A Level Physics CIE

6. Deformation of Solids

CONTENTS

6.1 Deformation: Stress & Strain

6.1.1 Extension & Compression

Tensile Force

- Forces don't just change the motion of a body, but can change the size and **shape** of them too. This is known as **deformation**
- Forces in opposite directions stretch or compress a body
  - When two forces **stretch** a body, they are described as **tensile**
  - When two forces **compress** a body, they are known as **compressive**

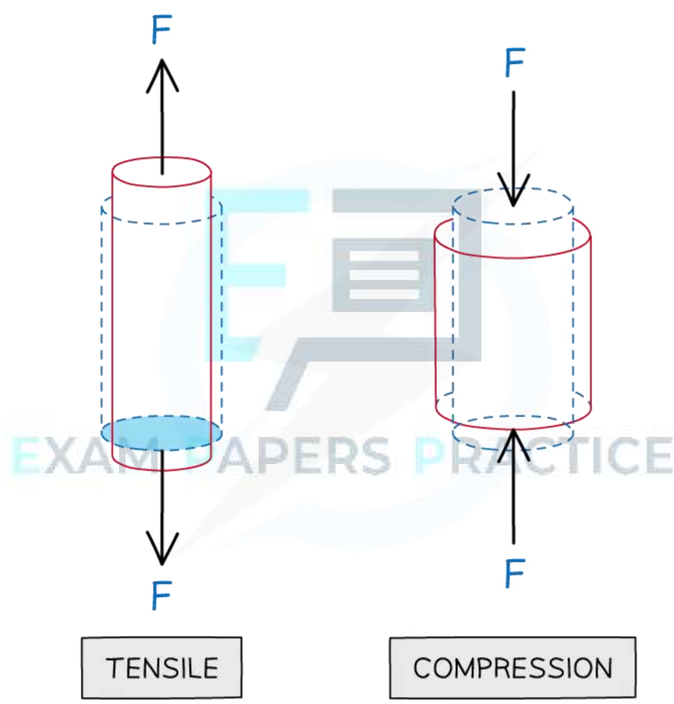


Diagram of tensile and compressive forces

Tensile Strength

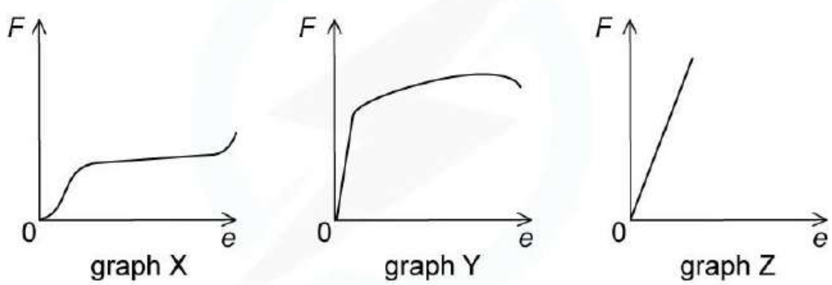
- Tensile strength is the amount of load or stress a material can handle until it stretches and breaks
- Here are some common materials and their tensile strength:

Tensile strength of various materials

Material	Tensile Strength (MPa)
Concrete	2–5
Rubber	16
Human skin	20
Glass	33
Human hair	200
Steel	840
Diamond	2800

Worked Example

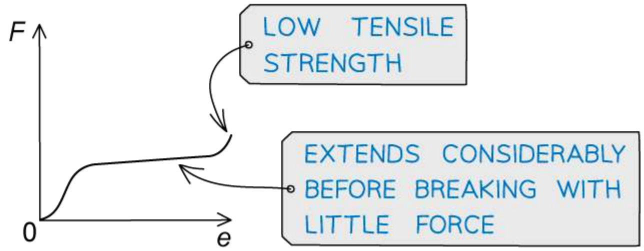
Cylindrical samples of steel, glass and rubber are each subjected to a gradually increasing tensile force F. The extensions e are measured and graphs are plotted as shown below.



Correctly label the graphs with the materials: steel, glass, rubber.

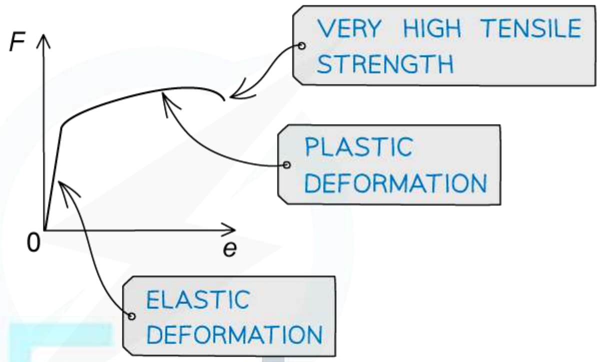
RUBBER

STRETCHY MATERIAL



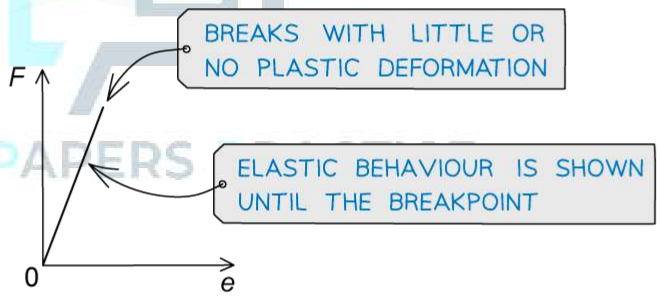
STEEL

DUCTILE MATERIAL



GLASS

BRITTLE MATERIAL

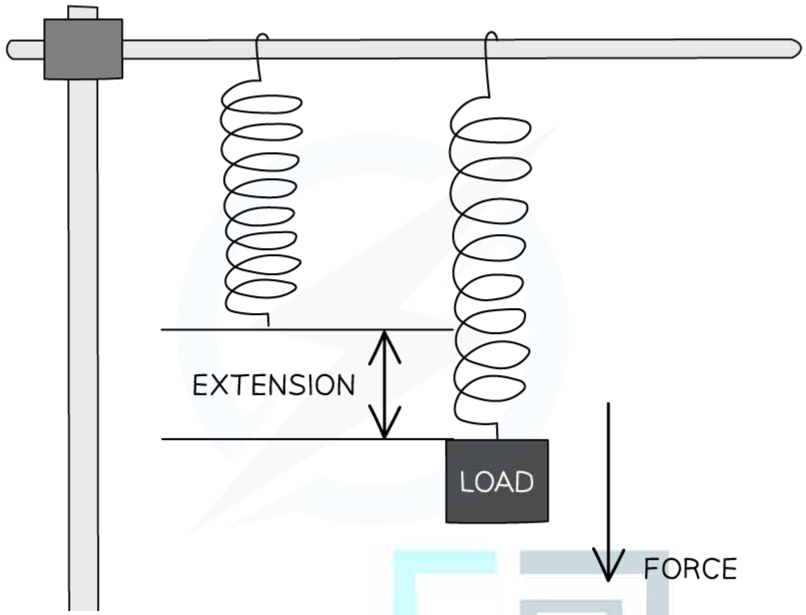


Exam Tip

Remember to read the questions carefully in order to not confuse the terms 'tensile stress' and 'tensile strain'.

