}C = \sin^{-1}\left(\frac{184}{\sqrt{83140 - 81696\cos 115}} \times \sin 115\right)$$

$$= 29.087649...$$

$$A\hat{C}B = 180 - 115 - A\hat{B}C = 35.912350...$$

To 1 d.p.,

$A\hat{B}C = 29.1°$ $A\hat{C}B = 35.9°$

$\sin A\hat{B}C = \frac{184}{BC}\sin 115$ } from part (d) working

e) $\sin A\hat{B}C = \frac{d}{222}$ SOHCAHTOA

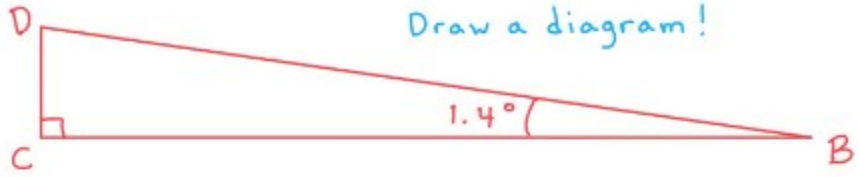
$$d = 222\sin A\hat{B}C$$

$$= 222\left(\frac{184}{\sqrt{83140 - 81696\cos 115}} \times \sin 115\right)$$

$$= 107.924639...$$

108 m (3 s.f.)

f)



$\tan(1.4) = \frac{CD}{BC}$ SOHCAHTOA

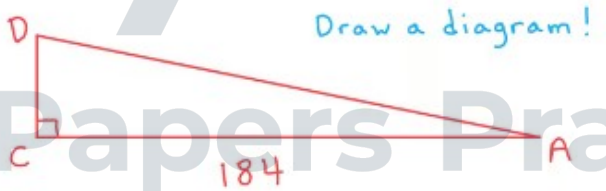
$CD = BC \tan(1.4)$

$= \left(\sqrt{83140 - 81696\cos 115}\right)\tan(1.4)$

$= 8.383352...$

8.38 m (3 s.f.)

g)



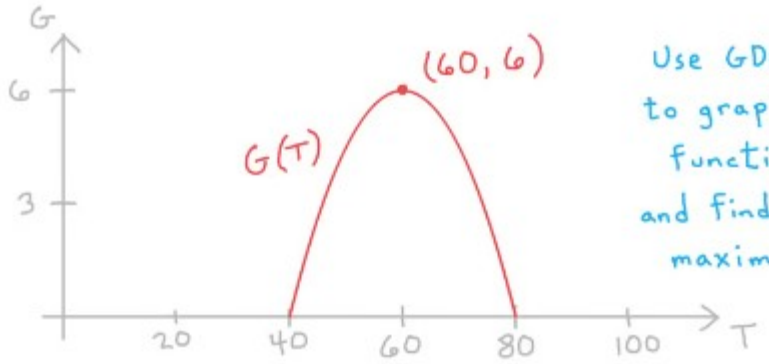
$AD = \sqrt{AC^2 + CD^2}$ Pythagoras

$= \sqrt{184^2 + \left(\underbrace{\sqrt{83140 - 81696\cos 115}\tan(1.4)}_{\text{exact value of CD}}\right)^2}$

$= 184.190880...$

184.2 m (1 d.p.)

h)



Use GDC to graph function and find the maximum

The maximum rate of growth is 6 inches per month, when the temperature is 60° F.

Question 4

a) (i) $\bar{x} = 2.03333333$ from GDC

$\bar{x} = 2.03$ baskets (3 s.f.)

(ii) $\sigma_x = 1.67099438$ from GDC

$\sigma_x = 1.67$ baskets (3 s.f.)

b) Median = 2 baskets from GDC

c) $IQR = Q_3 - Q_1$ } Interquartile range

$Q_1 = 0$ $Q_3 = 3$ from GDC

$Q_3 - Q_1 = 3 - 0 = 3$

IQR = 3 baskets

d) The upper outlier boundary is

$Q_3 + 1.5 \times IQR$

$3 + 1.5 \times 3 = 3 + 4.5 = 7.5$

$8 > 7.5$

A player scoring 8 baskets would be an outlier.

e) For that to be true... [3

... the first player must be one of the 17 players who scored one basket, out of the 92 who scored two or less...

$$\frac{17}{92} \times \frac{16}{149} = \frac{68}{3427} = 0.0198424...$$

... and then of the remaining 149 players, the second player must be one of the remaining 16 players who scored one basket.

