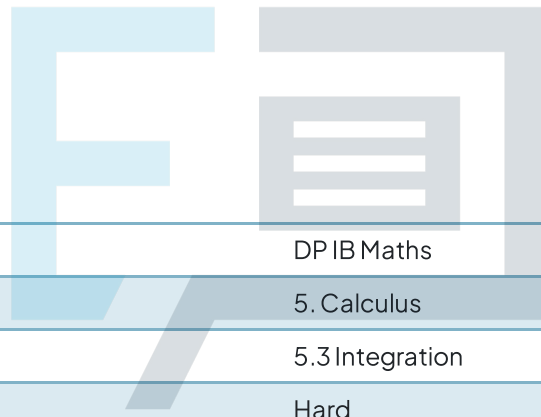




# 5.3 Integration

## Mark Schemes



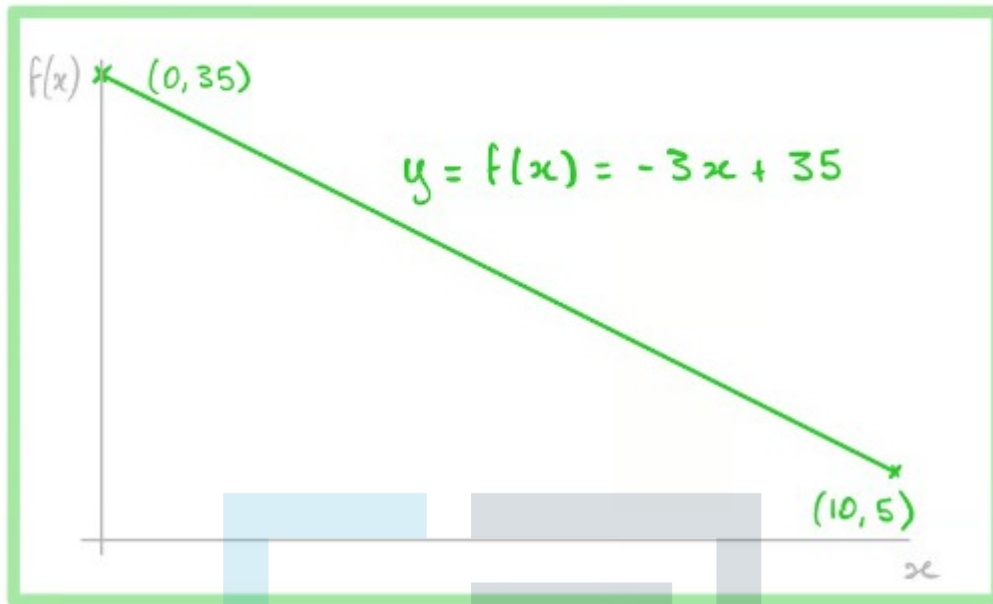
Course	DPIB Maths
Section	5. Calculus
Topic	5.3 Integration
Difficulty	Hard

# Exam Papers Practice

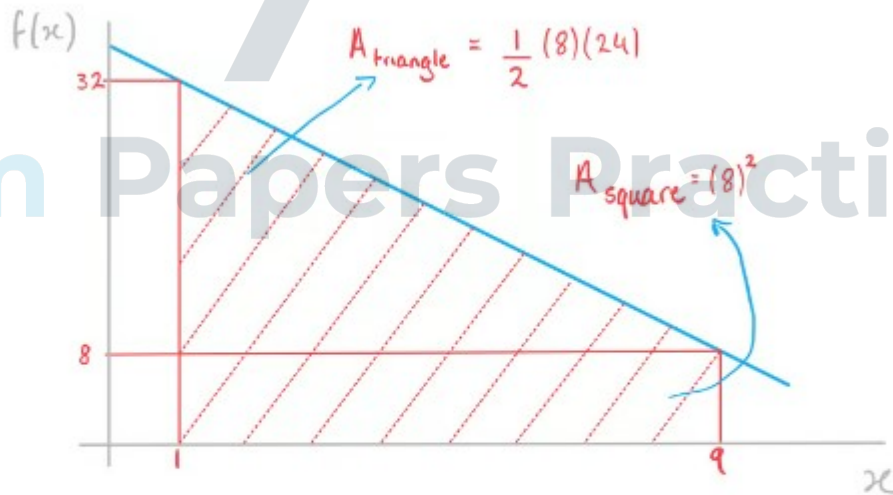
To be used by all students preparing for DP IB Maths AA SL  
Students of other boards may also find this useful