Extension and Compression

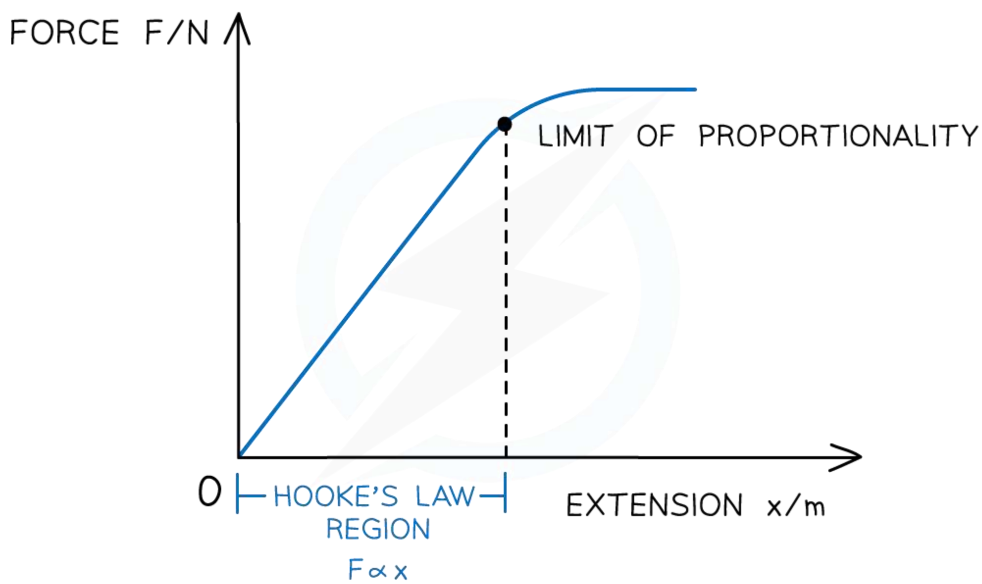
- When you apply a force (load) onto a spring, it produces a tensile force and causes the spring to extend



Stretching a spring with a load produces a force that leads to an extension

Hooke's Law

- If a material responds to tensile forces in a way in which the extension produced is proportional to the applied force (load), we say it obeys **Hooke's Law**
- This relationship between force and extension is shown in the graph below



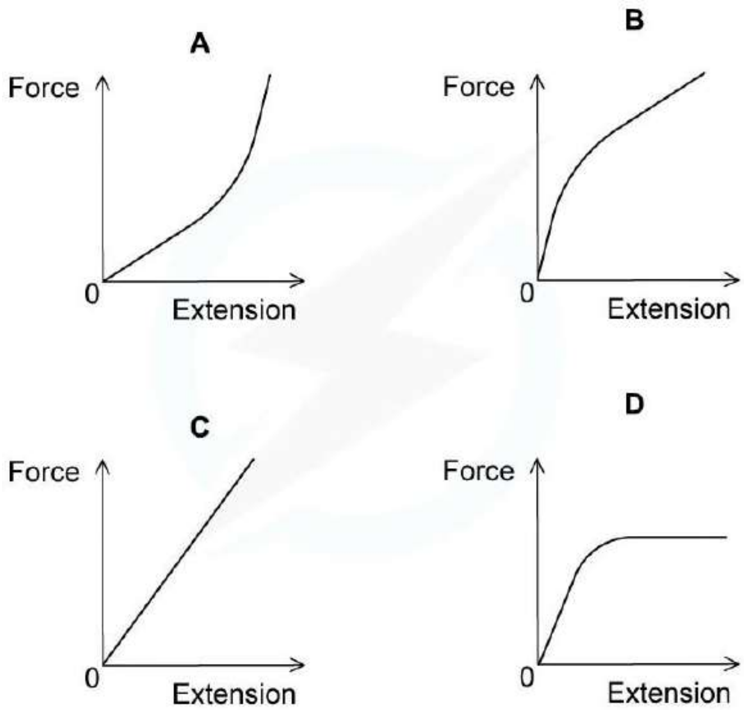
Force v extension graph for a spring

- The extension of the spring is determined by how much it has **increased** in length
- The **limit of proportionality** is the point beyond which Hooke's law is no longer true when stretching a material i.e. the extension is no longer proportional to the applied load
  - The point is identified on the graph where the line is no longer straight and starts to curve (flattens out)

- Hooke's law also applies to **compression** as well as extension. The only difference is that an applied force is now proportional to the **decrease** in length
- The gradient of this graph is equal to the **spring constant *k***. This is explored further in the revision notes "The Spring Constant"

Worked Example

Which graph represents the force-extension relationship of a rubber band that is stretched almost to its breaking point?



ANSWER: A

- Rubber bands obey Hooke's law until they're stretched up to twice their original size or more - this is because the long chain molecules become fully aligned and can no longer move past each other
- This is shown by graph A - after the section of linear proportionality (the straight line), the gradient increases significantly, so, a large force is required to extend the rubber band by even a small amount

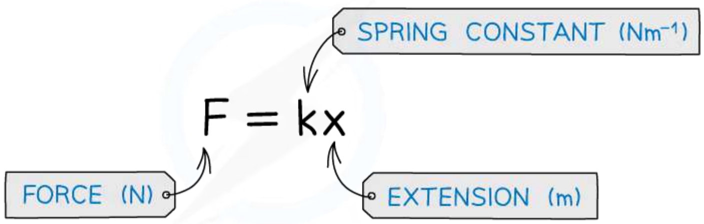
- Graph B is incorrect as the gradient decreases, suggesting that less force is required to cause a small extension
- Graph C is incorrect as this shows a material which obeys Hooke's Law and does not break easily, such as a metal
- Graph D is incorrect as the plateau suggests no extra force is required to extend the rubber as it has been stretched

Exam Tip

Exam questions may ask for the total length of a material after a load is placed on it and it has extended. Remember to add the extension to the original length of the material to get its final full length

Hooke's Law

- A material obeys Hooke's Law if **its extension is directly proportional to the applied force (load)**
- The Force v Extension graph is a straight line through the origin (see "Extension and Compression")
- This linear relationship is represented by the Hooke's law equation



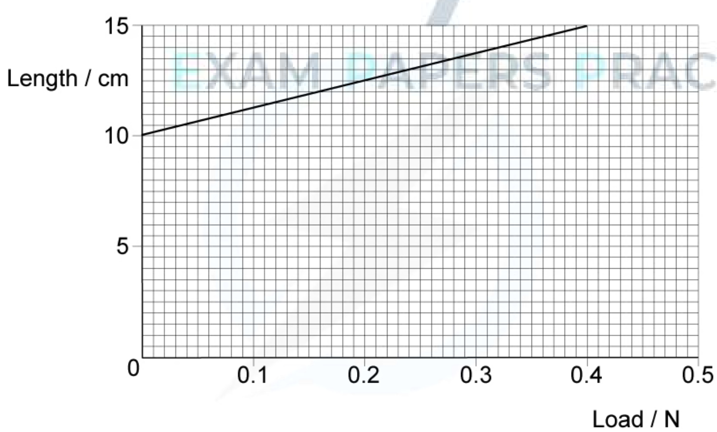
Hooke's Law

- The constant of proportionality is known as the **spring constant *k***

Worked Example

A spring was stretched with increasing load.

