

# A Level Physics CIE

## 5. Work, Energy & Power

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## 5.1 Energy: Conservation, Work, Power & Efficiency

### 5.1.1 Work & Energy

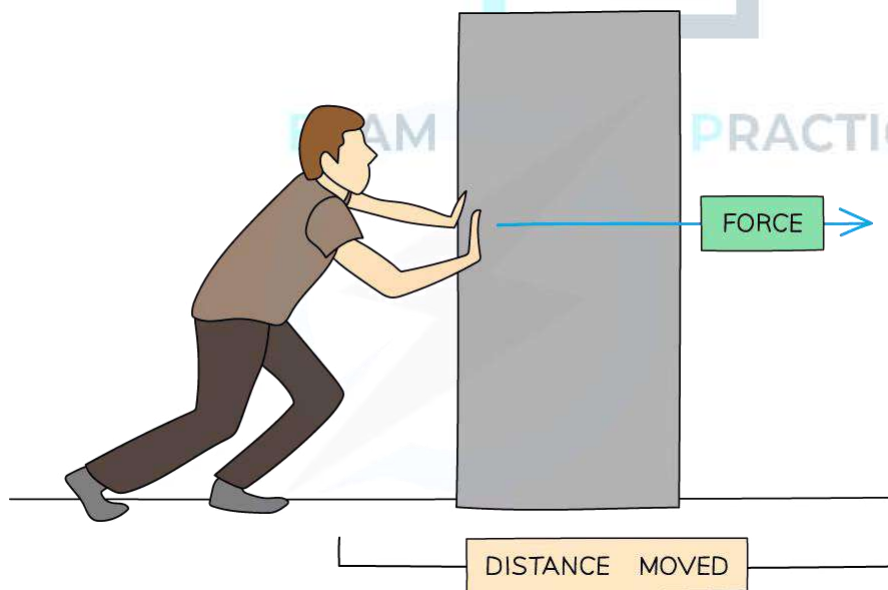
#### Work Done

- In Physics, **work is done** when an **object is moved** over a distance by an external force applied in the direction of its displacement

$$W = F \times d$$

Diagram illustrating the equation  $W = F \times d$ . The equation is centered, with three callout boxes pointing to its components: "WORK DONE (J)" points to  $W$ , "FORCE (N) APPLIED IN DIRECTION OF DISPLACEMENT" points to  $F$ , and "DISPLACEMENT (m)" points to  $d$ .

- In the diagram below, the man's pushing force on the block is doing work as it is transferring energy to the block (increasing its kinetic energy)



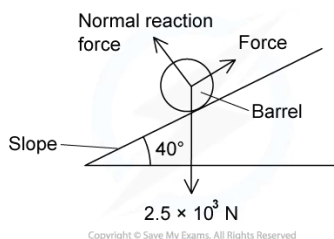
*Work is done when a force is used to move an object over a distance*

- When work is done, energy is transferred from one object to another

- Work done can be thought of as the amount of energy transferred, hence its units are in Joules (J)
- Usually, if a force acts in the direction that an object is moving then the object will gain energy
- If the force acts in the opposite direction to the movement then the object will lose energy

### ? Worked Example

The diagram shows a barrel of weight  $2.5 \times 10^3$  N on a frictionless slope inclined at  $40^\circ$  to the horizontal.



A force is applied to the barrel to move it up the slope at constant speed. The force is parallel to the slope. What is the work done in moving the barrel a distance of 6.0 m up the slope?

A.  $7.2 \times 10^3$  J      B.  $2.5 \times 10^4$  J  
 C.  $1.1 \times 10^4$  J      D.  $9.6 \times 10^3$  J

ANSWER: D

STEP 1

WORK DONE EQUATION

$$W = F \times d$$

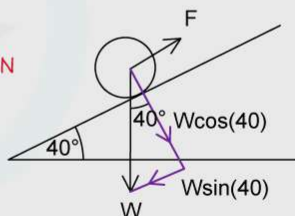
STEP 2

CALCULATE THE FORCE IN THE DIRECTION OF TRAVEL

THE FORCE NEEDED TO PUSH THE BARREL NEEDS TO OVERCOME THE COMPONENT OF THE BARREL'S WEIGHT. SINCE THE FORCE IS PARALLEL TO THE SLOPE, THE COMPONENT OF THE WEIGHT WE NEED IS THE ONE PARALLEL TO THE SLOPE.

$$F = W \sin(40) = 2.5 \times 10^3 \times \sin(40) = 1607 \text{ N}$$

THIS IS THE FORCE IN THE SAME DIRECTION AS THE DISPLACEMENT



STEP 3

SUBSTITUTE F AND d INTO THE WORK DONE EQUATION

$$W = 1607 \text{ N} \times 6.0 \text{ m} = 9.6 \times 10^3 \text{ J}$$



### Exam Tip

A common exam mistake is choosing the incorrect force which is not parallel to the direction of movement of an object. You may have to resolve the force vector to find the component that is parallel. The force does not have to be in the same direction as the movement, as shown in the worked example.



