5.6 Kinematics

Mark Schemes
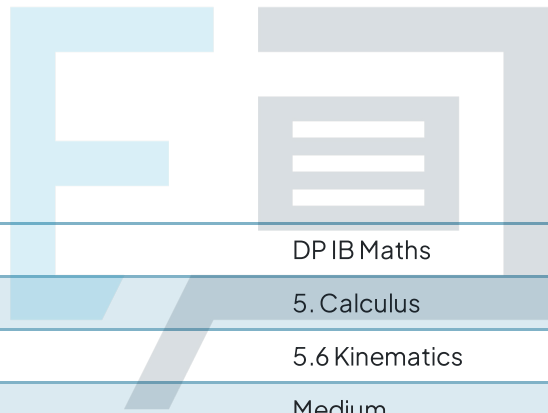
Course	DP IB Maths
Section	5. Calculus
Topic	5.6 Kinematics
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1

a)(i)
> 3200 m is the initial height of the skydiver.

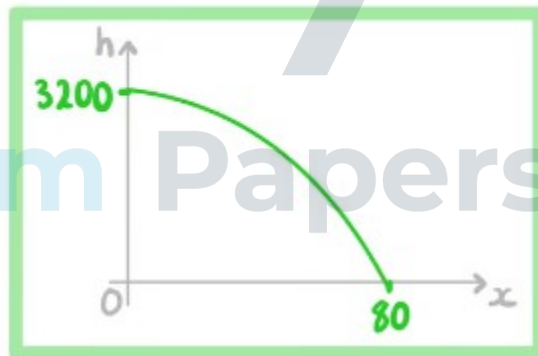(ii) Solve for $h=0$

$3200 - 0.5x^2 = 0$

$0.5x^2 - 3200 = 0$

$x = 80$ or $x = -80$

↑
Reject as $x \geqslant 0$

> Horizontal distance $= 80$ m

(b) This will be a negative parabola with a maximum at $(0, 3200)$ and a root at $(80, 0)$.

(c)
> The skydiver lands on the ground when $x = 80$, so for values of x larger than 80 the height above ground would be 0 m. However the model gives negative values when $x > 80$.

Question 2

(a)(i) $\boxed{5\,m}$

(ii) $\boxed{10\,m}$

b) Velocity is the gradient of the displacement-time graph.

$m = \dfrac{y_2 - y_1}{x_2 - x_1}$ (from formula booklet)

$m = \dfrac{0 - (-10)}{20 - 13}$

$= \dfrac{10}{7}$

$\boxed{Velocity = \dfrac{10}{7}\ ms^{-1}}$

c) Speed is the magnitude of velocity

$Velocity = \dfrac{0 - 10}{10 - 7}$

$= -\dfrac{10}{3}$

$\left| -\dfrac{10}{3} \right| = \dfrac{10}{3}$

$\boxed{Speed = \dfrac{10}{3}\ ms^{-1}}$

(d) Look at the distance for each section:

0-4s   4-7s   7-13s   13-20s

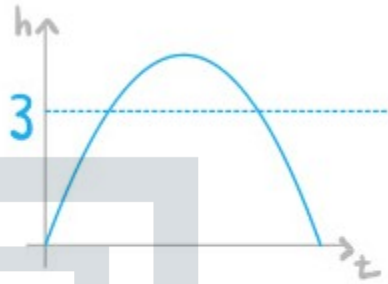10m + 0m + 20m + 10m

Total distance = 40m

Question 3

a) Solve $h = 3$

$13t - 4.9t^2 = 3$

$4.9t^2 - 13t + 3 = 0$

$t = 0.2553...$ $t = 2.3977...$
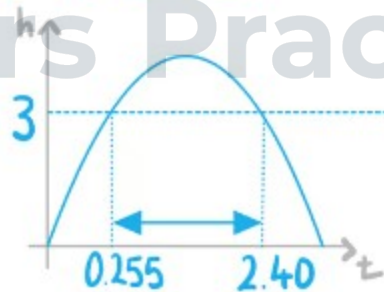
$t = 0.255s$ (3sf) $t = 2.40s$ (3sf)

b) Subtract the times when its at 3m.

