Pearson Edexcel International GCSE

Friday 15th November 2024

Morning (Time: 2 hours)

Paper reference **4PM1/02**

Further Pure Mathematics
PAPER 2

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
  – *there may be more space than you need*.
- You must **NOT** write anything on the formulae page.
  Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question*.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \dfrac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \dfrac{n}{2}\left[2a + (n-1)d\right]$

Geometric series

Sum to n terms, $S_n = \dfrac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \dfrac{a}{1-r} \quad |r| < 1$

Binomial series

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad$ for $\quad |x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule
In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$\tan\theta = \dfrac{\sin\theta}{\cos\theta}$

$\sin(A + B) = \sin A \cos B + \cos A \sin B \qquad \sin(A - B) = \sin A \cos B - \cos A \sin B$

$\cos(A + B) = \cos A \cos B - \sin A \sin B \qquad \cos(A - B) = \cos A \cos B + \sin A \sin B$

$\tan(A + B) = \dfrac{\tan A + \tan B}{1 - \tan A \tan B} \qquad \tan(A - B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

Logarithms

$\log_a x = \dfrac{\log_b x}{\log_b a}$

Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

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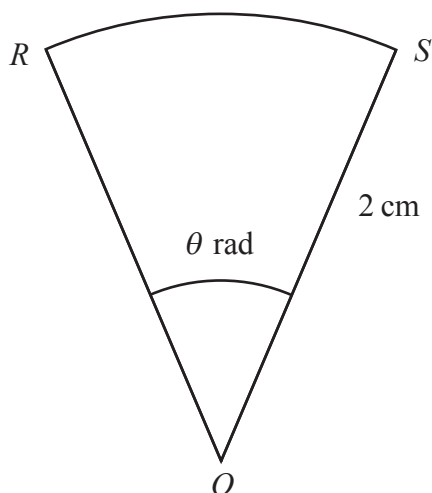
Diagram **NOT** accurately drawn

Figure 1

Figure 1 shows sector ROS of a circle with centre O and radius 2 cm
The size of angle ROS is θ radians.

The area of sector ROS is $\dfrac{\pi}{2}$ cm^2

(a) Find the exact value of θ

(2)

The perimeter of sector ROS is P cm

(b) Find the exact value of P

(3)

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(Total for Question 1 is 5 marks)

2   The length of rectangle R is 2 cm greater than its width.

The area of R is greater than 8 cm^2 and the perimeter of R is less than 30 cm.

Given that the width of R is w cm,

find the set of possible values of w
Give your answer in the form $a < w < b$ where a and b are rational numbers.

(6)

P 7 3 5 8 7 A 0 4 3 2

Question 2 continued

(Total for Question 2 is 6 marks)

3 The curve C has equation $y = e^{3x}(2x-1)^4$

Using calculus, find the exact value of the gradient of the tangent to C when $x = 1$

(5)

Question 3 continued

(Total for Question 3 is 5 marks)

4 (a) Complete the table of values for $y = \log_{10}(6x - 1) - x$ giving your answers to 2 decimal places.

x	0.25	0.5	1	1.5	2	2.5	3
y		−0.20	−0.30	−0.60			

(2)

(b) On the grid opposite, draw the graph of $y = \log_{10}(6x - 1) - x$ for $0.25 \leqslant x \leqslant 3$

(2)

(c) By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the root of the equation

$$10^{\frac{3x-4}{2}} = 6x - 1 \quad \text{in the interval } 0.25 \leqslant x \leqslant 3$$

(4)

