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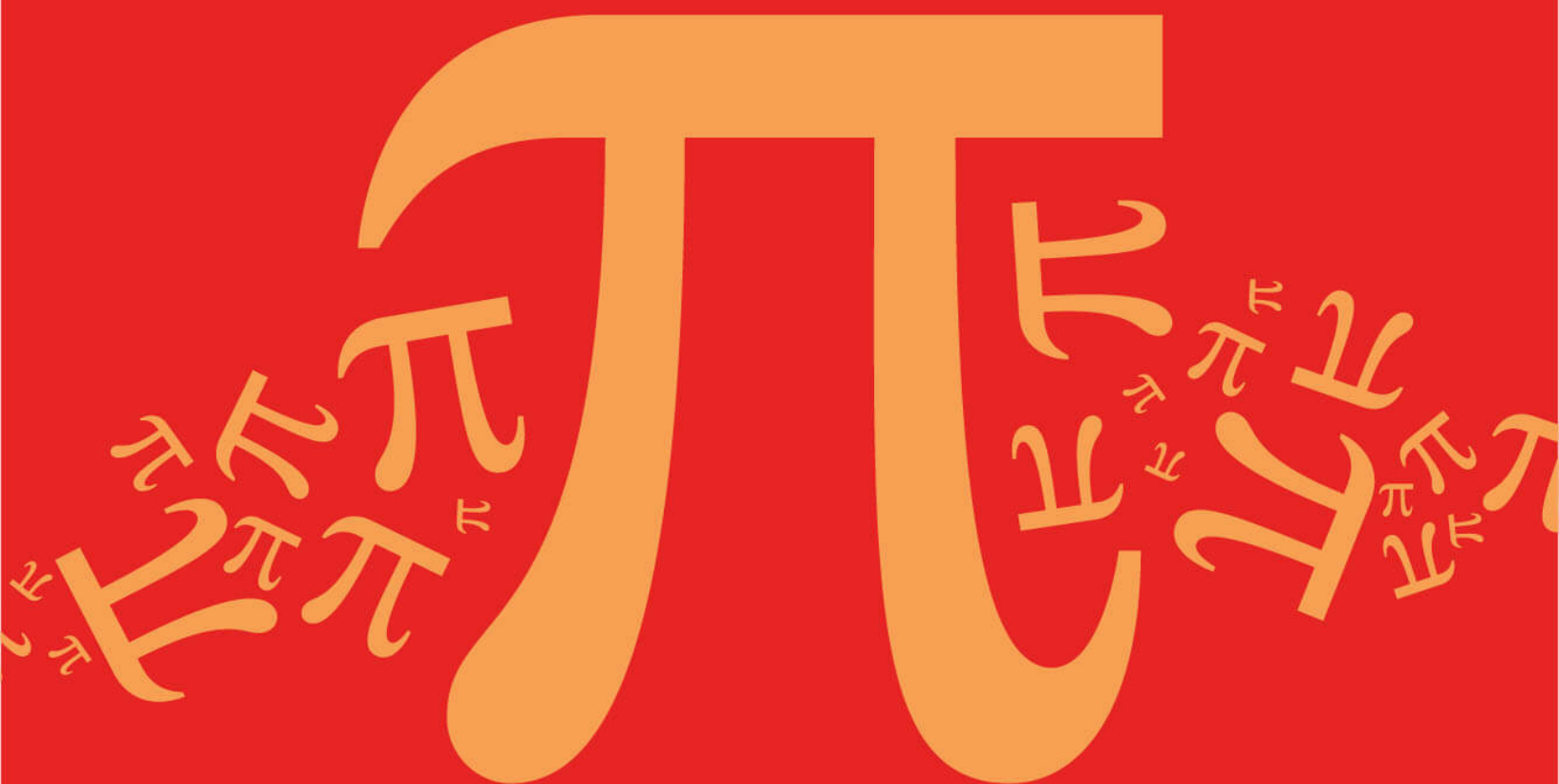
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## 5.6 Kinematics



# IB Maths - Revision Notes

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**AA HL**

## 5.6.1 Kinematics Toolkit

### Displacement, Velocity & Acceleration

#### What is kinematics?

- **Kinematics** is the branch of mathematics that models and analyses the **motion** of objects
- Common words such as **distance**, **speed** and **acceleration** are used in kinematics but are used according to their technical definition