## Question 1

a) Start =  $(0, 35)$  , End =  $(10, 5)$  ,  $m = -3$ .



b)  $A_{\text{triangle}} = \frac{1}{2}bh$        $A_{\text{square}} = x^2$



$$\int_1^9 (-3x + 35) dx = \frac{1}{2}(8)(24) + (8)^2$$

$$\int_1^9 (-3x + 35) dx = 160 \text{ units}^2$$

Question 2

$$\int f'(x)dx = f(x) + c$$

$$\int \left( \frac{9}{2}x^2 + 7x - 2 \right) dx = \frac{3}{2}x^3 + \frac{7}{2}x^2 - 2x + c$$

$$f(-3) = -\frac{11}{2}$$

$$\frac{3}{2}(-3)^3 + \frac{7}{2}(-3)^2 - 2(-3) + c = -\frac{11}{2}$$

$$-\frac{81}{2} + \frac{63}{2} + 6 + c = -3 + c = -\frac{11}{2}$$

$$\therefore c = -\frac{11}{2} + 3 = -\frac{5}{2}$$

$$f(x) = \frac{3}{2}x^3 + \frac{7}{2}x^2 - 2x - \frac{5}{2}$$

# Exam Papers Practice

## Question 3

$$a) \int f'(x) dx = f(x) + c$$

$$\int (4ax + 6) dx = 2ax^2 + 6x + c$$

$$f(-1) = f(4) \quad (\text{both equal } 0)$$

$$2a(-1)^2 + 6(-1) + \cancel{c} = 2a(4)^2 + 6(4) + \cancel{c}$$

$$2a - 6 = 32a + 24$$

$$\therefore a = -1, \quad c = 8$$

$$i) \quad a = -1$$

$$ii) \quad f(x) = -2x^2 + 6x + 8$$

iii) Graph  $f$  on your GDC to find  $V$ .

$$V \left( \frac{3}{2}, \frac{25}{2} \right) \text{ or } (1.5, 12.5)$$

b) Use your GDC to find the definite integral.

$$\int_{-1}^4 (-2x^2 + 6x + 8) dx = \frac{125}{3} \text{ units}^2$$

## Question 4

a) The bounds for the definite integral are  $x=2$  and  $x=1$ .

i)  $\int_1^2 \frac{1}{2}(x-1)(x+5) dx$

ii) Use your GDC to find the definite integral.

$$\int_1^2 \frac{1}{2}(x-1)(x+5) dx = \frac{5}{3} \text{ units}^2$$

b) Find  $y$  when  $x=2$

$$y = \frac{1}{2}(2-1)(2+5) = \frac{7}{2}$$

$R + S = \text{rectangle } \underline{w} \ b=2, \ h=\frac{7}{2}$

$$R + \frac{5}{3} = (2)\left(\frac{7}{2}\right) = 7$$

$$R = 7 - \frac{5}{3}$$

$$R = \frac{16}{3} \text{ units}^2$$

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Question 5

a) When  $x = 0$ 

$$y = \frac{6(0-3)}{2(0)-9} = \frac{-18}{-9} = 2$$

When  $y = 0$ 

$$0 = \frac{6(x-3)}{2x-9}, \quad x = 3$$

 $(0, 2)$  and  $(3, 0)$ b)  $V = \text{cross-sectional area} \times \text{length } (l)$ 

$$30 = \int_0^3 \frac{6(x-3)}{2x-9} dx \times l$$

$$l = \frac{30}{\int_0^3 \frac{6(x-3)}{2x-9} dx} = 7.396\dots$$

$$l = 7.40 \text{ cm (3 s.f.)}$$

Exam Papers Practice

## Question 6

a)  $f(x) = 0$  when  $2x + 1 = 0$

$$\therefore x = -\frac{1}{2}$$

The bounds for the definite integral are

$$x = 0 \text{ and } x = -\frac{1}{2}$$

i) 
$$\int_{-\frac{1}{2}}^0 (2x+1)(4x^2-10x+41) dx$$

ii) Use your GOC to find the definite integral.

$$\int_{-\frac{1}{2}}^0 (2x+1)(4x^2-10x+41) dx = \frac{257}{24} \text{ units}^2$$

b) Parallelogram area (A) = base  $\times$  height

$$\text{base} = a \quad \text{height} = \frac{7}{3}$$

$$A = \frac{257}{24} = a \left( \frac{7}{3} \right)$$

$$a = \frac{257}{24} \times \frac{3}{7} = \frac{257}{56}$$

$$a = \frac{257}{56}$$

The  $x$ -coordinate of C =  $a + 1$ .