The graph of the results is shown below.



What is the spring constant?

STEP 1

REARRANGE FROM HOOKE'S LAW, THE SPRING CONSTANT IS

$$k = \frac{F}{\Delta L}$$

STEP 2

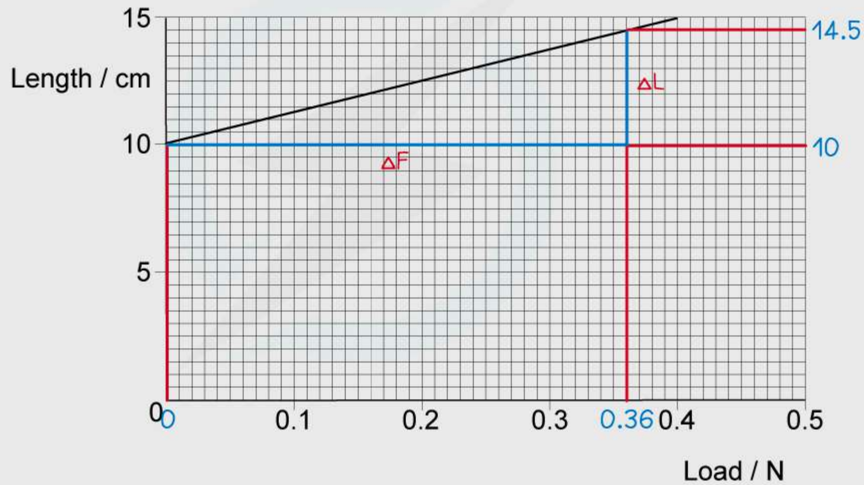
THE GRADIENT OF A FORCE-EXTENSION GRAPH IS THE SPRING CONSTANT

$$k = \frac{\Delta F}{\Delta L}$$

STEP 3

THIS PARTICULAR GRAPH HAS THE LENGTH ON THE y-AXIS AND THE FORCE ON THE x-AXIS.

THEREFORE THE SPRING CONSTANT IS $\frac{1}{\text{GRADIENT}}$



STEP 4

FIND THE GRADIENT

$$\frac{\Delta L}{\Delta F} = \frac{(0.145 - 0.10)\text{m}}{0.36\text{ N}} = \frac{1}{8.0}\text{mN}^{-1}$$

$\text{GRADIENT} = \frac{\Delta y}{\Delta x}$

STEP 5

$\text{SPRING CONSTANT} = \frac{1}{\text{GRADIENT}}$

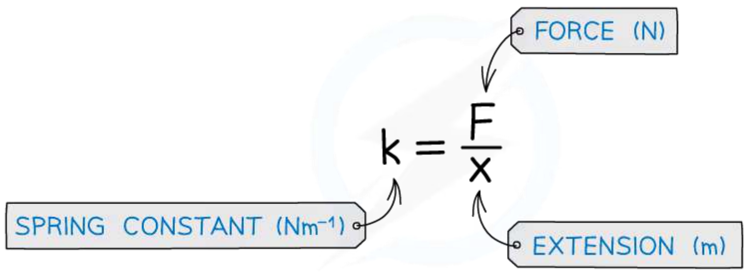
$$1 \div \frac{1}{8.0} = 8.0\text{ Nm}^{-1}$$

Exam Tip

Double check the axes before finding the spring constant as the gradient of a force-extension graph. Exam questions often swap the load onto the x-axis and length on the y-axis. In this case, the gradient is not the spring constant but 1 ÷ gradient is.

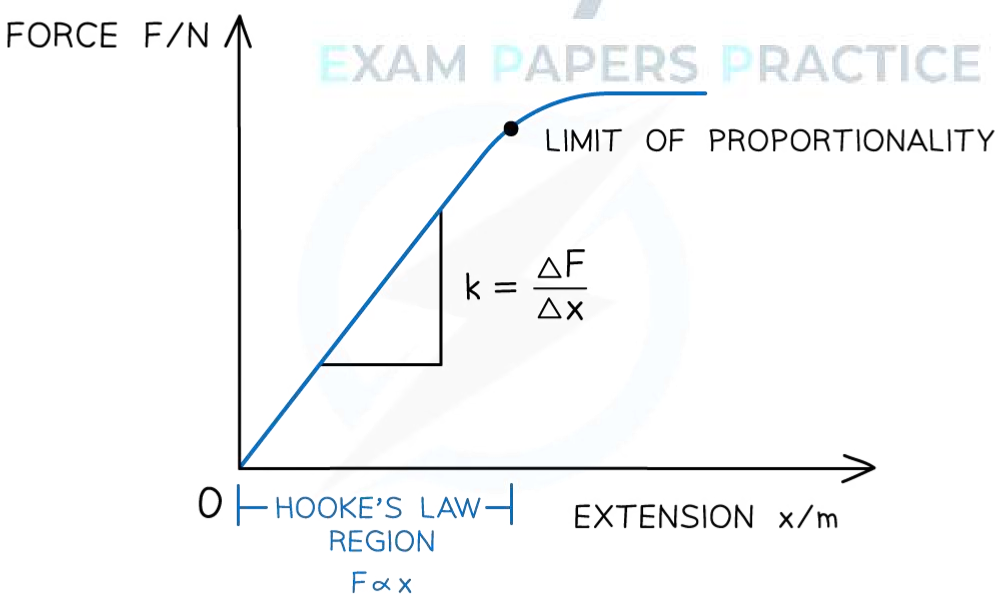
The Spring Constant

- *k* is the **spring constant** of the spring and is a measure of the **stiffness** of a spring
  - A stiffer spring will have a larger value of *k*
- It is defined as the **force per unit extension** up to the limit of proportionality (after which the material will not obey Hooke's law)
- The SI unit for the spring constant is **N m^{-1}**
- Rearranging the Hooke's law equation shows the equation for the spring constant is