$2.3977... - 0.2553...$
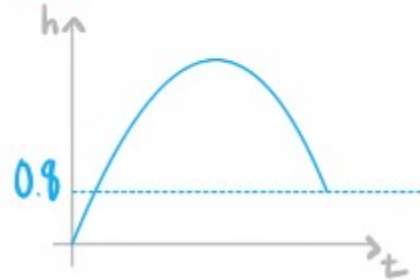
$= 2.142 ...$

2.14 seconds

(c) Solve $h = 0.8$

$13t - 4.9t^2 = 0.8$

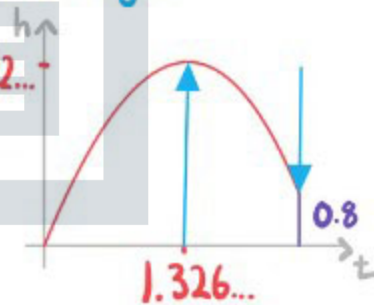$4.9t^2 - 13t + 0.8 = 0$

$t = 0.0630...$ $t = 2.5900...$

Reject as this is
on the way up

$\boxed{2.59 \text{ seconds}}$

(d) Method 1 – Find the maximum height.

Draw the graph on
the calculator and
find the coordinates
of the maximum.

Distance up + Distance down

$8.622... + (8.622... - 0.8)$

$= 16.44...$

$\boxed{16.4 \text{ m}}$

Method 2 – Calculus

Distance $= \int_{t_1}^{t_2} |v(t)| \, dt$ (from formula booklet)

$v(t) = h'(t)$

$v(t) = 13 - 9.8t$

$\int_0^{2.59...} |13 - 9.8t| \, dt = 16.444...$

$\boxed{16.4 \text{ m}}$

e) Velocity is the derivative of displacement.

$v(t) = h'(t)$

$v(t) = 13 - 9.8t$

Substitute $t = 1$

$v(1) = 13 - 9.8(1)$

$\qquad = 3.2$

$\boxed{v(1) = 3.2 \text{ ms}^{-1}}$

Question 4

(a) $t = 0 \Rightarrow h = H$

H is the initial height of the ball.

$\boxed{H = 10}$

(b) Substitute $t = 2$.

$h(2) = 10 + 7.8(2) - 4.9(2)^2$

$\boxed{h = 6m}$
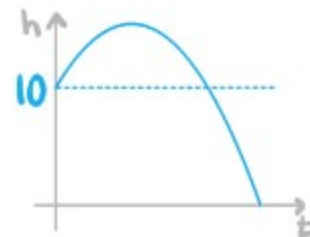
c) Solve $h(t) = 10$

$10 + 7.8t - 4.9t^2 = 10$

$4.9t^2 - 7.8t = 0$

$t = 0 \qquad t = \frac{78}{49} = 1.5918\ldots$

Reject as this
was when thrown

$\boxed{t = 1.59 s \text{ (3sf)}}$
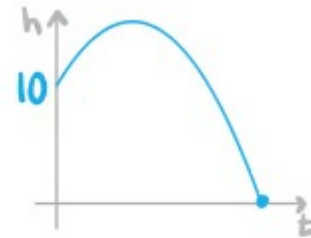
d) Solve $h(t) = 0$

$10 + 7.8t - 4.9t^2 = 0$

$4.9t^2 - 7.8t - 10 = 0$

$t = -0.8394...$ $t = 2.4312...$

Reject as time can't be negative

$\boxed{t = 2.43\,s}$

e) Differentiate twice

$h'(t) = 7.8 - 9.8t$

$\boxed{h''(t) = -9.8}$

Acceleration $a = \dfrac{d^2s}{dt^2}$ (from formula booklet)

$a(t) = h''(t)$

$\boxed{a(t) = -9.8 \text{ for all } t}$

Question 5

a) $a = \dfrac{dv}{dt}$ (from formula booklet)

Using the derivative function on the calculator.

$a = \dfrac{d}{dt}(2^t - 2)\Big|_{t=2}$

$= 2.7725...$

$\boxed{a(2) = 2.77\,ms^{-2} \ (3sf)}$

(b) Rest $\Rightarrow v = 0$

$2^t - 2 = 0$

$2^t = 2$

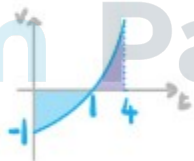$\boxed{t = 1 \text{ s}}$

c) Method 1 - Using formula booklet

Distance $= \int_{t_1}^{t_2} |v(t)| \, dt$ (from formula booklet)

Distance $= \int_0^4 |2^t - 2| \, dt$

$= 14.755...$

$\boxed{\text{Distance} = 14.8 \text{ m (3sf)}}$

Method 2 - Using a velocity-time graph

Displacement $= \int_{t_1}^{t_2} v(t) \, dt$ (from formula booklet)