## 5.1.2 The Principle of Conservation of Energy

### The Principle of Conservation of Energy

- ♦ The Principle of Conservation of Energy states that:
  - Energy cannot be created or destroyed, it can only change from one form to another
- ♦ This means the total amount of energy in a closed system remains constant, although how much of each form there is may change
- ♦ Common examples of energy transfers are:
  - A falling object (in a vacuum): gravitational potential energy → kinetic energy
  - A battery: chemical energy → electrical energy → light energy (if connected to a bulb)
  - Horizontal mass on a spring: elastic potential energy → kinetic energy

#### Types of energy

FORM	WHAT IS IT?
KINETIC	THE ENERGY OF A MOVING OBJECT.
GRAVITATIONAL POTENTIAL	THE ENERGY SOMETHING GAINS WHEN YOU LIFT IT UP, AND WHICH IT LOSES WHEN IT FALLS.
ELASTIC	THE ENERGY OF A STRETCHED SPRING OR ELASTIC BAND.(SOMETIMES CALLED STRAIN ENERGY)
CHEMICAL	THE ENERGY CONTAINED IN A CHEMICAL SUBSTANCE.
NUCLEAR	THE ENERGY CONTAINED WITHIN THE NUCLEUS OF AN ATOM.
INTERNAL	THE ENERGY SOMETHING HAS DUE TO ITS TEMPERATURE (OR STATE). (SOMETIMES REFERRED TO AS THERMAL OR HEAT ENERGY)

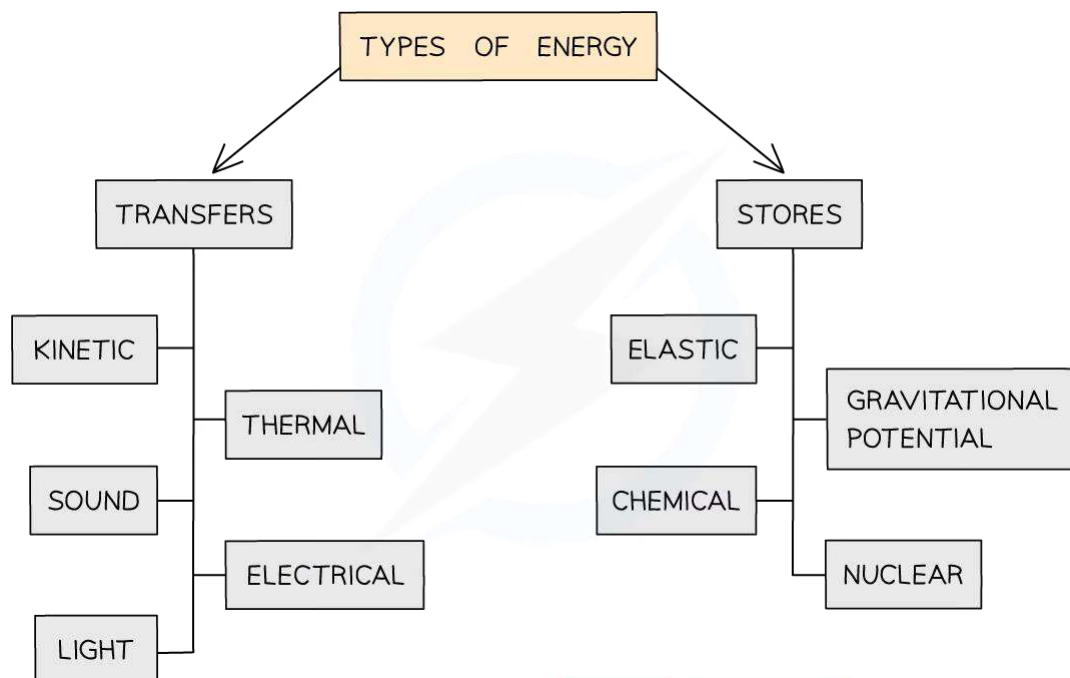


Diagram showing the forms of energy transfers and stores

### Energy dissipation

- When energy is transferred from one form to another, not all the energy will end up in the desired form (or place)
- Dissipation is used to describe ways in which energy is wasted
- Any energy not transferred to useful energy stores is wasted because it is lost to the surroundings
- These are commonly in the form of **thermal (heat)**, **light** or **sound** energy
- What counts as **wasted energy** depends on the system
- For example, in a **television**:

electrical energy → light energy + sound energy + thermal energy

- Light and sound energy are useful energy transfers whereas thermal energy (from the heating up of wires) is wasted

- Another example, in a **heater**:

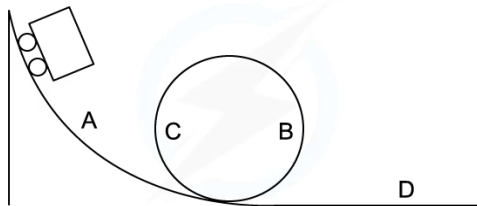
electrical energy → thermal energy + sound energy

- The thermal energy is useful, whereas sound is not

## ? Worked Example

The diagram shows a rollercoaster going down a track.

