5.4 Further Integration

Question Paper
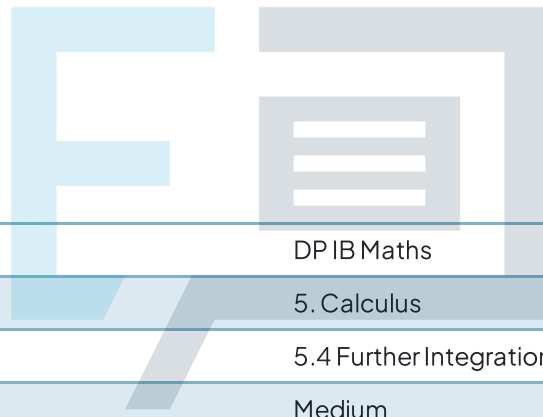
Course	DP IB Maths
Section	5. Calculus
Topic	5.4 Further Integration
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1a

Find the indefinite integral

$$\int \sin x \ \mathrm{d}x$$

[1 mark]

Question 1b

Find the exact value for

$$\int_1^4 \frac{1}{x} \ \mathrm{d}x$$

[3 marks]

Question 1c

Find the indefinite integral for

$$y = \int 7e^{7x} \ dx$$

[2 marks]

Question 2a

Integrate

$$\int \cos 2x \ \mathrm{d}x$$

[2 marks]

Question 2b

Find the definite integral

$$\int_0^2 (3x - 1)^3 \, dx$$

[4 marks]

Question 2c

Find an expression for y given that

$$\frac{dy}{dx} = e^{5x}$$

[2 marks]

Question 3

Using a suitable substitution, show that

$$\int_1^2 \frac{x}{x+4} \, dx = 1 + 4\ln\frac{5}{6}$$

[7 marks]

Question 4

Given that

$$\cos 2\theta \equiv 2\cos^2 \theta - 1$$

use calculus to find the exact value of

$$\int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \cos^2 \theta \ \mathrm{d}\theta$$
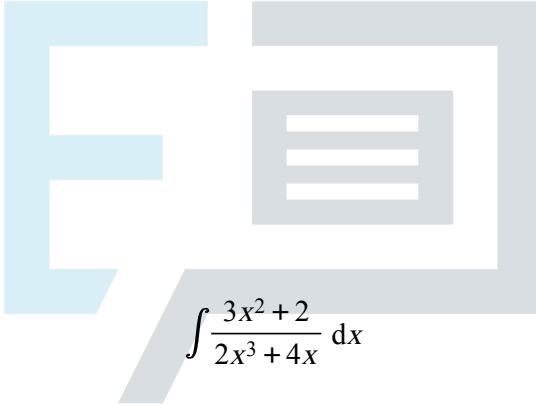
[6 marks]

Question 5a

Given that $f(x) = 2x^3 + 4x,$ find $f'(x)$.

[2 marks]

Question 5b

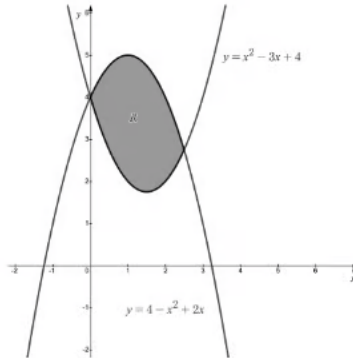Hence, or otherwise, find

$$\int \frac{3x^2 + 2}{2x^3 + 4x} \, dx$$

[4 marks]

Question 6a

The diagram below shows a sketch of the curves with equations

$$y = x^2 - 3x + 4 \text{ and } y = 4 - x^2 + 2x$$

Find the x-coordinates of the intersections of the two graphs.

[2 marks]

Question 6b

Show that the area of the shaded region labelled R is given by

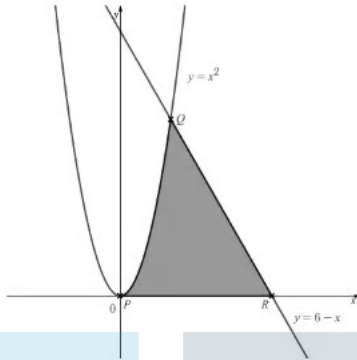$$\int_0^{\frac{5}{2}} \left(5x - 2x^2\right) \, \mathrm{d}x$$

[2 marks]

Question 6c

Use calculus to find the area of the shaded region labelled R.

[2 marks]

Question 7a

The diagram below shows the graphs of the line $y = 6 - x$ and the curve $y = x^2$.

Work out the x-coordinates of the points labelled P, Q and R.

[3 marks]

Question 7b

Work out the area of the shaded region.

[4 marks]

Question 8a
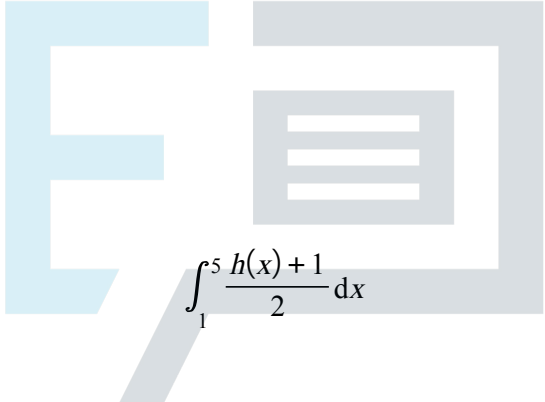
Consider the function $h(x)$ such that

$$\int_1^5 h(x)\mathrm{d}x = 2.$$

Find

$$\int_5^1 h(x)\,\mathrm{d}x$$

[2 marks]

Question 8b

Find

$$\int_1^5 \frac{h(x) + 1}{2}\,\mathrm{d}x$$

[3 marks]

Question 8c

Find

$$\int_1^5 (h(x) + 2x)\mathrm{d}x$$

[3 marks]

Question 9a

Consider the function $f(x) = \ln(2x^2 + 1)$.

Find $f'(x)$.

[3 marks]

Question 9b

Hence, find

$$\int \frac{x}{2x^2 + 1}\,dx$$

[3 marks]

Question 10

Let $f'(x) = x^2 \cos(x^3 + 1)$.

Find $f(x)$ given that $f(-1) = 1$.