0-1 s $\int_0^1 2^t - 2 \, dt = -0.5573...$

1-4 s $\int_1^4 2^t - 2 \, dt = 14.1977...$

Distance is magnitude of displacement

Total distance $= |-0.5573...| + 14.1977...$

$= 14.755...$

$\boxed{\text{Distance} = 14.8 \text{ m (3sf)}}$

Question 6

a)

$$a = \frac{dv}{dt} \quad \text{(from formula booklet)}$$

$$v = \int a \, dt$$

$$v = \int \frac{1}{t^2} + \sin t \, dt$$

$$v = \int t^{-2} + \sin t \, dt$$

$$v = -t^{-1} - \cos t + c$$

$$v(1) = 1$$

$$1 = -(1)^{-1} - \cos 1 + c$$

$$c = 1 + 1 + \cos 1 \quad \leftarrow \text{ Make sure calculator is in}$$

$$c = 2.5403... \qquad \text{radians.}$$

$$\boxed{v = -\frac{1}{t} - \cos t + 2.54}$$

b) Substitute $t = 2$.

$$v(2) = -\frac{1}{2} - \cos 2 + 2.54...$$

$$= 2.456... \qquad \text{Make sure calculator is in}$$
$$\text{radians.}$$

$$\boxed{v(2) = 2.46 \, ms^{-1} \ (3sf)}$$

Question 7

(a) Rest $\Rightarrow$ v = 0

$\sin t + \cos 2t = 0$

Solve using calculator (in radians).

$t = 1.5707...$ $t = 3.6651...$

$\boxed{t = 1.57s \ (3sf) \quad t = 3.67s \ (3sf)}$

b) Sketch the velocity-time graph

1.57 → Still moves forward

3.67 → Changes direction
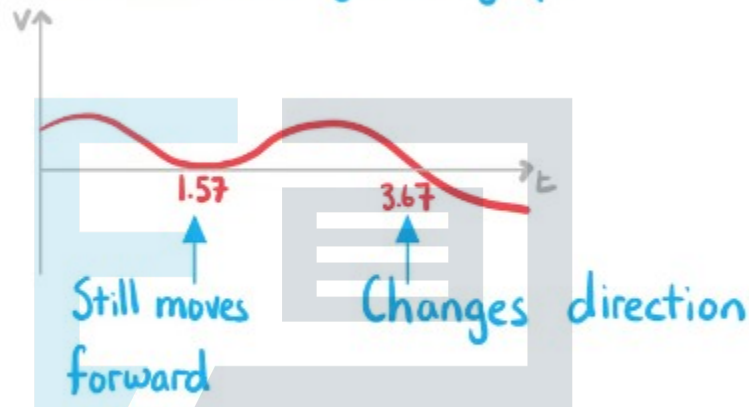
$\boxed{t = 3.67s \ (3sf)}$

(c) Distance $= \int_{t_1}^{t_2} |v(t)| \, dt$ (from formula booklet)

Distance $= \int_0^1 |\sin t + \cos 2t| \, dt$

$= 0.9143...$

$\boxed{\text{Distance} = 0.914 \ (3sf)}$

(d) $a = \dfrac{dv}{dt}$ (from formula booklet)

<u>Method 1</u> – Using calculator

$a = \dfrac{d}{dt}(\sin t + \cos 2t)\Big|_{t=3.665\ldots}$

$= -2.5977\ldots$

$\boxed{a = -2.60 \text{ ms}^{-2} \text{ (3sf)}}$

<u>Method 2</u> – By hand

$a(t) = \cos t - 2\sin 2t$

$a(3.665\ldots) = -2.5977\ldots$

$\boxed{a = -2.60 \text{ ms}^{-2} \text{ (3sf)}}$

e) Displacement = $\int_{t_1}^{t_2} v(t) dt$ (from formula booklet)

Displacement = $\int_0^5 \sin t + \cos 2t \, dt$

Method 1 - Using calculator
$\int_0^5 \sin t + \cos 2t \, dt = 0.4443...$

Displacement = 0.444m (3sf)

Method 2 - By hand
$\int_0^5 \sin t + \cos 2t \, dt = \left[-\cos t + \frac{1}{2}\sin 2t \right]_0^5$

$= \left(-\cos 5 + \frac{1}{2}\sin 10\right) - \left(-\cos 0 + \frac{1}{2}\sin 0\right)$

$= -0.5556... \quad - (-1)$

$= 0.4443...$

Displacement = 0.444m (3sf)