Exam Papers Practice

5.2 Integration

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.2 Integration
Difficulty	Medium

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful

Question 1a

A curve $y = f(x)$ passes through point $\mathrm{A}(4,2)$ and has a gradient of $f'(x) = 5x - 2$.

Find the gradient of the curve at point A.

[2 marks]

Question 1b

Find the equation of the tangent to the curve at point A.

Give your answer in the form $y = mx + c$.

[2 marks]

Question 1c

Determine the equation of the curve $y = f(x)$.

[3 marks]

Question 2a

A point $P(3,8)$ lies on the curve $y = f(x)$ that has a gradient of $f'(x) = -2x^2 + 11$.

Find the gradient of the curve at point P.

[2 marks]

Question 2b

Find the equation of the tangent to the curve at point P.

Give your answer in the form $y = mx + c$.

[2 marks]

Question 2c

Determine the equation of the curve $y = f(x)$.

[3 marks]

Question 3a

The following table shows the x and y coordinates of five points that lie on a curve $y = f(x)$.

x	0	0.25	0.5	0.75	1
$y = f(x)$	1	2.25	4	6.25	9

Estimate the area under the curve over the interval $0 \leq x \leq 1$.

[2 marks]

Question 3b

The equation of the curve was found to be $y = (2x + 1)^2$.

Find the exact value of the area under the curve over the interval $0 \leq x \leq 1$.

[2 marks]

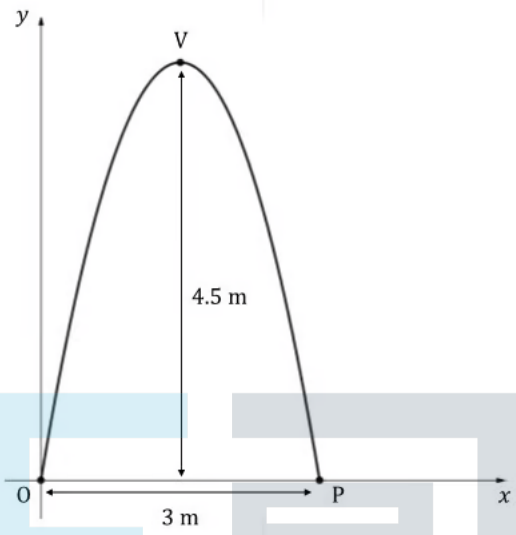
Question 3c

Find the percentage error between the estimation in part (a) and the exact value in part (b). Provide a reason for the difference.

[2 marks]

Question 4a

The following diagram shows an arch that is $4.5\ \text{m}$ tall and $3\ \text{m}$ wide. The arch crosses the x-axis at the origin, O, and at point P, and its vertex is at point V. The arch may be represented by a curve with an equation of the form $y = x(ax + 6)$, where all units are measured in metres.



Find

(i)
the coordinates of P

(ii)
the coordinates of V

(iii)
the value of a.

[4 marks]

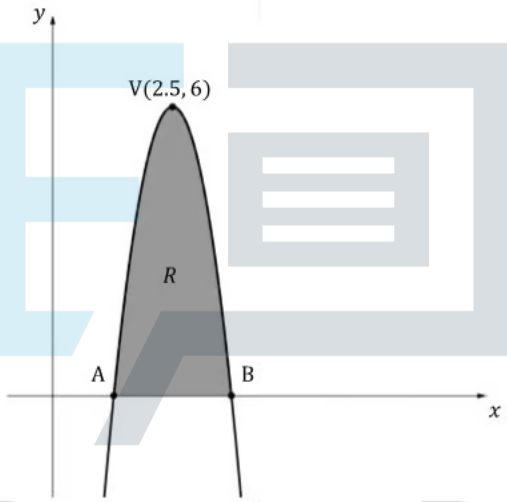
Question 4b

Find the cross-sectional area under the arch.

[2 marks]

Question 5a

The diagram below shows a part of the curve $y = -4x^2 + px + q$. Points A and B represent the x-intercepts, point V $(2.5,6)$ represents the vertex of the curve, and the shaded region R represents the area between the curve and the x-axis.



Find the values of p and q.

[2 marks]

Question 5b

Find the coordinates of points A and B.

[4 marks]

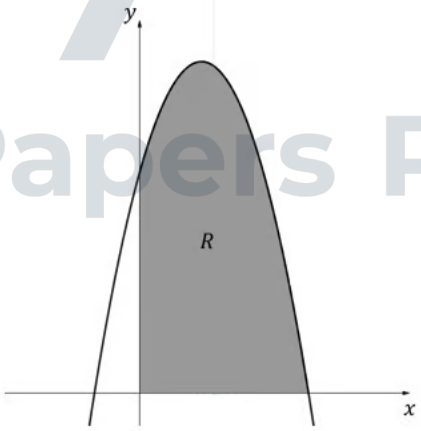
Question 5c

Find the area of region R.