The probability is $\frac{68}{3427} = 0.0198$ (4 d.p.)

f) $X \sim N(5, 0.8^2)$

(i) $P(X < 6) = 0.89435022$ from GDC

0.8944 (4 d.p.)

(ii) $P(X < 4) = 0.10564977$ from GDC

0.1056 (4 d.p.)

(iii) $P(4 \leq X \leq 6) = 0.78870045$ from GDC

$0.78870045 \times 150 = 118.305067...$

118 players

Question 5

a) (i) $V = \pi r^2 h$ } Volume of a cylinder

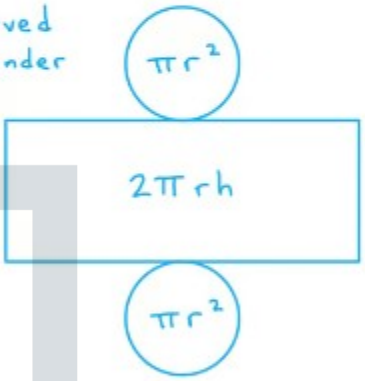
$V = \pi (4)^2 (15) = 240\pi = 753.982236...$

$V = 240\pi \text{ cm}^3 = 754 \text{ cm}^3 \text{ (3 s.f.)}$

exact answer

(ii) $A = 2\pi rh$ } Area of the curved surface of a cylinder

$A = \pi r^2$ } Area of a circle



$A = 2\pi(4)(15) + 2 \times \pi(4)^2$

$= 152\pi = 477.522083...$

$A = 152\pi \text{ cm}^2 = 478 \text{ cm}^2 \text{ (3 s.f.)}$

exact answer

$$\int x^n \, dx = \frac{x^{n+1}}{n+1} + c \quad \Big\} \text{ Integral of } x^n \ (n \neq -1)$$

b) $P(x) = \int \frac{dP}{dx} \, dx$

$P(x) = \int (-2x + 472) \, dx$

$= -2\left(\frac{x^2}{2}\right) + 472x + c = -x^2 + 472x + c$

$\frac{8000}{1000} = 8$ Don't forget that x is thousands of cans

So $P(8) = 2450$

$-(8)^2 + 472(8) + c = 2450$

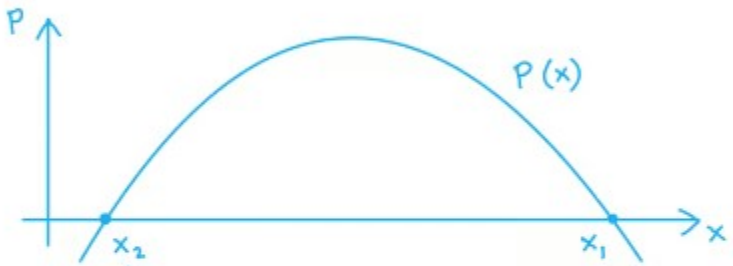
$c + 3712 = 2450 \Rightarrow c = -1262$

$$P(x) = -x^2 + 472x - 1262$$

c) We need to find when $P(x) = 0$

$-x^2 + 472x - 1262 = 0$ Solve with GDC

$x_1 = 469.3109513$ $x_2 = 2.689048693$



x_2 is the 'break even' point

$1000x_2 = 2689.048693$ Don't forget that x is thousands of cans

We need the smallest integer bigger than this

2690 cans

d) $P(x)$ has a maximum when $\frac{dP}{dx} = 0$

$$-2x + 472 = 0$$

$$2x = 472 \implies x = 236$$

Selling 236000 cans will maximise profit.

Don't forget that x is thousands of cans

So the max profit is $P(236)$

$$-(236)^2 + 472(236) - 1262 = 54434$$

Maximum profit is 54434 NZD

* You could also find these values by graphing $P(x)$ on your GDC and finding the max.

