Please check the examination details bel	ow before enter	ring your candidate information
Candidate surname		Other names
Centre Number Candidate No Pearson Edexcel Inter		al GCSE
Time 2 hours	Paper reference	4PM1/01R
Further Pure Mat	hemat	tics
Calculators may be used.		Total Marks

Instructions

- Use black ink or ball-point pen.
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided
 - there may be more space than you need.
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
 - use this as a guide as to how much time to spend on each question.

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶







International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere = $4\pi r^2$

Curved surface area of cone = $\pi r \times \text{slant height}$

Volume of sphere = $\frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to *n* terms, $S_n = \frac{n}{2} [2a + (n-1)d]$

Geometric series

Sum to *n* terms,
$$S_n = \frac{a(1-r^n)}{(1-r)}$$

Sum to infinity, $S_{\infty} = \frac{a}{1-r} |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots + \frac{n(n-1)\dots(n-r+1)}{r!}x^r + \dots$$
 for $|x| < 1, n \in \mathbb{Q}$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x} \left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)} \right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{\left[\mathrm{g}(x)\right]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc \cos A$

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\sin(A+B) = \sin A \cos B + \cos A \sin B \qquad \sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\sin(A-B) = \sin A \cos B - \cos A \sin B$$

$$\cos(A+B) = \cos A \cos B - \sin A \sin B$$

$$\cos(A - B) = \cos A \cos B + \sin A \sin B$$

$$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A \tan B}$$

$$\tan(A - B) = \frac{\tan A - \tan B}{1 + \tan A \tan B}$$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$



Answer all TEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 Given that $\frac{a-\sqrt{48}}{\sqrt{3}+1}$ can be written in the form $b\sqrt{3}-9$ where a and b are integers,

find the value of *a* and the value of *b* Show your working clearly.

(4)



(Total for Question 1 is 4 marks)

2	In $\triangle ABC$,				
		$\angle BAC = 50^{\circ}$	$AB = 10 \mathrm{cm}$	BC = 9 cm	
	Given that $\angle BCA = x^{\circ}$				
	find the two possible val	ues, to one decima	l place, of x		
					(3)





3	An arithmetic series has first term 16 and common difference -5 The sum to n terms of this series is S_n	
	Given that $S_n < -450$	
	find the least value of <i>n</i>	(4)



4 O, A and B are fixed points such that

$$\overrightarrow{OA} = p\mathbf{i} + 2p\mathbf{j}$$
 $\overrightarrow{OB} = 5\mathbf{i} + 9p\mathbf{j}$

Given that \overrightarrow{AB} is parallel to $(\mathbf{i} - 2\mathbf{j})$

(a) find the value of p

(6)

(b) Hence find \overrightarrow{AB} as a simplified expression in terms of **i** and **j**

(2)

(c) Find a unit vector parallel to \overrightarrow{OA}

Give your answer in the form $\frac{\sqrt{a}}{5}(b\mathbf{i}+c\mathbf{j})$ where a, b and c are integers to be found.









(5)

5	$f(x) = 2ax^3 + x^2 - bx + 3a$ where a and b are integers.	
	Given that $(x + 2)$ and $(x - 1)$ are both factors of $f(x)$	
	(a) show that $a = 2$ and find the value of b	(5)
	(b) Hence factorise $f(x)$ completely.	(2)
	Hence, given that $h(y) = 2^{(3y+2)} + 2^{2y} - 11(2^y) + 6$	

(c)	solve the equation	h(y) = 0	
	Where appropriate	give your answers to 3 decimal places	





- 6 The curve C has equation $y = \frac{e^{(x^2+1)}}{x^2+1}$
 - (a) Show that $\frac{dy}{dx} = \frac{Kx^3e^{(x^2+1)}}{(x^2+1)^2}$ where *K* is a constant whose value is to be found.

(5)

(b) Find an equation of the tangent to C at the point on C where x = -1 Simplify your answer.