#### What definitions do I need to be aware of?

- Firstly, only motion of an object in a **straight line** is considered
  - this could be a **horizontal** straight line
    - the **positive** direction would be to the **right**
  - or this could be a **vertical** straight line
    - the **positive** direction would be **upwards**

#### Particle

- A **particle** is the general term for an **object**
  - some questions may use a **specific** object such as a **car** or a **ball**

#### Time $t$ seconds

- **Displacement**, **velocity** and **acceleration** are all **functions** of time  $t$
- **Initially** time is zero  $t = 0$

#### Displacement $s$ m

- The **displacement** of a particle is its **distance relative** to a **fixed point**
  - the fixed point is often (but not always) the particle's **initial position**

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- **Displacement** will be **zero**  $s = 0$  if the object is at or has returned to its initial position
- **Displacement** will be negative if its **position relative** to the **fixed point** is in the **negative direction** (left or down)

#### Distance $d$ m

- Use of the word **distance** needs to be considered carefully and could refer to
  - the distance **travelled** by a particle
  - the **(straight line)** distance the particle is from a **particular point**
- Be careful not to confuse **displacement** with **distance**
  - if a bus route starts and ends at a bus depot, when the bus has returned to the depot, its **displacement** will be **zero** but the distance the bus has travelled will be the length of the route

- **Distance** is always **positive**

**Velocity**  $v \text{ ms}^{-1}$

- The **velocity** of a particle is the **rate of change** of its **displacement** at time  $t$
- **Velocity** will be **negative** if the **particle** is moving in the **negative direction**
- A **velocity** of **zero** means the particle is **stationary**  $v = 0$

**Speed**  $|v| \text{ ms}^{-1}$

- **Speed** is the **magnitude** (a.k.a. absolute value or modulus) of **velocity**
  - as the particle is **moving** in a **straight line**, **speed** is the **velocity ignoring the direction**
    - if  $v = 4$ ,  $|v| = 4$
    - if  $v = -6$ ,  $|v| = 6$

**Acceleration**  $a \text{ ms}^{-2}$

- The **acceleration** of a particle is the **rate of change** of its **velocity** at time  $t$
- Acceleration can be **negative** but this alone cannot fully describe the particle's motion
  - if **velocity** and **acceleration** have the **same** sign the particle is **accelerating** (speeding up)
  - if **velocity** and **acceleration** have **different** signs then the particle is **decelerating** (slowing down)
  - if **acceleration** is **zero**  $a = 0$  the particle is moving with **constant** velocity
  - in all cases the **direction of motion** is determined by the **sign of velocity**

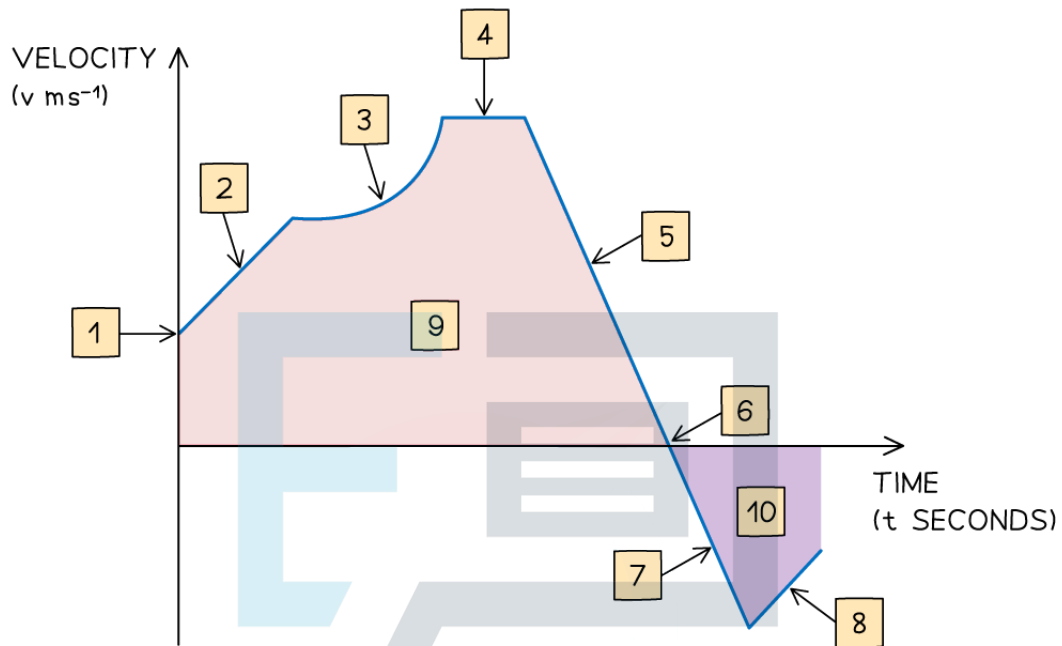
**Are there any other words or phrases in kinematics I should know?**