Spring constant equation

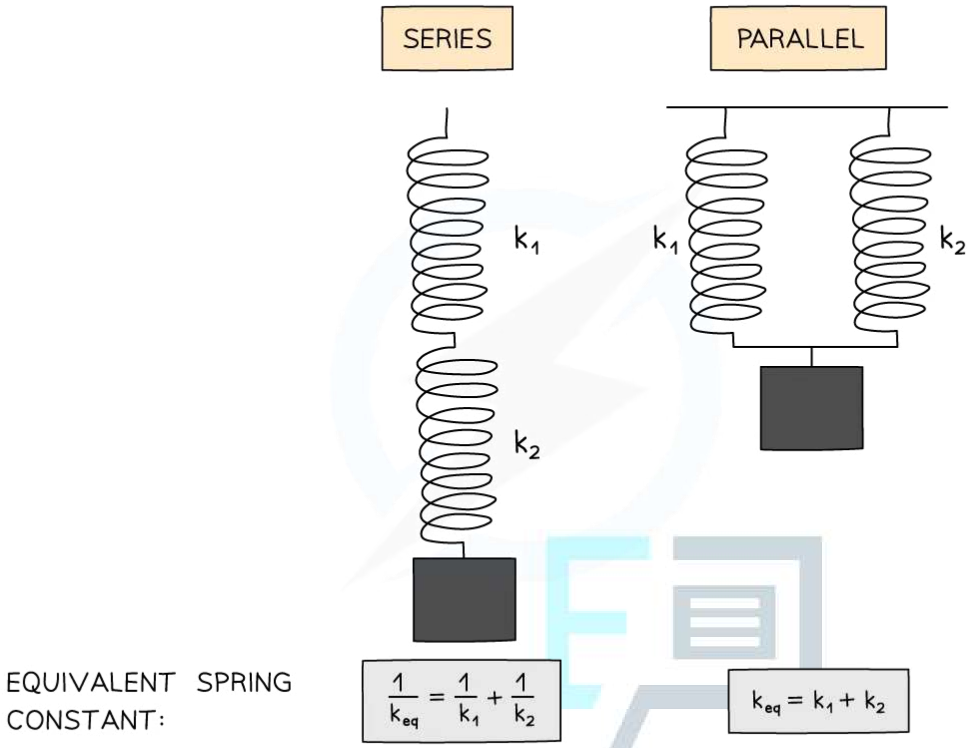
- The spring constant is the **force per unit extension** up to the limit of proportionality (after which the material will not obey Hooke's law)
- Therefore, the spring constant *k* is the gradient of the linear part of a Force v Extension graph



Spring constant is the gradient of a force v extension graph

Combination of springs

- Springs can be combined in different ways
  - In **series** (end-to-end)
  - In **parallel** (side-by-side)

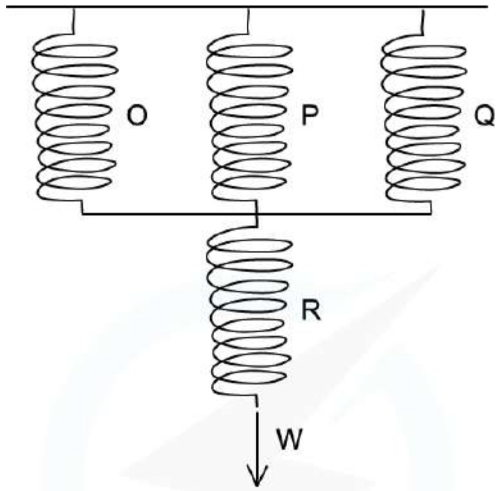


Spring constants for springs combined in series and parallel

- This is assuming k_1 and k_2 are different spring constants
- The equivalent spring constant for combined springs are summed up in different ways depending on whether they're connected in parallel or series

Worked Example

Three springs are arranged vertically as shown.



Springs P,Q and O are identical and have spring constant k. Spring R has spring constant 4k.What is the increase in the overall length of the arrangement when a force W is applied as shown?

A. $\frac{12k}{7W}$ **B.** $\frac{6W}{5k}$ **C.** $\frac{7W}{12k}$ **D.** $\frac{2W}{5}$

ANSWER: C

STEP 1

EQUATION FOR EXTENSION x

REARRANGE FROM HOOKE'S LAW

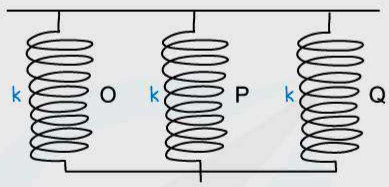
$$x = \frac{F}{k} = \frac{W}{k}$$

STEP 2

FIND THE VALUE OF k:

EQUIVALENT k FROM PARALLEL SPRINGS P AND Q

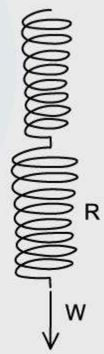
$$k_{OPQ} = k_O + k_P + k_Q = k + k + k = 3k$$



EQUIVALENT k FROM SPRING R AND THE COMBINED SPRINGS P AND Q ARE CONNECTED IN SERIES

$$\frac{1}{k_{OPQR}} = \frac{1}{3k} + \frac{1}{4k} = \frac{7}{12k}$$

$$k_{OPQR} = 1 \div \frac{7}{12k} = \frac{12k}{7}$$



STEP 3

SUBSTITUTE BACK INTO THE EXTENTION EQUATION

$$x = \frac{W}{k_{OPQR}} = W \div \frac{12k}{7} = \frac{7W}{12k}$$

Exam Tip

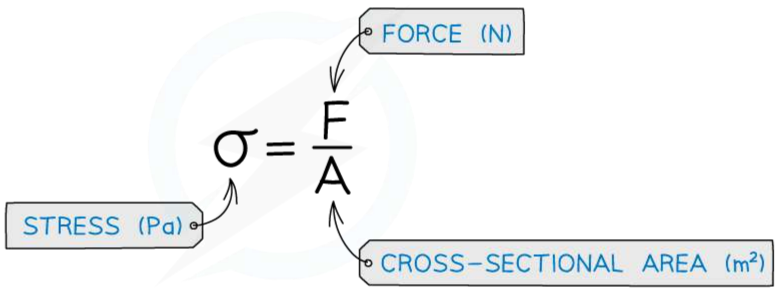
The equivalent (or effective) spring constant equations for combined springs work for any number of springs e.g. if there are 3 springs in parallel k_1 , k_2 and k_3 , the equivalent spring constant would be $k_{eq} = k_1 + k_2 + k_3$.