The rollercoaster takes the path A – B – C – D.



Which statement is true about the energy changes that occur for the rollercoaster down this track?

- A. KE – GPE – GPE – KE
- B. KE – GPE – KE – GPE
- C. GPE – KE – KE – GPE
- D. GPE – KE – GPE – KE

ANSWER: D

- At point A:
  - The rollercoaster is raised above the ground, therefore it has GPE
  - As it travels down the track, GPE is converted to KE and the roller coaster speeds up
- At point B:
  - KE is converted to GPE as the rollercoaster rises up the loop
- At point C:
  - This GPE is converted back into KE as the rollercoaster travels back down the loop
- At point D:
  - The flat terrain means the rollercoaster only has KE



### Exam Tip

You may not always be given the energy transfers happening in the system in exam questions. By familiarising yourself with the transfers and stores of energy, you will be expected to relate these to the situation in question. For example, a ball rolling down a hill is transferring gravitational potential energy to kinetic energy whilst a spring converts elastic potential energy into kinetic energy.



## 5.1.3 Efficiency

**Efficiency of a System**

- The efficiency of a system is the ratio of the useful energy output from the system to the total energy input
  - If a system has **high** efficiency, this means most of the energy transferred is **useful**
  - If a system has **low** efficiency, this means most of the energy transferred is **wasted**
- Multiplying this ratio by 100 gives the efficiency as a percentage
- The efficiency is calculated using the equation:

$$\text{EFFICIENCY} = \frac{\text{USEFUL ENERGY OUTPUT}}{\text{TOTAL ENERGY INPUT}} \times 100\%$$

*Efficiency equation in terms of energy*

- Efficiency can also be written in terms of power (the energy per second):

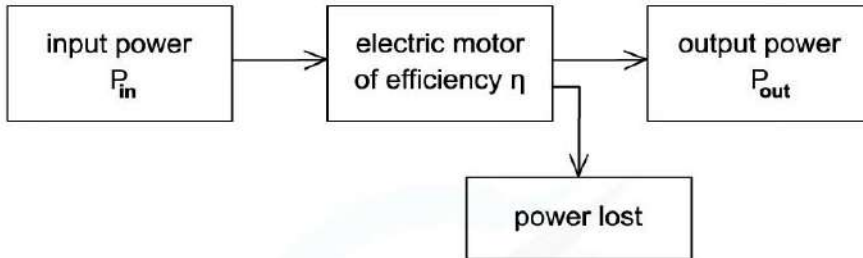
$$\text{EFFICIENCY} = \frac{\text{USEFUL POWER OUTPUT}}{\text{TOTAL POWER INPUT}} \times 100\%$$

*Efficiency equation in terms of power*



## ? Worked Example

An electric motor has an input power  $P_{in}$ , useful output power  $P_{out}$  and efficiency  $\eta$ .



What is the output power  $P_{out}$  of the motor?

- A.  $\eta P_{in}$       B.  $\frac{-\eta P_{lost}}{\eta - 1}$       C.  $\eta P_{lost}$       D.  $-\eta P_{lost} (\eta - 1)$

ANSWER: B

$$\text{EFFICIENCY} = \frac{\text{USEFUL POWER OUTPUT}}{\text{TOTAL POWER INPUT}}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{P_{out}}{P_{out} + P_{lost}}$$

MULTIPLY BY  $P_{out} + P_{lost}$  ON BOTH SIDES       $\eta (P_{out} + P_{lost}) = P_{out}$

EXPAND THE BRACKETS       $\eta P_{out} + \eta P_{lost} = P_{out}$

$-P_{out}$  FROM BOTH SIDES       $\eta P_{out} - P_{out} = -\eta P_{lost}$

TAKE  $P_{out}$  AS A FACTOR       $P_{out}(\eta - 1) = -\eta P_{lost}$

DIVIDE BY  $\eta - 1$        $P_{out} = \frac{-\eta P_{lost}}{\eta - 1}$



### Exam Tip

Efficiency can be in a ratio or percentage format. If the question asks for an efficiency as a ratio, give your answer as a fraction or decimal. If the answer is required as a percentage, remember to multiply the ratio by 100 to convert it, e.g. Ratio = 0.25, Percentage =  $0.25 \times 100 = 25\%$