- Certain words and phrases can imply values or directions in kinematics
  - a particle described as "at **rest**" means that its velocity is zero,  $v = 0$
  - a particle described as moving "**due east**" or "**right**" or would be moving in the **positive horizontal** direction
    - this also means that  $v > 0$
  - a particle "**dropped from the top of a cliff**" or "**down**" would be moving in the **negative vertical** direction
    - this also means that  $v < 0$

**What are the key features of a velocity-time graph?**

- The **gradient** of the graph equals the **acceleration** of an object
- A **straight line** shows that the object is **accelerating** at a **constant rate**
- A **horizontal** line shows that the object is moving at a **constant velocity**
- The **area** between graph and the x-axis tells us the **change in displacement** of the object
  - Graph **above** the x-axis means the object is moving **forwards**
  - Graph **below** the x-axis means the object is moving **backwards**

- The **total displacement** of the object from its starting point is the sum of the **areas above** the x-axis **minus** the sum of the **areas below** the x-axis
- The **total distance travelled** by the object is the sum of **all the areas**
- If the graph **touches** the **x-axis** then the object is **stationary** at that time
- If the graph is **above** the **x-axis** then the object has positive velocity and is **travelling forwards**
- If the graph is **below** the **x-axis** then the object has negative velocity and is **travelling backwards**



- |   |   |    |   |
|---|---|----|---|
| 1 | INITIAL VELOCITY                                      | 7  | SPEEDING UP BUT MOVING BACKWARDS        |
| 2 | CONSTANT ACCELERATION                                 | 8  | SLOWING DOWN BUT STILL MOVING BACKWARDS |
| 3 | VARIABLE ACCELERATION                                 | 9  | DISTANCE TRAVELLED FORWARDS             |
| 4 | CONSTANT VELOCITY                                     | 10 | DISTANCE TRAVELLED BACKWARDS            |
| 5 | DECELERATING (SLOWING DOWN BUT STILL MOVING FORWARDS) |    |   |
| 6 | INSTANTANEOUSLY AT REST (STATIONARY FOR AN INSTANT)   |    |   |

### 💡 Exam Tip

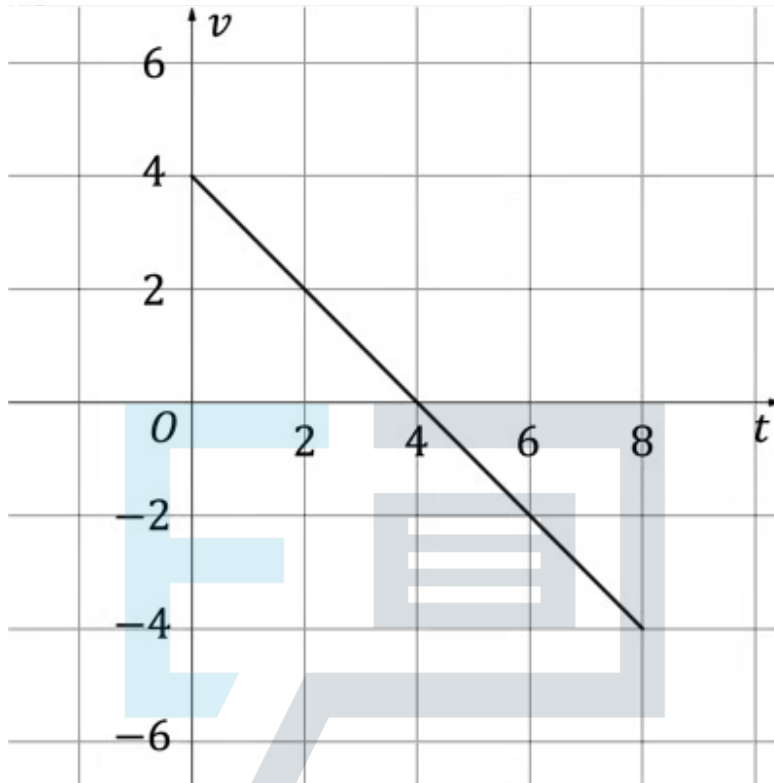
- In an exam if you are given an expression for the velocity then sketching a velocity-time graph can help visualise the problem