6.1.3 The Young Modulus

Stress, Strain & the Young Modulus

Stress

- Tensile stress is the **applied force per unit cross sectional area** of a material

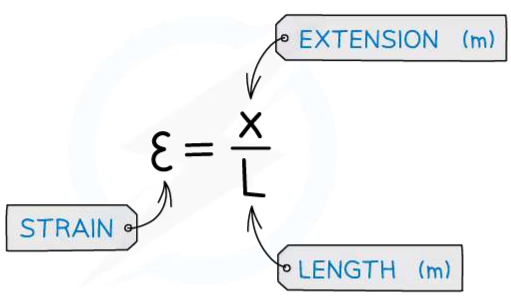


Stress equation

- The **ultimate tensile stress** is the **maximum** force per original cross-sectional area a wire is able to support until it breaks

Strain

- Strain is the **extension per unit length**
- This is a deformation of a solid due to stress in the form of elongation or contraction
- Note that strain is a **dimensionless** unit because it's the ratio of lengths



Strain equation

Young's Modulus

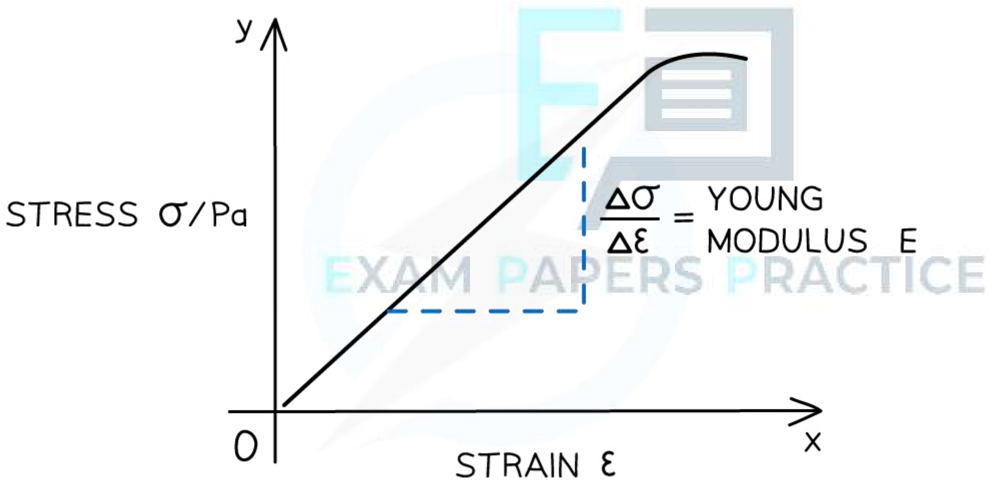
- The Young modulus is the measure of the ability of a material to withstand changes in length with an added load ie. how stiff a material is
- This gives information about the elasticity of a material
- The Young Modulus is defined as the **ratio of stress and strain**

$$\text{YOUNG MODULUS } E = \frac{\text{STRESS } \sigma}{\text{STRAIN } \varepsilon} = \frac{FL}{Ax}$$

(Pa)

Young Modulus equation

- Its unit is the same as stress: **Pa** (since strain is unitless)
- Just like the Force-Extension graph, stress and strain are directly proportional to one another for a material exhibiting elastic behaviour



A stress-strain graph is a straight line with its gradient equal to Young modulus

- The **gradient** of a stress-stress graph when it is linear is the **Young Modulus**

Worked Example

A metal wire that is supported vertically from a fixed point has a load of 92 N applied to the lower end.

The wire has a cross-sectional area of 0.04 mm^2 and obeys Hooke's law.

The length of the wire increases by 0.50%.What is the Young modulus of the metal wire?

A. 4.6×10^7Pa B. 4.6×10^{12} Pa C. 4.6×10^9 Pa D. 4.6×10^{11} Pa

ANSWER: D

STEP 1

YOUNG MODULUS EQUATION

$$E = \frac{\text{STRESS}}{\text{STRAIN}} = \frac{FL}{A\Delta L}$$

STEP 2

CALCULATE STRESS

$$\text{STRESS} = \frac{F}{A} = \frac{92\text{ N}}{0.04 \times 10^{-6}\text{ m}^2} = 2.3 \times 10^9\text{ Pa}$$

$1\text{ mm}^2 = 1 \times 10^{-6}\text{ m}^2$

STEP 3

CALCULATE STRAIN

$$\text{STRAIN} = \frac{\Delta L}{L} = 0.5\% = 0.005$$

EXTENSION

STEP 4

SUBSTITUTE INTO YOUNG MODULUS EQUATION

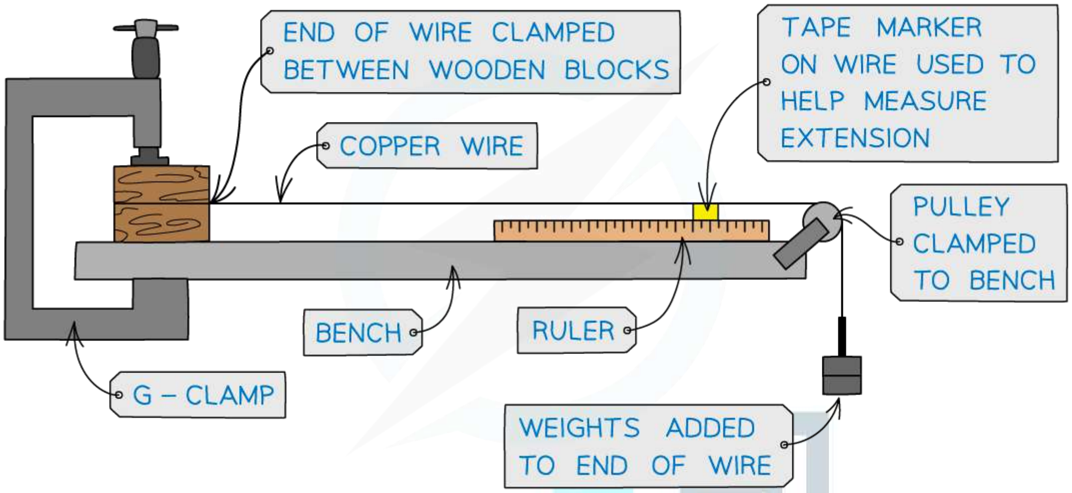
$$E = \frac{\text{STRESS}}{\text{STRAIN}} = \frac{2.3 \times 10^9\text{ Pa}}{0.005} = 4.6 \times 10^{11}\text{ Pa}$$

Exam Tip

To remember whether stress or strain comes first in the Young modulus equation, try thinking of the phrase 'When you're stressed, you show the strain' ie. Stress ÷ strain.

Young's Modulus Experiment

- To measure the Young's Modulus of a metal in the form of a wire requires a clamped horizontal wire over a pulley (or vertical wire attached to the ceiling with a mass attached) as shown in the diagram below



- A reference marker is needed on the wire. This is used to accurately measure the extension with the applied load
- The independent variable is the **load**
- The dependent variable is the **extension**

Method

1. Measure the original length of the wire using a metre ruler and mark this reference point with tape
2. Measure the diameter of the wire with micrometer screw gauge or digital calipers
3. Measure or record the mass or weight used for the extension e.g. 300 g
4. Record initial reading on the ruler where the reference point is
5. Add mass and record the new scale reading from the metre ruler
6. Record final reading from the new position of the reference point on the ruler
7. Add another mass and repeat method

Improving experiment and reducing uncertainties:

- Reduce uncertainty of the cross-sectional area by measuring the diameter d in several places along the wire and calculating an average
- Remove the load and check wire returns to original limit after each reading
- Take several readings with different loads and find average
- Use a Vernier scale to measure the extension of the wire

Measurements to determine Young's modulus

1. Determine extension x from final and initial readings

Example table of results:

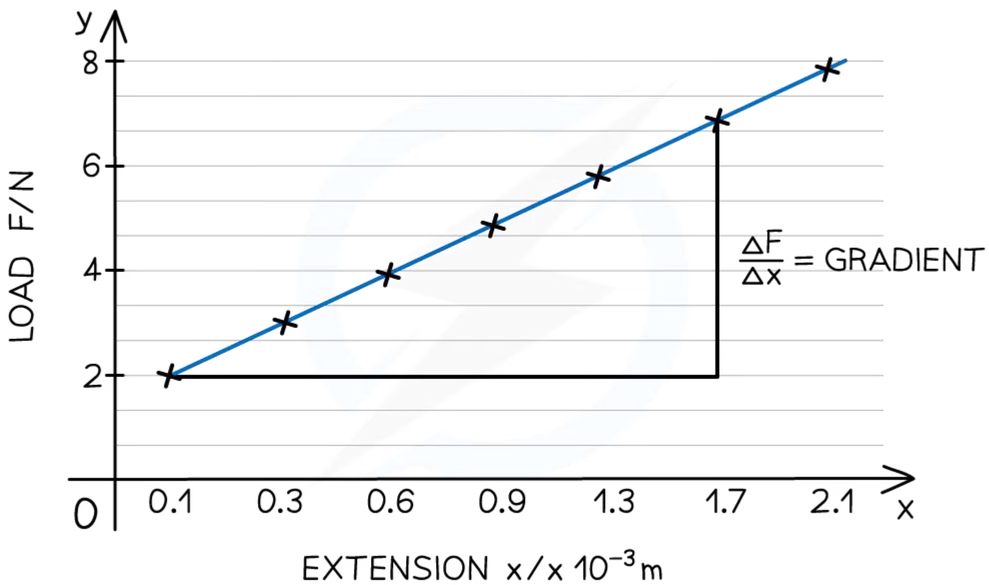
Mass m / g	Load F / N	Initial length / mm	Final length / mm	Extension x ($\times 10^{-3}$) / m
200	2.0	500	500.1	0.1
300	2.9	500.1	500.4	0.3
400	3.9	500.4	501.0	0.6
500	4.9	501.0	501.9	0.9
600	5.9	501.9	503.2	1.3
700	6.9	503.2	504.9	1.7
800	7.8	504.9	507.0	2.1

Table with additional data

Length l / m	1.382
Diameter 1 / mm	0.277
Diameter 2 / mm	0.280
Diameter 3 / mm	0.275
Average Diameter d / mm	0.277
Cross-sectional area A / m^2	6.03×10^{-8}

2. Plot a graph of force against extension and draw line of best fit

3. Determine gradient of the force v extension graph



4. Calculate cross-sectional area from:

DIAMETER OF THE WIRE (m)

$$\text{CROSS-SECTIONAL AREA } A = \frac{\pi d^2}{4}$$

5. Calculate the Young's modulus from:

FORCE / LOAD (N)

LENGTH OF WIRE (m)

$$\text{YOUNG'S MODULUS } E = \frac{\text{STRESS}}{\text{STRAIN}} = \frac{FL}{Ax} = \text{GRADIENT} \times \frac{L}{A}$$

CROSS-SECTIONAL AREA (m^2)

EXTENSION (m)

Exam Tip

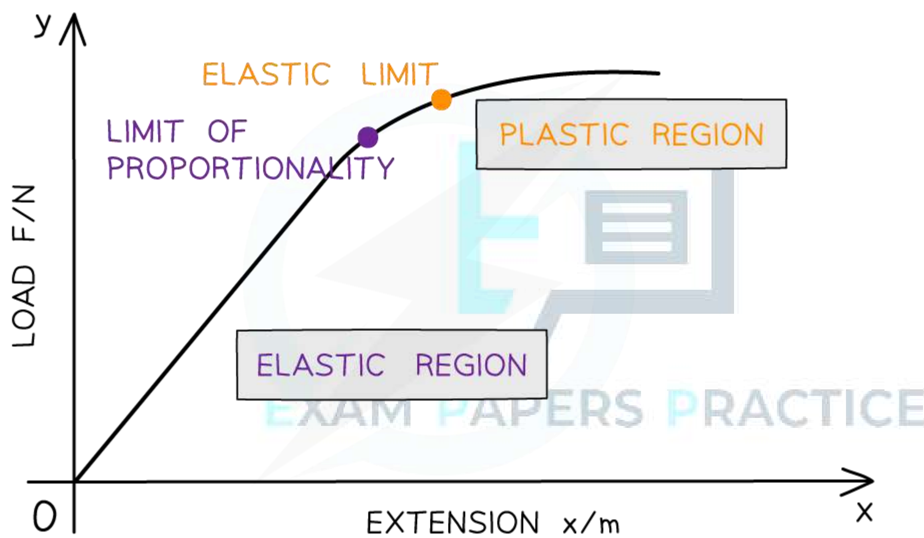
Although every care should be taken to make the experiment as reliable as possible, you will be expected to suggest improvements in producing more accurate and reliable results (e.g. repeat readings and use a longer length of wire)

6.2 Deformation: Elastic & Plastic Behaviour

6.2.1 Elastic & Plastic Behaviour

Elastic & Plastic Deformation

- **Elastic deformation:** when the load is removed, the object **will** return to its original shape
- **Plastic deformation:** when the load is removed, the object **will not** return to its original shape or length. This is beyond the elastic limit
- **Elastic limit:** the point beyond which the object does not return to its original length when the load is removed
- These regions can be determined from a Force-Extension graph:



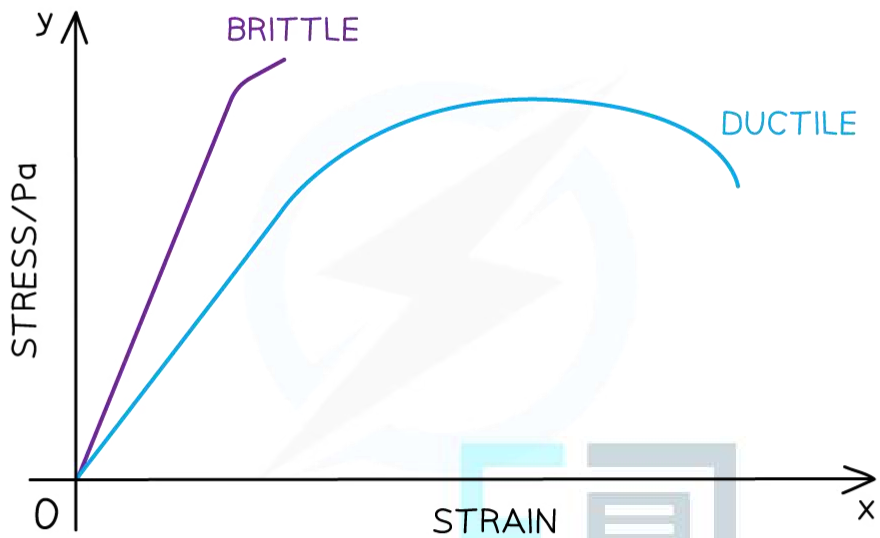
Below the elastic limit, the material exhibits elastic behaviour