$$\therefore C \left( \frac{313}{56}, \frac{7}{3} \right)$$



Question 7 a) Graph the line and curve on your GDC.

i) point of intersection:  $(3, 20)$

ii) the line's  $x$ -intercept:  $(\frac{1}{2}, 0)$

iii) the curve's  $x$ -intercept:  $(4, 0)^*$

\* This is the  $x$ -intercept in the diagram.

b) Area = RA triangle +  $\int_3^4 (-x^3 + x^2 + 10x + 8) dx$

triangle area =  $\frac{1}{2}bh$

$b = 2.5$        $h = 20$

$A = \frac{1}{2}(2.5)(20) + \int_3^4 (-x^3 + x^2 + 10x + 8) dx$

$A = 25 + \frac{139}{12}$

$A = \frac{439}{12} \text{ units}^2$



## Question 8

a) i) Expand  $f(x)$ 

$$f(x) = x^3 - 12x + 16$$

$$y = x^n \rightarrow \frac{dy}{dx} = nx^{n-1} \quad (\text{in formula booklet})$$

$$\therefore f'(x) = 3x^2 - 12$$

$$\text{ii) } f'(x) = 3x^2 - 12 = 0$$

$$\therefore x^2 - 4 = 0$$

$$x = \pm 2 \quad \therefore a = -2, \quad b = 2.$$

$$f(-2) = (-2)((-2)^2 - 12) + 16 = (-2)(-8) + 16$$

$$f(-2) = 32$$

$$f(2) = (2)((2)^2 - 12) + 16 = (2)(-8) + 16$$

$$f(2) = 0$$

$$\therefore A(-2, 32), \quad B(2, 0)$$



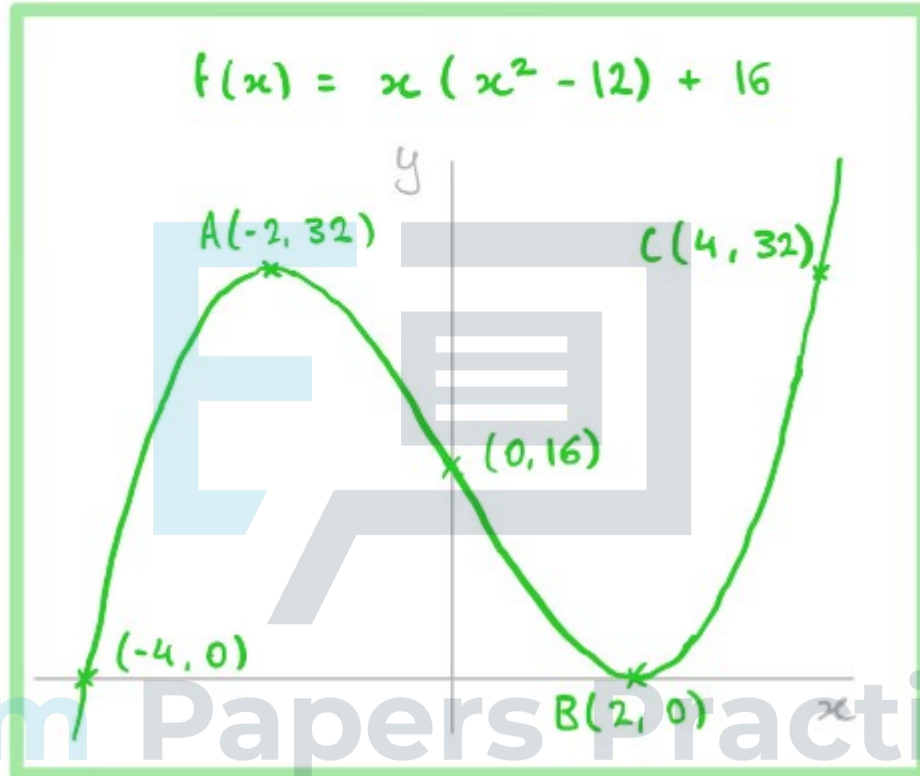
b)i)  $f(x) = x(x^2 - 12) + 16 = 32$

$\therefore x = -2$  or  $4$  (GDC)