### Worked example

A particle is projected vertically upwards from ground level, taking 8 seconds to return to the ground.

The velocity-time graph below illustrates the motion of the particle for these 8 seconds.



- i) How many seconds does the particle take to reach its maximum height?  
Give a reason for your answer.
- ii) State, with a reason, whether the particle is accelerating or decelerating at time  $t = 3$ .

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i. At maximum height, velocity is zero

$$v = 0 \text{ at } t = 4$$

∴ The particle takes 4 seconds to reach its maximum height. This is because its velocity is  $0 \text{ m s}^{-1}$  at 4 seconds.

- ii. At  $t = 3$ , velocity is POSITIVE  
Acceleration is the gradient of velocity  
At  $t = 3$ , acceleration is NEGATIVE

∴ At 3 seconds the particle is decelerating as its velocity and acceleration have different signs.



## 5.6.2 Calculus for Kinematics

### Differentiation for Kinematics

#### How is differentiation used in kinematics?

- **Displacement, velocity and acceleration** are related by calculus
- In terms of differentiation and derivatives
  - **velocity** is the **rate of change** of **displacement**
    - $v = \frac{ds}{dt}$  or  $v(t) = s'(t)$
  - **acceleration** is the **rate of change** of **velocity**
    - $a = \frac{dv}{dt}$  or  $a(t) = v'(t)$
  - so **acceleration** is also the **second derivative** of **displacement**
    - $a = \frac{d^2s}{dt^2}$  or  $a(t) = s''(t)$
- If a graph is not given you can use your GDC to draw one
  - you can then use your GDC's graphing features to find **gradients**
    - **velocity** is the **gradient** on a **displacement** (-time) graph
    - **acceleration** is the **gradient** on a **velocity** (-time) graph

#### Worked example

The displacement,  $s$  m, of a particle at  $t$  seconds, is modelled by  $s(t) = 2t^3 - 27t^2 + 84t$

- i. Find  $v(t)$  and  $a(t)$ .
- ii. Find the times at which the particle is at rest.

$$i. \quad v(t) = s'(t) = 6t^2 - 54t + 84 = 6(t^2 - 9t + 14)$$

$$a(t) = v'(t) = 12t - 54 = 6(2t - 9)$$

$$\therefore v(t) = 6(t-7)(t-2)$$

$$a(t) = 6(2t-9)$$

It's not essential to factorise the final answers

- ii. The particle is at rest when  $v(t) = 0$ 

$$6(t-7)(t-2) = 0$$

$$t = 7, t = 2$$

$\therefore$  The particle is at rest at 2 seconds and 7 seconds

## Integration for Kinematics

### How is integration used in kinematics?

- Since **velocity** is the **derivative** of **displacement** ( $v = \frac{ds}{dt}$ ) it follows that

$$s = \int v dt$$

- Similarly, **velocity** will be an **antiderivative** of **acceleration**

$$v = \int a dt$$

### How would I find the constant of integration in kinematics problems?

- A **boundary** or **initial** condition would need to be known
  - phrases involving the word “**initial**”, or “**initially**” are referring to **time** being **zero**, i.e.  $t = 0$
  - you might also be given information about the object at some other time (this is called a **boundary condition**)
  - substituting** the values in from the **initial or boundary condition** would allow the **constant of integration** to be found

### How are definite integrals used in kinematics?

- Definite integrals can be used to find the displacement of a particle between two points in time