Question 4 continued

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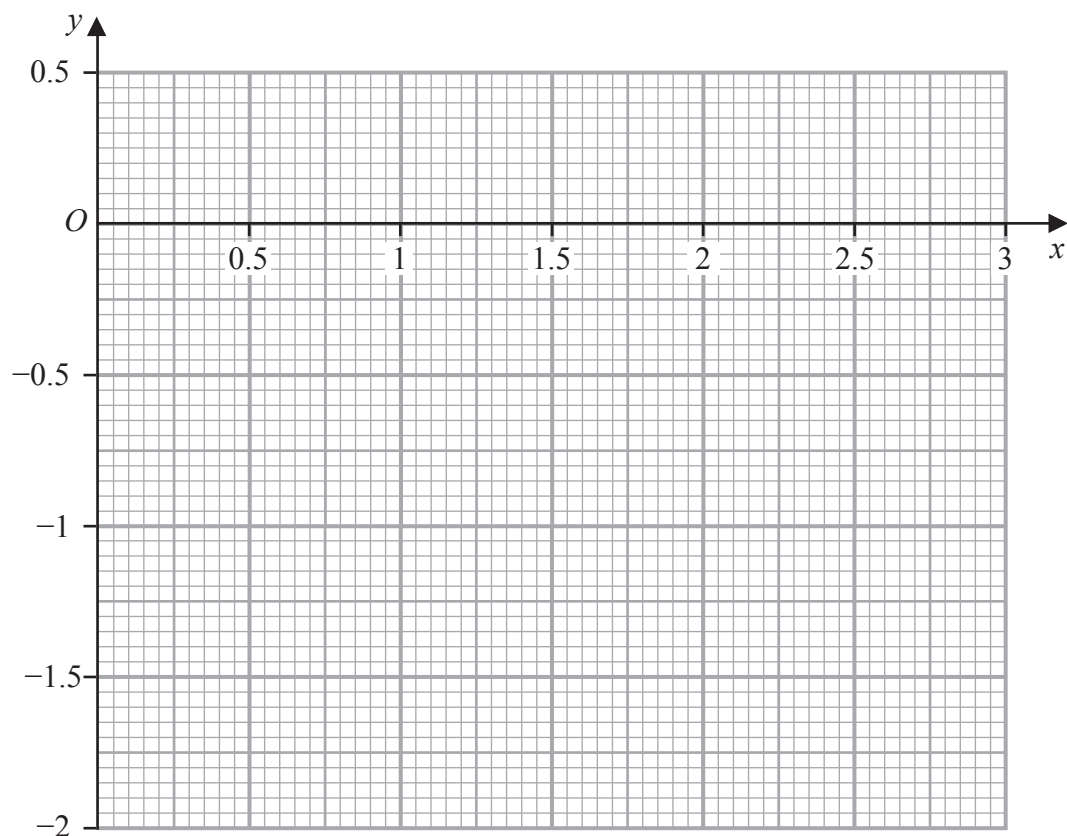
Turn over for a spare grid if you need to redraw your graph.

Question 4 continued

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Only use this grid if you need to redraw your graph.

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(Total for Question 4 is 8 marks)

5 The height of liquid in a vessel P is h

The volume, V, of the liquid in P is given by $V = 6h^3$

Liquid is leaking from P at a constant rate of $36\,\text{cm}^3/\text{s}$

Find the exact rate of change, in cm/s, of h when $V = 384\ \text{cm}^3$

(5)

(Total for Question 5 is 5 marks)

6 (i) Solve the equation $5\left(\log_b 9 + \log_b 3\right) = 3$

(4)

(ii) Solve the equation $3\log_3 x + 3\log_x 27 = 8\log_4 128$
Give your answers in exact form.

(7)

(Total for Question 6 is 11 marks)

P 7 3 5 8 7 A 0 1 5 3 2

7 (a) Use the factor theorem to show that $(4x-1)$ is a factor of

$$f(x) = 64x^3 - 64x^2 + 3$$

(2)

(b) Hence, or otherwise, find the exact roots of the equation

$$f(x) = 0$$

(4)

A geometric series G has first term a and common ratio r
The third term of G is 9 and the sum to infinity of G is 192

(c) Show that $64r^3 - 64r^2 + 3 = 0$

(3)

Given that r is a rational number

(d) write down the value of r

(1)

(e) show that $a = 144$

(2)

The sum to n terms of G is S_n

(f) Using logarithms, find the least value of n such that $S_n > 191.9$

(4)

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Question 7 continued

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(Total for Question 7 is 16 marks)

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8

D   Diagram **NOT** accurately drawn
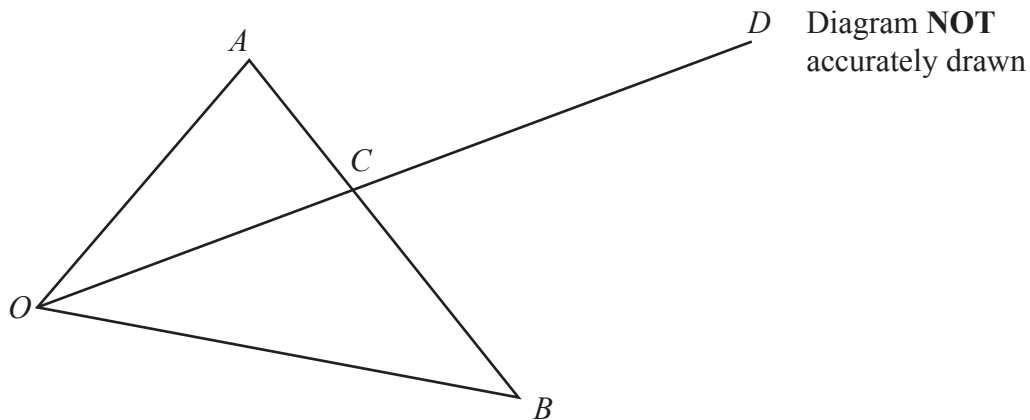
Figure 2

Figure 2 shows triangle *AOB*

$$\overrightarrow{OA} = 4\mathbf{a} + 5\mathbf{b} \qquad \overrightarrow{OB} = 8\mathbf{a} - \mathbf{b} \qquad \overrightarrow{OD} = 15\mathbf{a} + 10\mathbf{b} \qquad \text{where } |\mathbf{a}| = |\mathbf{b}| = 1$$

(a) (i) Find $\overrightarrow{AB}$ in terms of **a** and **b**

(ii) Find, in its simplest form, the exact value of $|\overrightarrow{AB}|$

(3)

(b) Find the area of triangle *AOB*

(4)

The point *C* lies on *AB* and *OD* such that *O*, *C* and *D* are collinear.