Maximum profit is 54434 NZD } from part (d)

e) Find 60% of the maximum profit

$0.6 \times 54434 = 32660.4$

Use the compound interest function on your GDC

$n = 24$ (24 months in two years) $I\% = 5.5$

$PV = 0$ $PMT = -32660.40$

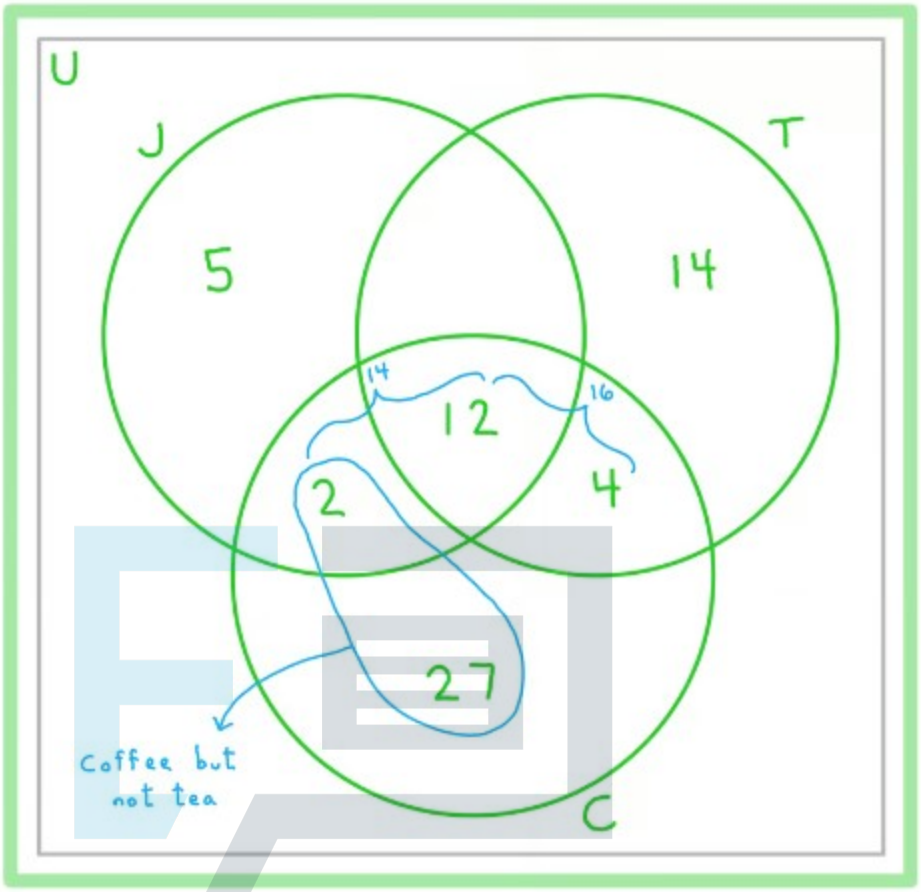
$P/Y = 12$ $C/Y = 12$

$\Rightarrow FV = 826587.6989$

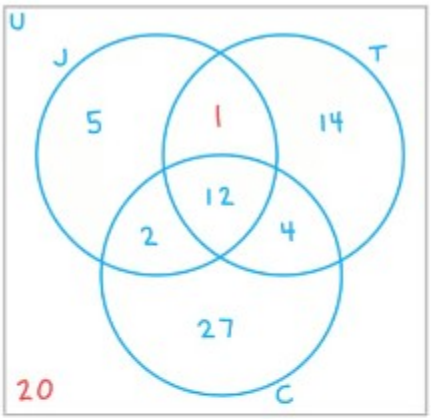
FV > 800000, so yes they will have enough.

Question 6

a) (i)



(ii) $2 + 27 = 29$



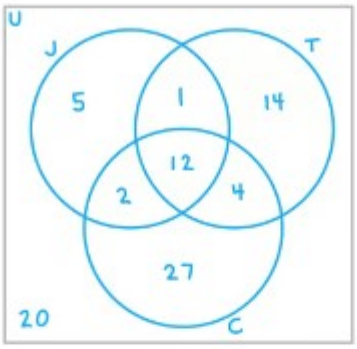
$12 + 4 + 14 + 1 = 31$

b) (i) This is the total number inside the 'T' and 'J' circles.

$5 + 2 + 1 + 12 + 14 + 4 = \boxed{38}$

(ii) This is the number out of 85 not in any of the circles.

$85 - (5 + 2 + 1 + 12 + 14 + 4 + 27) = \boxed{20}$



c) (i) $\frac{2+12+4+27}{85} = \frac{45}{85} = \frac{9}{17}$

(ii) $\frac{4}{85}$

(iii) $\frac{27+20}{85} = \frac{47}{85}$

(iv) $\frac{5+20}{5+20+2+27} = \frac{25}{54}$

number that don't like coffee and don't like tea / number that don't like tea