Above the elastic limit, the material exhibits plastic behaviour

- Elastic deformation occurs in the 'elastic region' of the graph. The extension is proportional to the force applied to the material (straight line)
- Plastic deformation occurs in the 'plastic region' of the graph. The extension is no longer proportional to the force applied to the material (graph starts to curve)
- These regions are divided by the elastic limit

Brittle and ductile materials

* **Brittle** materials have very little to no plastic region e.g. glass, concrete. The material breaks with little elastic and insignificant plastic deformation
* **Ductile** materials have a larger plastic region e.g. rubber, copper. The material stretches into a new shape before breaking

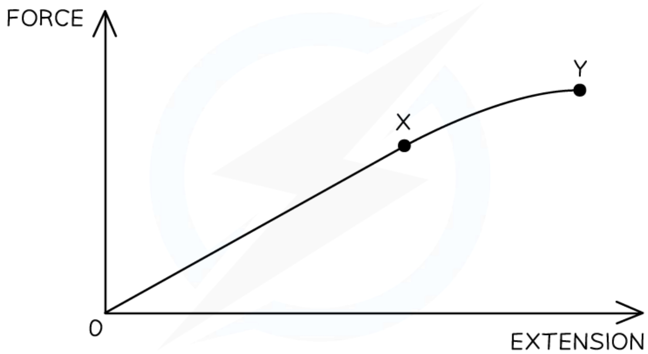


Stress-strain curve for a brittle and ductile material

* To identify these materials on a stress-strain or force-extension graph up to their breaking point:
  * A brittle material is represented by a straight line through the origins with no or negligible curved region
  * A ductile material is represented with a straight line through the origin then curving towards the x-axis

Worked Example

A sample of metal is subjected to a force which increases to a maximum value and then fractures. A force-extension graph for the sample is shown.



What is the behaviour of the metal between X and Y?**A.** both elastic and plastic

B. not elastic and not plastic

C. plastic but not elastic

D. elastic but not plastic

ANSWER: C

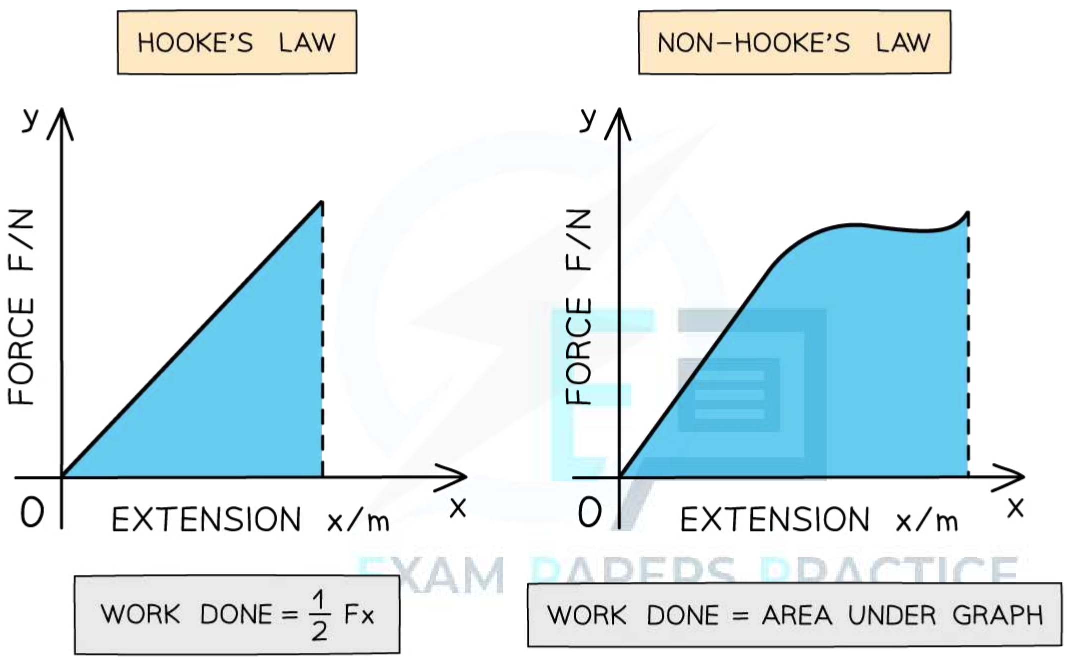
- Since the graph is a straight line and the metal fractures, the point after X must be its elastic limit
- The graph starts to curve after this and fractures at point Y
- This curve between X and Y denotes plastic behaviour
- Therefore, the correct answer is **C**

Exam Tip

Although similar definitions, the elastic limit and limit of proportionality are not the same point on the graph. The limit of proportionality is the point beyond which the material is no longer defined by Hooke's law. The elastic limit is the furthest point a material can be stretched whilst still able to return to its previous shape. This is at a slightly higher extension than the limit of proportionality. Be sure not to confuse them.

Area under a Force-Extension Graph

- The work done in stretching a material is equal to the force multiplied by the distance moved
- Therefore, the **area under a force-extension graph is equal to the work done** to stretch the material
- The work done is also equal to the **elastic potential energy** stored in the material

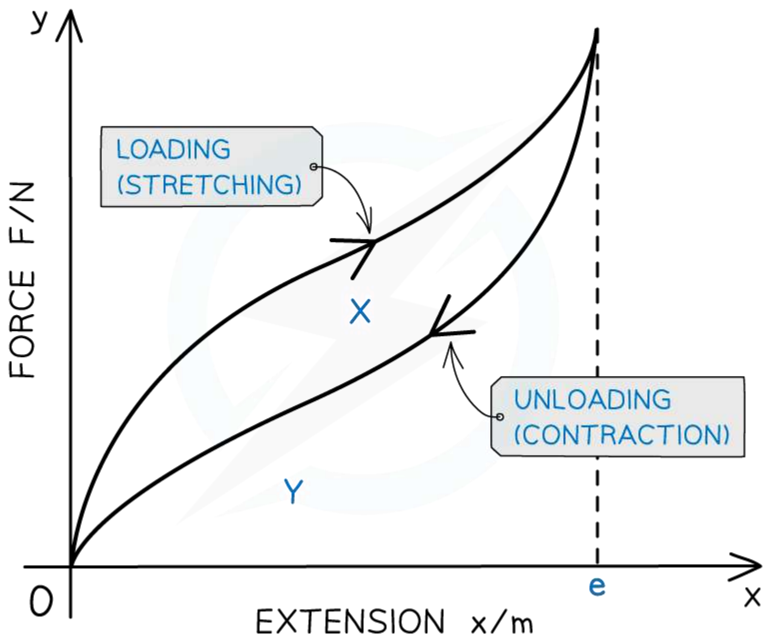


Work done is the area under the force - extension graph

- This is true for whether the material obeys Hooke's law or not
  - For the region where the material obeys Hooke's law, the work done is the area of a right angled triangle under the graph
  - For the region where the material doesn't obey Hooke's law, the area is the full region under the graph. To calculate this area, split the graph into separate segments and add up the individual areas of each