## Solving Problems Involving Efficiency

- Recall the two equations for calculating efficiency are:

$$\text{EFFICIENCY} = \frac{\text{USEFUL ENERGY OUTPUT}}{\text{TOTAL ENERGY INPUT}} \times 100\%$$

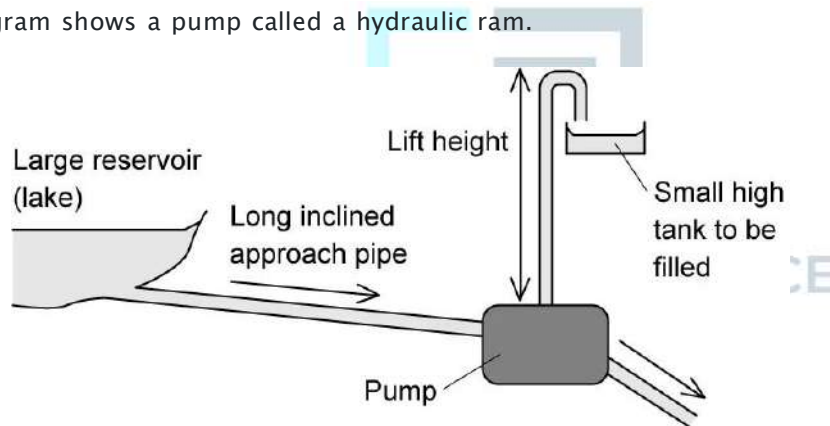
$$\text{EFFICIENCY} = \frac{\text{USEFUL POWER OUTPUT}}{\text{TOTAL POWER INPUT}} \times 100\%$$

- Which to use will depend on whether you're given a system calculating energies or power as shown in the examples below



### Worked Example

The diagram shows a pump called a hydraulic ram.



In one such pump, the long approach pipe holds 700 kg of water. A valve shuts when the speed of this water reaches  $3.5 \text{ m s}^{-1}$  and the kinetic energy of this water is used to lift a small quantity of water by a height of 12m. The efficiency of the pump is 20%. Which mass of water could be lifted 12 m?

- A. 6.2 kg      B. 4.6 kg      C. 7.3 kg      D. 0.24 kg

ANSWER: C

THE KINETIC ENERGY OF THE WATER IS CONVERTED TO GRAVITATIONAL POTENTIAL ENERGY WHEN LIFTED BY 12m

$$KE = GPE$$

$$\frac{1}{2}mv^2 = mgh$$

SINCE EFFICIENCY IS 20% ONLY 20% OF THE KINETIC ENERGY WILL BE CONVERTED.

$$0.2 \times \frac{1}{2}mv^2 = mgh$$

$$0.2 \times \frac{1}{2} \times 700 \times (3.5)^2 = m \times 9.81 \times 12$$

$$857.5 = m \times 117.72$$

$$\frac{857.5}{117.72} = m$$

$$m = 7.3 \text{ kg (2 s.f.)}$$

- The pump is what converts the water's **kinetic energy** into **gravitational potential energy**
- Since its efficiency is 20%, you would multiply the kinetic energy by 0.2 since only 20% of the **kinetic energy** will be converted (**not** 20% of the gravitational potential energy)



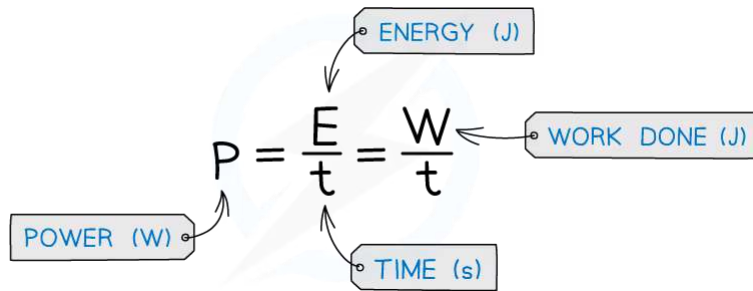
#### Exam Tip

Equations for kinetic and potential energies are important for these types of questions. Also, familiarise yourself with the different equations for power depending on what quantities are given.

## 5.1.4 Power

### Defining Power

- The power of a machine is the **rate at which it transfers energy**
- Since work done is equal to the energy transferred, power can also be defined as the **rate of doing work** or **the work done per unit time**
- The SI unit for power is **Watts (W)** where  $1 \text{ W} = 1 \text{ J s}^{-1}$


$$P = \frac{E}{t} = \frac{W}{t}$$

The diagram shows the equation  $P = \frac{E}{t} = \frac{W}{t}$  with four callout boxes: 'ENERGY (J)' pointing to 'E', 'WORK DONE (J)' pointing to 'W', 'TIME (s)' pointing to 't', and 'POWER (W)' pointing to 'P'.