[2 marks]

Question 6a

The following diagram shows part of the graph of $f(x) = (5 - 2x)(2 + 3x),\ x \in R$. The shaded region R is bounded by the x-axis, the y-axis and the graph of f.



Write down an integral for the area of region R

[2 marks]

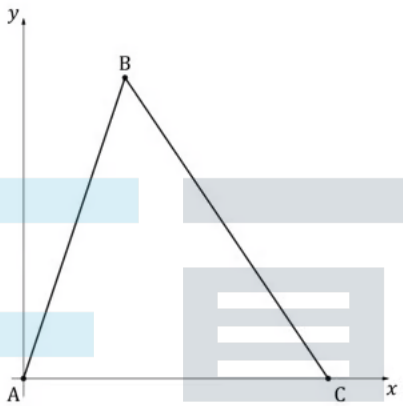
Question 6b

Find the area of region R.

[1 mark]

Question 6c

The three points $\mathrm{A}(0,0)$, $\mathrm{B}(4,h)$ and $\mathrm{C}(9,0)$ define the vertices of a triangle.



Find the value of h, the y-coordinate of B, given that the area of the triangle is equal to the area of region R.

[2 marks]

Question 7a

A rice farm sells x kg of rice every week.

It is known that $\frac{\mathrm{d}P}{\mathrm{d}x} = -0.02x + 6,\ x \geq 0,$ where P is the weekly profit, in dollars ($), from the sale of x kg of rice.

Find the amount of rice, in kg, that should be sold each week to maximise the profit.

[3 marks]

Question 7b

The profit from selling 250 kg of rice is $480.

Find $P(x)$.

[5 marks]

Question 8a

A paint company sells x hundred of litres of paint every week.

It is known that $\frac{dP}{dx} = -1.9x + 145,\ x \geq 0,$ where P is the weekly profit, in euros (€), from the sale of x hundred litres of paint.

Find the number of litres that should be sold each week to maximise the profit.

[3 marks]

Question 8b

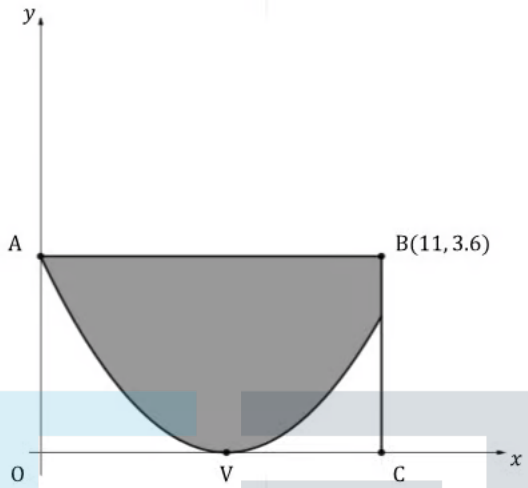
The profit from selling 7000 litres of paint is €5000.

Find $P(x)$.

[5 marks]

Question 9a

A river has a cross-sectional area shown by the shaded region of the diagram below, where the x and y values are in metres. The riverbed (the curved part of the region shown) has an equation of the form $y = q(x-6)^2$. Point O is the origin, and points O, A, B and C are the vertices of a rectangle. Point V, the deepest point of the riverbed, is situated on the x-axis.



Find

(i)
the coordinates of V

(ii)
the area of the rectangle $OABC$.

[3 marks]

Question 9b

Determine the value of q.

[2 marks]

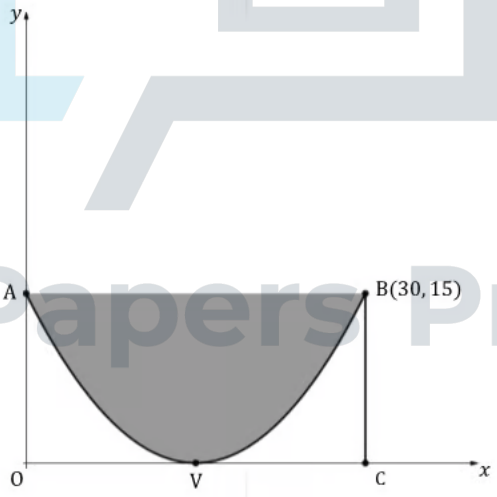
Question 9c

Find the cross-sectional area of the riverbed.

[3 marks]

Question 10a

A trough has a cross-sectional area shown by the shaded region of the diagram below, where the x and y values are in centimetres. The curved bottom of the trough has an equation in the form $y = r(x - 15)^2$. Point O is the origin, and points $\mathrm{O}, \mathrm{A}, \mathrm{B}$ and C are the vertices of a rectangle. Point V, the deepest point of the trough, is situated on the x-axis.



Determine the value of r.

[2 marks]

Question 10b

Find the cross-sectional area of the trough.

[4 marks]

Question 10c

The length of the trough is 1.2 m.

Find the volume of the trough. Give your answer in cm^3

[2 marks]