Loading and unloading

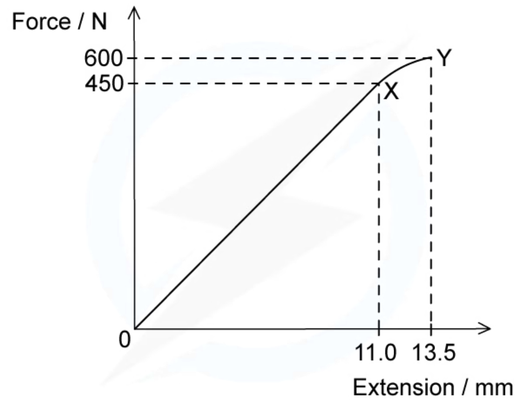
- The force-extension curve for stretching and contraction of a material that has exceeded its elastic limit, but is not plastically deformed is shown below



- The curve for contraction is always below the curve for stretching
- The area **X** represents the **net work done** or the **thermal energy** dissipated in the material
- The area **X + Y** is the **minimum energy required** to stretch the material to extension *e*

Worked Example

The graph shows the behaviour of a sample of a metal when it is stretched until it starts to undergo plastic deformation.



What is the total work done in stretching the sample from zero to 13.5 mm extension?

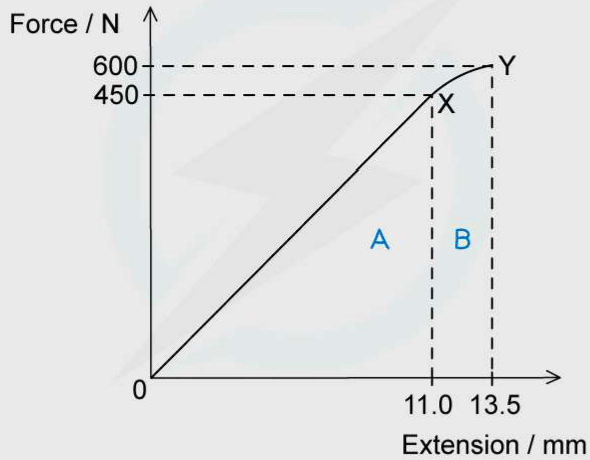
Simplify the calculation by treating the curve XY as a straight line.

STEP 1

WORK DONE = AREA UNDER THE FORCE–EXTENSION GRAPH

STEP 2

SPLIT GRAPH INTO THE TWO AREAS



STEP 3

CALCULATE AREA A

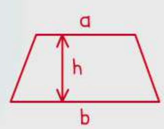
AREA OF A RIGHT ANGLED TRIANGLE = $\frac{1}{2}$ × BASE × HEIGHT

AREA = $\frac{1}{2} \times 11 \times 10^{-3} \times 450 = 2.475$ J

STEP 4

CALCULATE AREA B

AREA OF TRAPEZIUM = $\left(\frac{a+b}{2}\right) \times h$



AREA = $\left(\frac{450+600}{2}\right) \times 2.5 \times 10^{-3} = 1.313$ J

STEP 5

TOTAL AREA = 2.475 + 1.313 = 3.79 J (3 s.f.)

Exam Tip

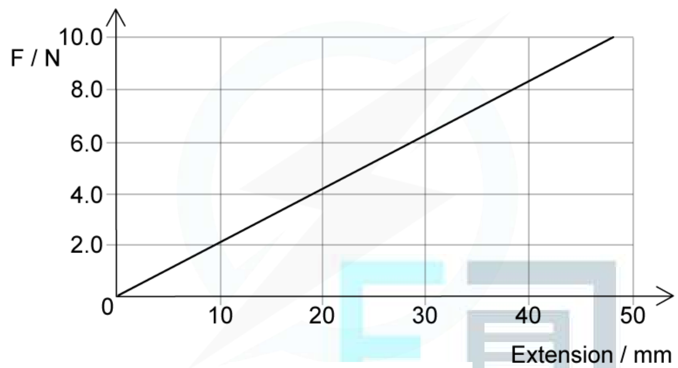
Make sure to be familiar with the formula for the area of common 2D shapes such as a right angled triangle, trapezium, square and rectangles.

Elastic Potential Energy

- Elastic potential energy is defined as the energy stored within a material (e.g. in a spring) when it is stretched or compressed
- It can be found from the **area under the force–extension graph** for a material deformed within its limit of proportionality

Worked Example

A spring is extended with varying forces; the graph below shows the results.



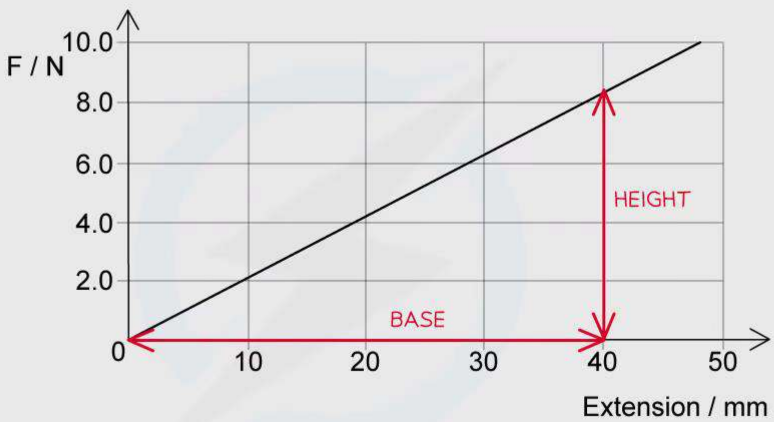
What is the energy stored in the spring when the extension is 40 mm?

STEP 1

ENERGY STORED = AREA UNDER THE GRAPH

STEP 2

CALCULATE AREA UNDER GRAPH FOR EXTENSION OF 40mm



AREA OF TRIANGLE = $\frac{1}{2}$ × BASE × HEIGHT

AREA = $\frac{1}{2} \times 40 \times 10^{-3}\,m \times 8.1\,N = 0.16\,J$

STEP 3

ENERGY STORED = 0.16 J

Calculating Elastic Potential Energy

- A material within it's limit of proportionality obeys Hooke's law. Therefore, for a material obeying Hooke's Law, elastic potential energy can be calculated using:

$$\text{HOOKE'S LAW: } F = kx$$

$$EPE = \frac{1}{2}Fx = \frac{1}{2}(kx)x$$

$$\text{ELASTIC POTENTIAL ENERGY} = \frac{1}{2}kx^2$$

Elastic potential energy can be derived from Hooke's law

- Where k is the **spring constant (N m^{-1})** and x is the **extension (m)**

Exam Tip

The formula for EPE = ½ kx^2 is only the area under the force-extension graph when it is a straight line i.e. when the material obeys Hooke's law and is within its elastic limit.