*Power is the rate of change of work*

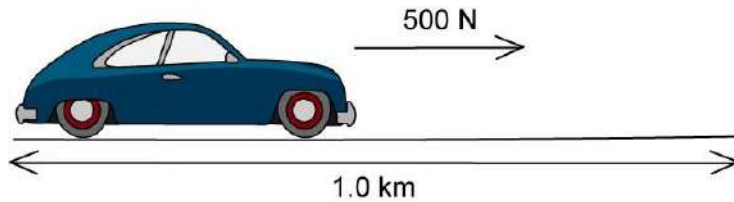
- You may be familiar with labels on lightbulbs which indicate their power such as 60 W or 100 W. These tell you about an energy transferred by an electrical current rather than by a force doing work

## Solving Problems Involving Power



### Worked Example

A car engine exerts the following force for 1.0 km in 200 s.



What is the average power developed by the engine?

STEP 1 EQUATION FOR POWER

$$\text{POWER} = \frac{\text{WORK DONE}}{\text{TIME}}$$

STEP 2 CALCULATE WORK DONE

$$\begin{aligned} W &= F \times d \\ &= 500 \text{ N} \times 1.0 \times 10^3 \text{ m} \\ &= 5 \times 10^5 \text{ J} \end{aligned}$$

STEP 3 SUBSTITUTE VALUES INTO POWER EQUATION

$$\text{POWER} = \frac{5 \times 10^5 \text{ J}}{200 \text{ s}} = 2500 \text{ W} = 2.5 \text{ kW}$$



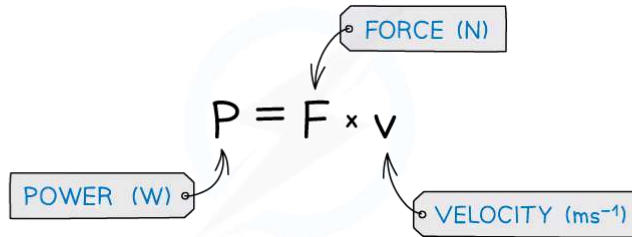
### Exam Tip

Think of power as “energy per second”. Thinking of it this way will help you to remember the relationship between power and energy: “Watt is the unit of power?”

### 5.1.5 Derivation of $P = Fv$

#### Derivation of $P = Fv$

- Moving power is defined by the equation:


$$P = F \times v$$

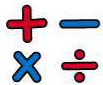
POWER (W)      FORCE (N)      VELOCITY ( $\text{ms}^{-1}$ )

- This equation is only relevant where a **constant force** moves a body at **constant velocity**. Power is required in order to produce an acceleration
- The force must be applied in the **same** direction as the velocity

#### Derivation

- The derivation for this equation is shown below:





Derivation of  $P = F \times v$

POWER IS THE RATE OF CHANGE OF WORK

$$\text{POWER} = \frac{W}{t}$$

WORK DONE = FORCE  $\times$  DISTANCE

$$W = F \times d$$

AT CONSTANT VELOCITY,  $d = v \times t$  THEREFORE

$$W = F \times v \times t$$

$$P = \frac{W}{t} = \frac{F \times v \times t}{t}$$

CANCELLING  $t$

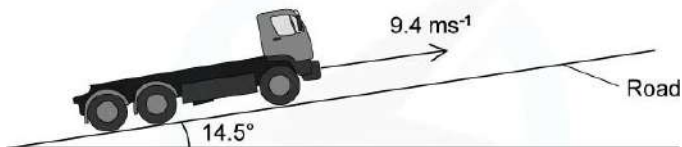
$$P = F \times v$$

*Derivation of  $P = F \times v$*



### Worked Example

A lorry moves up a road that is inclined at  $14.5^\circ$  to the horizontal.



The lorry has mass  $3500 \text{ kg}$  and is travelling at a constant speed of  $9.4 \text{ m s}^{-1}$ . The force due to air resistance is negligible. Calculate the useful power from the engine to move the lorry up the road.

STEP 1

EQUATION FOR POWER

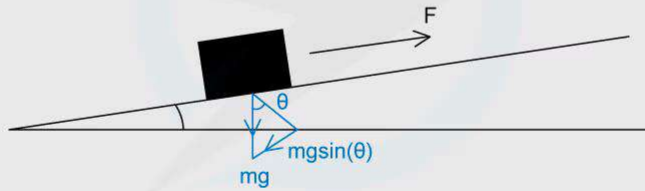
$$P = F \times v$$

STEP 2

CALCULATE THE FORCE

THE FORCE NEEDED TO MOVE THE LORRY UP THE ROAD IS THAT WHICH OVERCOMES THE COMPONENT OF ITS WEIGHT ACTING DOWN THE SLOPE

$$F = mg \sin\theta = 3500 \times 9.81 \times \sin(14.5) = 8596.8 \text{ N}$$



STEP 3

SUBSTITUTE INTO POWER EQUATION

$$P = 8596.8 \times 9.4 = 80809.9 \text{ W} = 81000 \text{ W} = 81 \text{ kW} \text{ (2.s.f.)}$$



### Exam Tip

The force represented in exam questions will often be a drag force. Whilst this is in the opposite direction to its velocity, remember the force needed to calculate the power is equal to (or above) this drag force to overcome it therefore you equate it to that value.