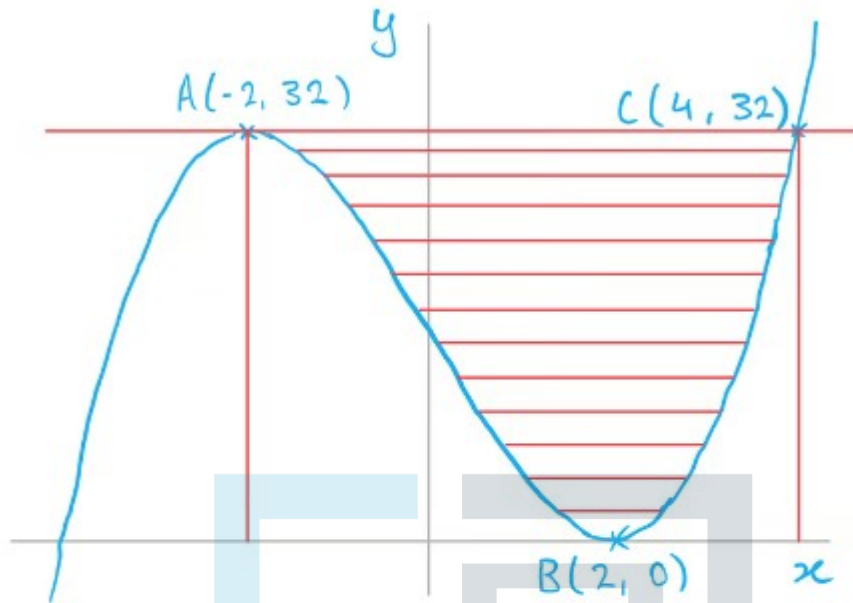
$c > 0$

$\therefore c = 4$

ii) plot  $f(x)$  on your GDC to help



c) Use your sketch from part b to help.



$$\text{Area R} = \text{rectangle} - \int_{-2}^4 f(x) dx$$

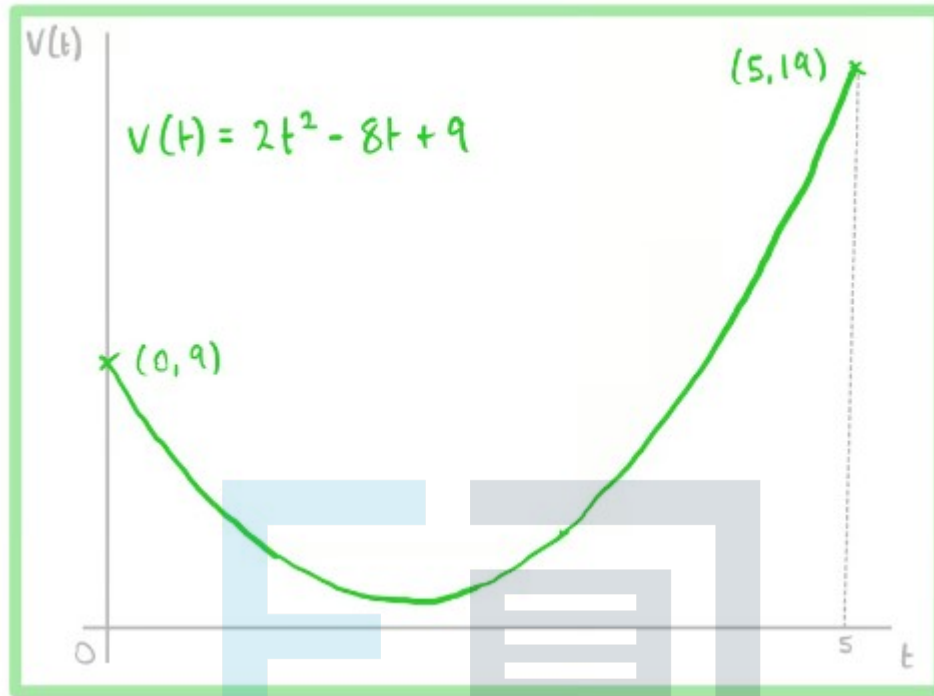
$$\text{Area R} = (6 \times 32) - \int_{-2}^4 (x(x^2 - 12) + 16) dx$$

$$\text{Area R} = 108 \text{ units}^2$$

Exam Papers Practice

Question 9

a)



b) Find the integral between  $t=1$  and  $t=4.5$ .

$$\text{distance} = \int_1^{4.5} (2t^2 - 8t + 9) dt$$

$$\text{distance} = \frac{175}{12} \text{ m}$$

$$\text{distance} = 14.6 \text{ m (3sf)}$$