(c) Use a vector method to find vector $\overrightarrow{OC}$ as a simplified expression in terms of **a** and **b**

(5)

P 7 3 5 8 7 A 0 2 1 3 2

P 7 3 5 8 7 A 0 2 2 3 2

(Total for Question 8 is 12 marks)

9 (a) Using a formula given on page 2, show that

$$\cos 2\theta = 2\cos^2 \theta - 1$$

(2)

(b) Hence show that

$$\int_{\frac{\pi}{3}}^{\frac{3\pi}{4}} \left(2\cos^2 \theta - 1 \right) d\theta = -\frac{a + \sqrt{b}}{c}$$

where *a, b* and *c* are integers to be found.

(4)

Diagram **NOT**
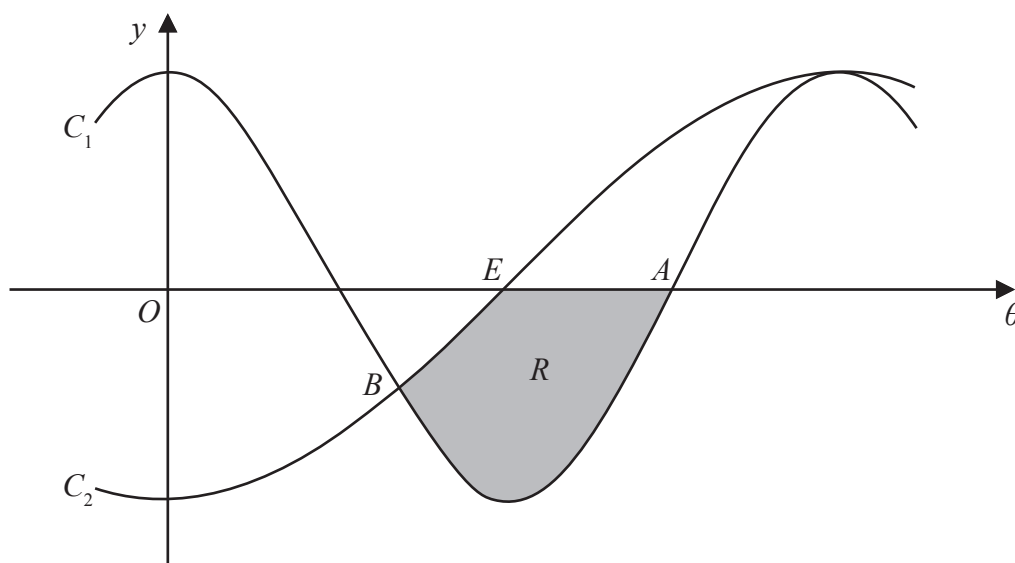accurately drawn

Figure 3

Figure 3 shows part of the curve C_1 with equation $y = 2\cos^2 \theta - 1$ and part of the curve C_2 with equation $y = -\cos\theta$

Point *B* is the intersection of C_1 and C_2 as shown in Figure 3

Point $A\left(\dfrac{3\pi}{4}, 0 \right)$ is the intersection of C_1 with the θ-axis as shown in Figure 3

Point $E\left(\dfrac{\pi}{2}, 0 \right)$ is the intersection of C_2 with the θ-axis as shown in Figure 3

The finite region *R*, shown shaded in Figure 3, is bounded by the θ-axis, C_1 and C_2

(c) Use calculus to find, in its simplest form, the exact area of *R*

(8)

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Question 9 continued

(Total for Question 9 is 14 marks)

10 A curve C has equation

$$y = \frac{5x-2}{3x+2} \qquad x \neq -\frac{2}{3}$$

(a) Find the coordinates of the point where C intersects the

 (i) x-axis
 (ii) y-axis

(2)

(b) Write down an equation of the asymptote to C that is

 (i) parallel to the x-axis
 (ii) parallel to the y-axis

(2)

(c) Sketch C on the opposite page.
 Show and label the asymptotes and the coordinates of the points where C crosses the coordinate axes.

(3)

Point A lies on C such that the gradient of C at A is parallel to the line with equation $4y - x = 7$

The normal to C at A intersects the x-axis at point D and the y-axis at point E
Given that the x coordinate of A is positive,

(d) find, in its simplified form, the exact length of line DE

(11)

Question 10 continued

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(Total for Question 10 is 18 marks)

TOTAL FOR PAPER IS 100 MARKS

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