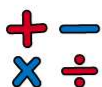


## 5.2 Energy: GPE & KE

### 5.2.1 Gravitational Potential Energy

#### Derivation of $GPE = mgh$

- Gravitational potential energy is energy stored in a mass due to its position in a gravitational field
- When a heavy object is lifted, work is done since the object is provided with an upward force against the downward force of gravity
  - Therefore **energy is transferred to the object**
- This equation can therefore be derived from the work done

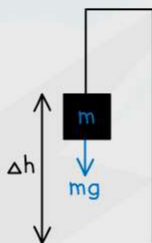


Derivation of  $GPE = mgh$

CONSIDER A MASS  $m$  LIFTED THROUGH HEIGHT  $h$

THE WEIGHT OF THE MASS IS  $mg$  WHERE  $g$  IS THE GRAVITATIONAL FIELD STRENGTH

$$W = F \times d = mg \times \Delta h$$



DUE TO ITS NEW POSITION, THE BODY IS NOW ABLE TO DO EXTRA WORK EQUAL TO  $mg\Delta h$

$$\text{CHANGE IN POTENTIAL ENERGY} = mg\Delta h$$

IF WE CONSIDER THE MASS TO HAVE 0 POTENTIAL ENERGY AT GROUND LEVEL

$$\Delta GPE = mg\Delta h$$

◦ " $\Delta$ " REFERS TO "CHANGE IN"

*Derivation of  $GPE = mgh$*





## Gravitational Potential Energy

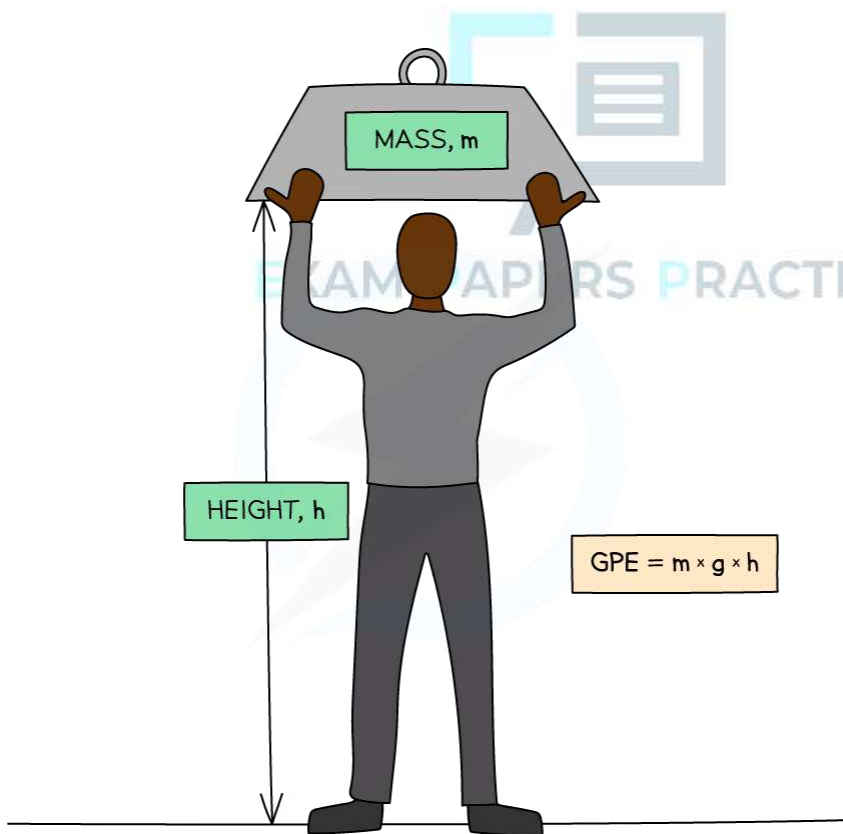
- Gravitational potential energy (GPE) is energy stored in a mass due to its position in a gravitational field
  - If a mass is **lifted** up, it will **gain** GPE (converted from other forms of energy)
  - If a mass **falls**, it will **lose** GPE (and be converted to other forms of energy)
- The equation for gravitational potential energy for energy changes in a **uniform gravitational field** is:

$$\Delta GPE = mg\Delta h$$

Diagram illustrating the equation for GPE with labels:

- $\Delta GPE$ : CHANGE IN GRAVITATIONAL POTENTIAL ENERGY (J)
- $m$ : MASS (kg)
- $g$ : GRAVITATIONAL FIELD STRENGTH ( $9.81 \text{ Nkg}^{-1}$ )
- $\Delta h$ : CHANGE IN HEIGHT (m)

Equation for GPE

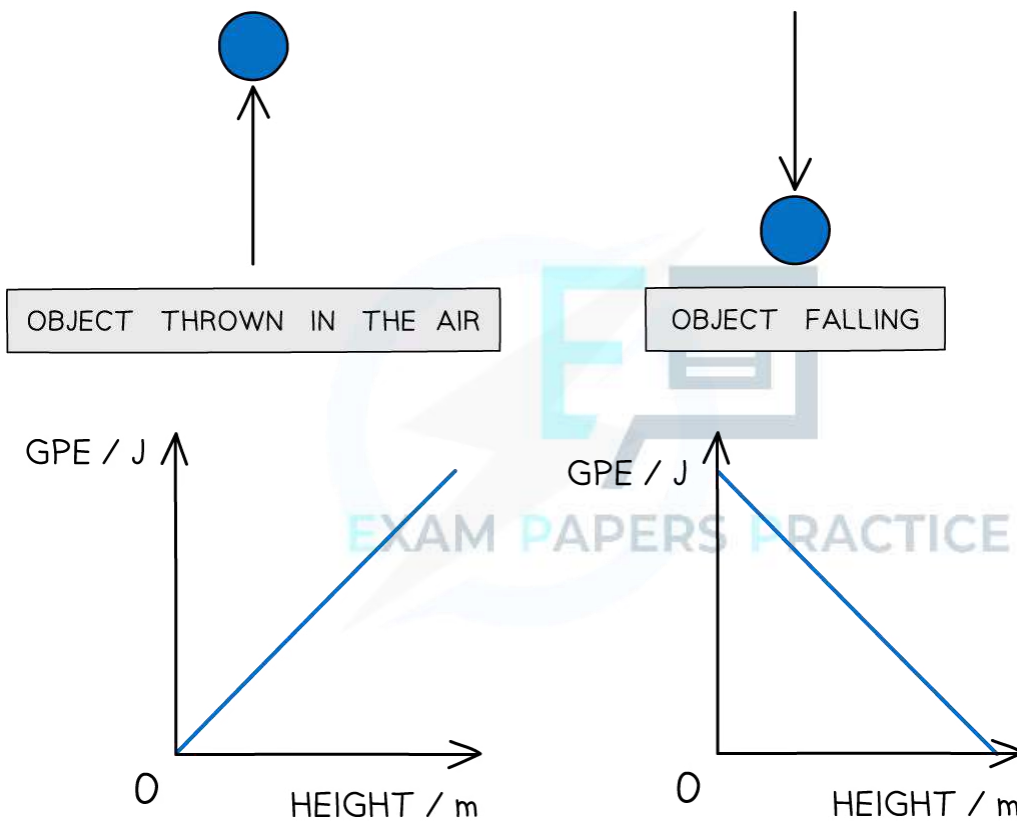


*GPE: The energy an object has when lifted up*

- The potential energy on the Earth's surface at ground level is taken to be equal to 0
- This equation is only relevant for energy changes in a **uniform gravitational field** (such as near the Earth's surface)

### GPE v Height graphs

- The two graphs below show how GPE changes with height for a ball being thrown up in the air and when falling down



*Graphs showing the linear relationship between GPE and height*

- Since the graphs are straight lines, GPE and height are said to have a **linear** relationship
- These graphs would be identical for GPE against time instead of height

## ? Worked Example

To get to his apartment a man has to climb five flights of stairs.

The height of each flight is 3.7 m and the man has a mass of 74 kg.

What is the approximate gain in the man's gravitational potential energy during the climb?

- A. 13 000 J      B. 2700 J      C. 1500 J      D. 12 500 J

ANSWER: A




Diagram showing a man climbing stairs. The height of one flight is labeled as  $\Delta h = 3.7 \text{ m}$ .

STEP 1    GPE EQUATION  
 $\Delta \text{GPE} = mg\Delta h$

STEP 2    FIND  $h$   
 $\Delta h = 5 \times 3.7 \text{ m} = 18.5 \text{ m}$

5 FLIGHTS OF STAIRS

STEP 3    SUBSTITUTE VALUES INTO GPE EQUATION  
 $\Delta \text{GPE} = 74 \times 9.81 \times 18.5 = 13000 \text{ J (2 s.f.)}$



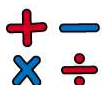
### Exam Tip

This equation only works for objects close to the Earth's surface where we can consider the gravitational field to be uniform. In A2 level, you will consider examples where the gravitational field is not uniform such as in space, where this equation for GPE will not be relevant.