(5)

|
 |
|------|------|------|------|------|------|------|
|
 |
|
 |
 |
 |
 |
 | | |
|
 |





A particle P is moving along the x-axis. At time t seconds $(t \ge 0)$ the velocity of P is v m/s where

$$v = t^2 - 10t + 28$$

(a) Find the velocity of P when t = 1

(1)

Given that the distance of P from the origin is 24 m when t = 3

(b) find the distance of P from the origin when t = 5

(5)

(c) Find the acceleration of P when t = 9

(2)

- (d) (i) Show that there are no values of t for which P is instantaneously at rest.
 - (ii) Find the least magnitude of the velocity of P

(3)

|
 |
|------|------|------|------|------|------|------|
|
 |
|
 |
|
 |
|
 | | | | |
 | |
| | | | | | | |



Question 7 continued	



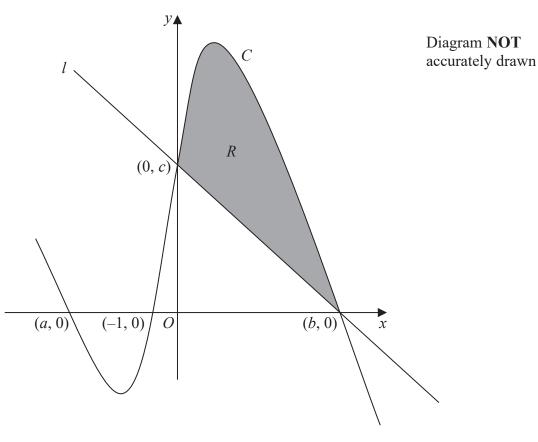


Figure 1

Figure 1 shows part of the curve C with equation y = f(x)

The curve C passes through the points with coordinates

$$(a, 0), (-1, 0), (b, 0)$$
 and $(0, c)$

Given that $f'(x) = 17 + 2x - 3x^2$

(a) show that the equation of C is $y = 15 + 17x + x^2 - x^3$

(4)

(b) Find the value of a, the value of b and the value of c

(6)

The straight line l intersects C at the points with coordinates (b, 0) and (0, c)

The region R, shown shaded in Figure 1, is bounded by l and C

(c) Use algebraic integration to find the exact area of region R

(5)



\cdots

$\times\!\!\times\!\!\times\!\!\times$

\times

\times

$\times\!\!\!/\!\!\!/$
XXXX

×33×
\times
\times 111

XX
\times
\times
\times

$\times \omega \times$

30
× in

\times
\times

\times
\times
-XXXXXXX
5
VR.
VRITE
VR.
VRITE
VRITE IN
VRITE
VRITE IN
VRITE IN TH
VRITE IN
VRITE IN TH
VRITE IN THI
VRITE IN THIS A
VRITE IN THIS A
VRITE IN THIS AR
VRITE IN THIS ARE
VRITE IN THIS ARE
VRITE IN THIS AR
VRITE IN THIS ARE
VRITE IN THIS AREA
VRITE IN THIS AREA
VRITE IN THIS AREA D
VRITE IN THIS AREA
VRITE IN THIS AREA DO
VRITE IN THIS AREA DO
VRITE IN THIS AREA DO NO
VRITE IN THIS AREA DO N
VRITE IN THIS AREA DO NOT
VRITE IN THIS AREA DO NOT
VRITE IN THIS AREA DO NOT V
VRITE IN THIS AREA DO NOT W
VRITE IN THIS AREA DO NOT W
VRITE IN THIS AREA DO NOT WRI
VRITE IN THIS AREA DO NOT WRIT
VRITE IN THIS AREA DO NOT WRIT
VRITE IN THIS AREA DO NOT WRITE
VRITE IN THIS AREA DO NOT WRITE I
VRITE IN THIS AREA DO NOT WRITE IN
VRITE IN THIS AREA DO NOT WRITE IN
VRITE IN THIS AREA DO NOT WRITE IN
VRITE IN THIS AREA DO NOT WRITE IN TH
VRITE IN THIS AREA DO NOT WRITE IN TH
VRITE IN THIS AREA DO NOT WRITE IN THI
VRITE IN THIS AREA DO NOT WRITE IN TH
VRITE IN THIS AREA DO NOT WRITE IN THIS .
VRITE IN THIS AREA DO NOT WRITE IN THIS A
VRITE IN THIS AREA DO NOT WRITE IN THIS AI
VRITE IN THIS AREA DO NOT WRITE IN THIS AR
VRITE IN THIS AREA DO NOT WRITE IN THIS ARE
VRITE IN THIS AREA DO NOT WRITE IN THIS AREA
VRITE IN THIS AREA DO NOT WRITE IN THIS ARE
VRITE IN THIS AREA DO NOT WRITE IN THIS AREA
VRITE IN THIS AREA DO NOT WRITE IN THIS AREA
VRITE IN THIS AREA DO NOT WRITE IN THIS AREA
VRITE IN THIS AREA DO NOT WRITE IN THIS AREA
VRITE IN THIS AREA DO NOT WRITE IN THIS AREA

