Exam Papers Practice

5.1 Differentiation

Question Paper

Course	DP IB Maths
Section	5. Calculus
Topic	5.1 Differentiation
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1a

The equation of a curve is $y = \frac{3}{2}x^2 - 15x + 2.$

Find $\frac{dy}{dx}$.

[2 marks]

Question 1b

The gradient of the tangent to the curve at point A is -3.

Find

(i)
the coordinates of A

the equation of the tangent to the curve at point A

(ii)
Give your answer in the form $y = mx + c$.

[4 marks]

Question 2a

Consider the function $f(x) = 3x^7 - 12x.$

Find $f'(x).$

[1 mark]

Question 2b

Find the gradient of the graph of f at $x = 0$.

[2 marks]

Question 2c

Find the coordinates of the points at which the normal to the graph of f has a gradient of 4.

[3 marks]

Question 3a

The equation of a curve is $y = 4 - \frac{4}{x}$.

Find the equation of the tangent to the curve at $x = 2$.

Give your answer in the form $y = mx + c$.

[3 marks]

Question 3b

Find the coordinates of the points on the curve where the gradient is 16.

[3 marks]

Question 4a

Consider the function $f(x) = \frac{4}{x} + \frac{2x^4}{5} - \frac{2}{5}, \quad x \neq 0.$

Calculate

(i)
$f(2)$

(ii)
$f'(2).$

[3 marks]

Question 4b

A line, l, is tangent to the graph of $y = f(x)$ at the point $x = 2$.

Find the equation of l. Give your answer in the form $y = mx + c$.

[3 marks]

Question 4c

The graph of $y = f(x)$ and l have a second intersection at point A.

Use your graphic display calculator to find the coordinates of A.

[2 marks]

Question 5a

Consider the function $f(x) = x^2 - bx + c$.

Find $f'(x)$.

[1 mark]

Question 5b

The equation of the tangent line to the graph $y = f(x)$ at $x = 2$ is $y = x - 1$.

Calculate the value of b.

[2 marks]

Question 5c

Calculate the value of c and write down the function $f(x)$.

[3 marks]

Question 6a

The curve with equation $y = ax^2 + bx + c$ has a gradient of -7 at the point $(-1, 13)$, and a gradient of -3 at the point $(1, 3)$.

By considering $\frac{dy}{dx}$ show that $2a + b = -3$ and $-2a + b = -7$.

[2 marks]

Question 6b

Hence find the values of a and b.

[1 mark]

Question 6c

By considering a point that you know to be on the curve, find the value of c.

[2 marks]

Question 7a

The curve C has equation $y = 3x^2 - 6 + \frac{4}{x}$. The point $P(1, 1)$ lies on C.

Find an expression for $\frac{dy}{dx}$.

[2 marks]

Question 7b

Show that an equation of the normal to C at point P is $x + 2y = 3$.

[3 marks]

Question 7c

This normal cuts the x-axis at the point Q.

Find the length of PQ, giving your answer as an exact value.

[2 marks]

Question 8

Find the values of x for which $f(x) = -9x^2 + 5x - 3$ is an increasing function.

[3 marks]

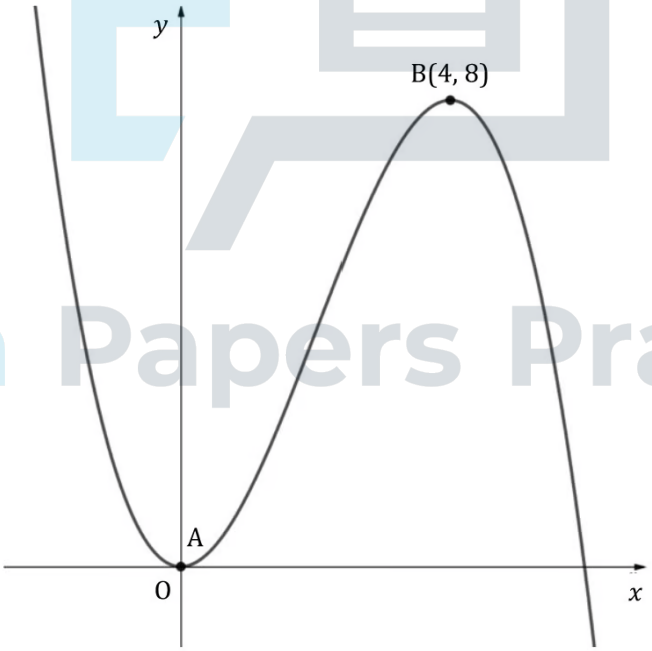
Question 9

Show that the function $f(x) = x^3 - 3x^2 + 6x - 7$ is increasing for all $x \in \mathbb{R}$.