## 5.2.2 Kinetic Energy

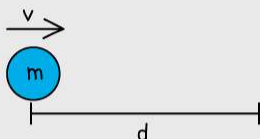
### Derivation of $KE = \frac{1}{2}mv^2$

- Kinetic energy is energy an object has due to its **motion** (or velocity)
- A force can make an object accelerate; work is done by the force and energy is transferred to the object
- Using this concept of work done and an equation of motion, the extra work done due to an object's speed can be derived
- The derivation for this equation is shown below:



Derivation of  $KE = \frac{1}{2}mv^2$

CONSIDER A MASS  $m$  AT REST WHICH ACCELERATES TO A SPEED  $v$  OVER A DISTANCE  $d$



WORK DONE IN ACCELERATING THE MASS

$$W = F \times d$$

AND  $F = ma$  FROM NEWTON'S SECOND LAW

RECALL THE SUVAT EQUATION

$$v^2 = u^2 + 2as$$

IF  $u = 0$  AND  $s = d$

$$v^2 = 2ad$$

REARRANGING FOR  $a$

$$a = \frac{v^2}{2d}$$

SUBSTITUTE BACK INTO  $F = ma$

$$F = ma = \frac{mv^2}{2d}$$

SUBSTITUTE THIS FORCE F INTO THE WORK DONE EQUATION

$$W = \frac{mv^2}{2d} \times d = \frac{1}{2}mv^2$$

THE MASS IS NOW ABLE TO DO EXTRA WORK =  $\frac{1}{2}mv^2$   
DUE TO ITS SPEED

IT HAS KINETIC ENERGY =  $\frac{1}{2}mv^2$

*Derivation for Kinetic Energy equation*



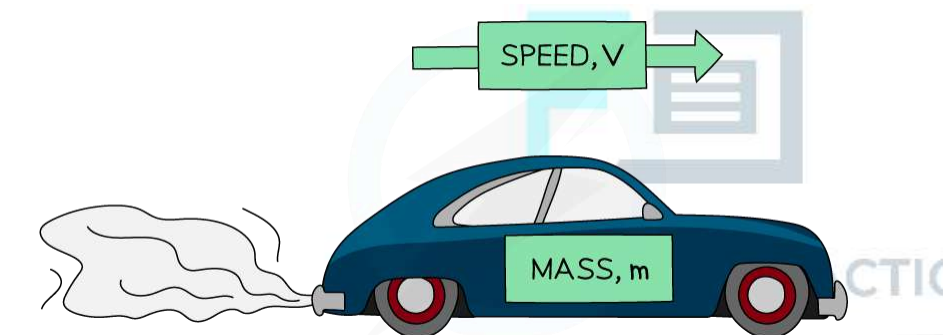
## Kinetic Energy

- Kinetic energy is energy an object has due to its **motion** (or velocity)
  - The faster an object is moving, the greater its kinetic energy
- When an object is falling, it is **gaining** kinetic energy since it is gaining speed. This energy transferred from the gravitational potential energy it is losing
- An object will maintain this kinetic energy unless its speed changes

$$KE = \frac{1}{2}mv^2$$

KINETIC ENERGY (J)      MASS (kg)      VELOCITY (ms<sup>-1</sup>)

*Equation for Kinetic energy*

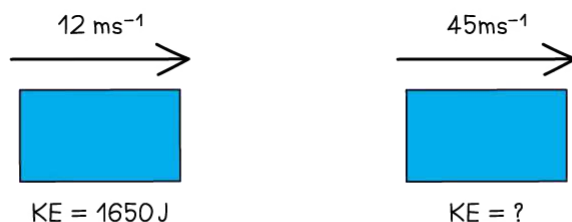


*KE: The energy an object has when its moving*

### ? Worked Example

A body travelling with a speed of  $12 \text{ m s}^{-1}$  has kinetic energy  $1650 \text{ J}$ . If the speed of the body is increased to  $45 \text{ m s}^{-1}$ , what is its new kinetic energy?





STEP 1

EQUATION FOR KINETIC ENERGY

$$KE = \frac{1}{2}mv^2$$

STEP 2

MASS WILL NOT CHANGE, SO CAN BE CALCULATED FROM ITS INITIAL KINETIC ENERGY

REARRANGE FOR MASS  $m$

$$m = \frac{2 \times KE}{v^2} = \frac{2 \times 1650}{12^2} = 23 \text{ kg}$$

STEP 3

SUBSTITUTE INTO KINETIC ENERGY EQUATION

USING VALUE OF MASS AND NEW VALUE OF VELOCITY

$$KE = \frac{1}{2} \times 23 \times 45^2 = 23000 \text{ J (2 s.f.)}$$



### Exam Tip

When using the kinetic energy equation, note that only the speed is squared, not the mass or the  $\frac{1}{2}$ . If a question asks about the 'loss of kinetic energy', remember not to include a negative sign since energy is a scalar quantity.