G	ė		ù	į
				¢
	S			Į
Š	Ì	2	S	į
á	ľ	2		į
j		į		į
j	Š			į
Í	2			
Í				
	>			

Question 8 continued	
	•••
	•••
	•••
	•••
	•••
	•••





 $A \longrightarrow B$

Diagram **NOT** accurately drawn

Figure 2

Figure 2 shows the right pyramid *OABCD* with a square base *ABCD* of side 12 cm.

$$OA = OB = OC = OD = x \text{ cm}$$
 and $\angle OAC = \angle ODB = \angle OCA = \angle OBD = 30^{\circ}$

(a) Find the exact length of AC

(2)

(b) Show that $x = 4\sqrt{6}$

(2)

(c) Find the total surface area, to the nearest cm², of the pyramid.

(5)

(d) Find the size of the obtuse angle, to the nearest degree, between the plane OAB and the plane OBC

(4)



2	2
>	>
S	D
S	S
	Ċ
	2
>	>
>	>
	<
	<
	2
S	>
	ς
	2
	>
	7
>	>
	ς
	2
S	>
	>
,	ς
>	Þ
S	5
	<
	Ċ
	2
S	>
>	5
	<
	Ċ
>	>
>	5
	<
	d
	2
>	>
S	5
	<
2	Ċ
>	2
S	>
ς	ς
	2
>	2
>	>
	S
	<
	2
>	2
>	3
	3
	3
	3
	?
	?
	3
	3
	?
	?
	3
>>>>>>	

DO

'n				
ì				
		Ħ		
				ì
2	<	2	ζ	Ì
Ì			Ş	ì
9		2	3	j
Ì	5	2	3	į
Ì				ĺ
3				ĺ
}				į
				į
3	>			
3	>			
3				
3				

Question 9 continued	





10 Using formulae from page 2

(a) show that cos(A - B) - cos(A + B) = 2 sin A sin B

(2)

(b) Hence show that $\cos 5\theta - \cos 9\theta = 2\sin 7\theta \sin 2\theta$

(1)

(c) Solve the equation

$$\cos 5\theta - \cos 9\theta = \sqrt{3} \sin 7\theta$$
 for $0 < \theta \leqslant \frac{1}{3}\pi$

Give your solutions in terms of π

(7)

(d) Using calculus and showing your working, evaluate

$$\int_0^{\frac{\pi}{7}} 8\sin 7x \cos 2x \tan 2x \, \mathrm{d}x$$

Give your answer to 3 decimal places.





Question 10 continued	
	(Total for Question 10 is 16 marks)
	TOTAL FOR PAPER IS 100 MARKS