- $\int_{t_1}^{t_2} v(t) dt$  would give the **displacement** of the particle **between** the times  $t = t_1$  and

$$t = t_2$$

- This can be found using a velocity-time graph by **subtracting** the **total area below** the horizontal axis from the **total area above**

- $\int_{t_1}^{t_2} |v(t)| dt$  gives the **distance** a particle has **travelled** between the times  $t = t_1$  and

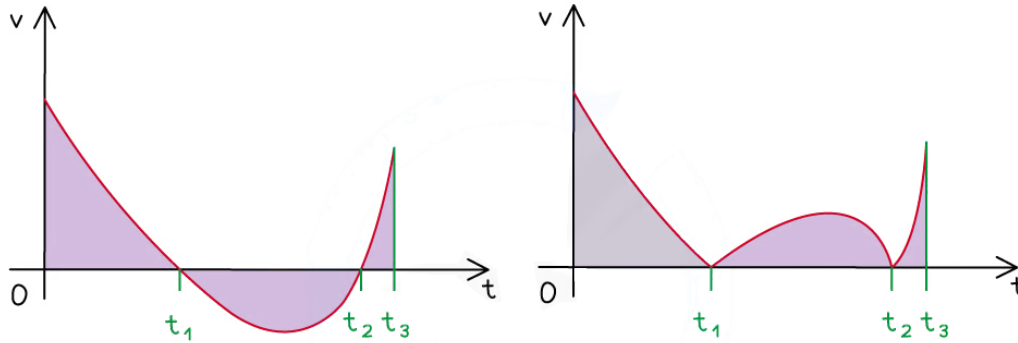
$$t = t_2$$

- This can be found using a velocity-time graph by **adding** the **total area below** the horizontal axis to the **total area above**

- Use a GDC to plot the modulus graph  $y = |v(t)|$

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$\int_0^{t_3} v(t) dt$  IS THE  
DISPLACEMENT OF THE  
PARTICLE FROM ITS INITIAL  
POSITION AT TIME  $t_3$

$\int_0^{t_3} |v(t)| dt$  IS THE  
DISTANCE THE PARTICLE  
HAS TRAVELLED AT TIME  $t_3$

**Exam Tip**

- Sketching the velocity-time graph can help you visualise the distances travelled using areas between the graph and the horizontal axis

**Worked example**

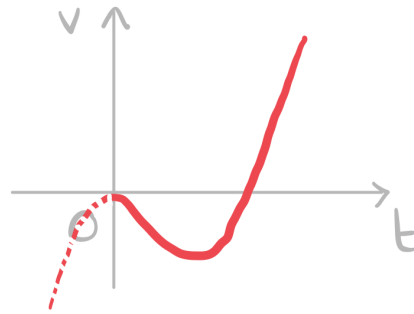
A particle moving in a straight horizontal line has velocity ( $v \text{ m s}^{-1}$ ) at time  $t$  seconds modelled by  $v(t) = 8t^3 - 12t^2 - 2t$ .

- Given that the initial position of the particle is at the origin, find an expression for its displacement from the origin at time  $t$  seconds.
- Find the displacement of the particle from the origin in the first five seconds of its motion.
- Find the distance travelled by the particle in the first five seconds of its motion.





Use your GDC to sketch a velocity(-time) graph and use it to check to see if your answers are sensible.



i. "initial" -  $t=0$ , "origin" -  $s=0$

$$s(t) = \int v(t) dt = \int (8t^3 - 12t^2 - 2t) dt$$

$$s(t) = 2t^4 - 4t^3 - t^2 + c$$

$$\text{at } t=0, s=0, \therefore c=0$$

$$\therefore s(t) = 2t^4 - 4t^3 - t^2$$

where  $c$  is a constant

ii. "first five seconds" -  $t_1=0, t_2=5$

Using a GDC this would be

$$s = \int_0^5 (8t^3 - 12t^2 - 2t) dt$$

$$s = 725 \text{ m}$$

iii. Using a GDC this would be

$$d = \int_0^5 |8t^3 - 12t^2 - 2t| dt$$

$d$  for distance

$$d = 736.734020\dots$$

$$\therefore d = 737 \text{ m (3 s.f.)}$$