[3 marks]

Question 10a

The graph of the cubic function $y = f(x)$ is shown below. Point A, a local minimum, is located at the origin and point B, a local maximum, sits at the point $(4, 8)$.



State the equations of the horizontal tangent to the curve.

[2 marks]

Question 10b

Write down the value of x where the point of inflection is located.

[1 mark]

Question 10c

Find the intervals where f is decreasing.

[2 marks]

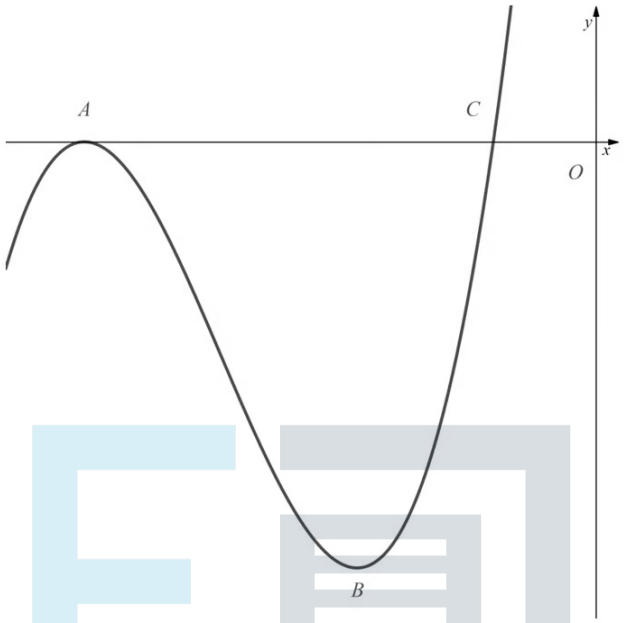
Question 10d

Sketch the graph of $f'(x)$, labelling clearly any intercepts and axis of symmetry.

[3 marks]

Question 11a

The diagram below shows part of the curve with equation $y = x^3 + 11x^2 + 35x + 25$. The curve touches the x-axis at C and cuts the x-axis at C. The points A and B are stationary points on the curve.



Using calculus, and showing all your working, find the coordinates of A and B.

[5 marks]

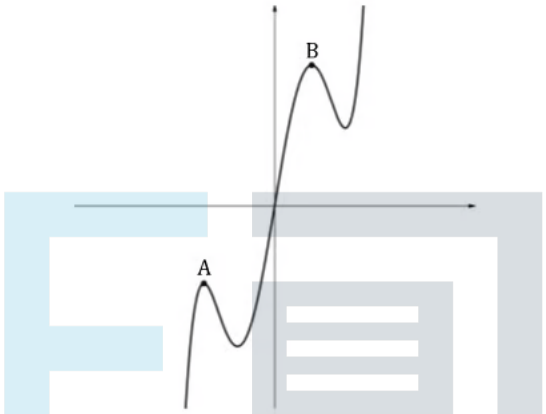
Question 11b

Show that $(-1, 0)$ is a point on the curve and explain why those must be the coordinates of point C.

[2 marks]

Question 12a

The equation of the curve C is $y = \frac{1}{35}x^5 - \frac{3}{4}x^3 + 6x$. A section of the curve C is shown on the diagram below.



Find $\frac{dy}{dx}$.

[2 marks]

Question 12b

There are two points, R and S, along the curve C at which the gradient of the normal to the curve C is equal to $-\frac{1}{10}$.

Calculate the x-coordinates of points R and S.

[4 marks]

Question 13a

Find the x-coordinates of the stationary points on the graph with equation $y = x^3 - 6x^2 + 9x - 1$.

[4 marks]

Question 13b

Find the nature of the stationary points found in part (a).

[3 marks]

Question 13c

Determine the x-coordinate of the point of inflection on the graph with equation $y = x^3 - 6x^2 + 9x - 1$.

[3 marks]

Question 13d

Explain why, in this case, the point of inflection is not a stationary point.

[1 mark]

Question 14

The graph of a continuous function has the following properties:

The function is concave down in the interval $(-\infty, a)$.

The function is concave up in the interval (a, ∞).

The graph of the function intercepts the -axis at the points $(b, 0)$, $(c, 0)$ and $(d, 0)$, where b, c and d are such that $d > c > b > 0$.

The $x-$ coordinates of the turning points of the function are e and f, which are such that $f > e$.

The graph of the function intercepts the y-axis at $(0, g)$

Given that the value of the function is positive when $x = a$, sketch a graph of the function. Be sure to label the x-axis with the x-coordinates of the stationary points and the point of inflection, and also to label the points where the graph crosses the coordinate axes.

[4 marks]