Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number | | | | | |

Candidate Number | | | | |

Pearson Edexcel International GCSE

Friday 26 May 2023

Afternoon (Time: 2 hours)

Paper reference **4PM1/01**

Further Pure Mathematics
PAPER 1

Calculators may be used.

Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Without sufficient working, correct answers may be awarded no marks.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You must **NOT** write anything on the formulae page. Anything you write on the formulae page will gain NO credit.

Information

- The total mark for this paper is 100.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Check your answers if you have time at the end.

Turn over ▶

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P 7 2 8 6 4 A 0 1 3 2

Pearson

International GCSE in Further Pure Mathematics Formulae sheet

Mensuration

Surface area of sphere $= 4\pi r^2$

Curved surface area of cone $= \pi r \times$ slant height

Volume of sphere $= \frac{4}{3}\pi r^3$

Series

Arithmetic series

Sum to n terms, $S_n = \frac{n}{2}[2a + (n-1)d]$

Geometric series

Sum to n terms, $S_n = \frac{a(1-r^n)}{(1-r)}$

Sum to infinity, $S_\infty = \frac{a}{1-r} \quad |r| < 1$

Binomial series

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \quad \text{for} \quad |x| < 1, n \in \mathbb{Q}$$

Calculus

Quotient rule (differentiation)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\mathrm{f}(x)}{\mathrm{g}(x)}\right) = \frac{\mathrm{f}'(x)\mathrm{g}(x) - \mathrm{f}(x)\mathrm{g}'(x)}{[\mathrm{g}(x)]^2}$$

Trigonometry

Cosine rule

In triangle ABC: $a^2 = b^2 + c^2 - 2bc\cos A$

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$\sin(A+B) = \sin A\cos B + \cos A\sin B$

$\sin(A-B) = \sin A\cos B - \cos A\sin B$

$\cos(A+B) = \cos A\cos B - \sin A\sin B$

$\cos(A-B) = \cos A\cos B + \sin A\sin B$

$\tan(A+B) = \frac{\tan A + \tan B}{1 - \tan A\tan B}$

$\tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\tan B}$

Logarithms

$$\log_a x = \frac{\log_b x}{\log_b a}$$

Answer all ELEVEN questions.

Write your answers in the spaces provided.

You must write down all the stages in your working.

1 (a) Show that $\sum_{r=1}^{n}(3r+2) = \frac{n}{2}(3n+7)$

(3)

(b) Hence, or otherwise, evaluate $\sum_{r=10}^{40}(3r+2)$

(2)

(Total for Question 1 is 5 marks)

2

$$y = (\sin 2x)\sqrt{3 + 2x}$$

Show that $\dfrac{dy}{dx} = \dfrac{\sin 2x + (A + Bx)\cos 2x}{\sqrt{3 + 2x}}$ where A and B are integers to be found.

(5)

Question 2 continued

(Total for Question 2 is 5 marks)

3

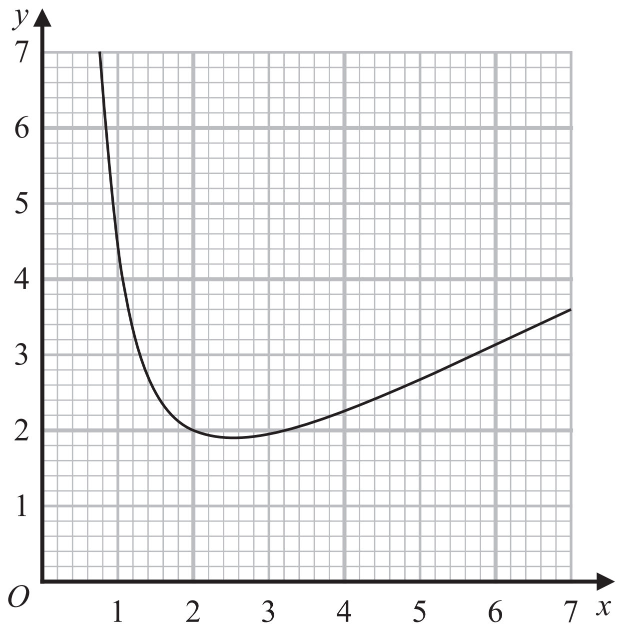

Figure 1

Figure 1 shows part of the curve with equation $y = \frac{x}{2} + \frac{4}{x^2}$ in the interval $0.8 < x < 7$

By drawing a suitable straight line on the grid, obtain an estimate, to one decimal place, of the roots of the equation $3x^3 - 12x^2 + 8 = 0$ in the interval $0.8 < x < 7$

(5)

Question 3 continued

(Total for Question 3 is 5 marks)

4 A particle P is moving along the x-axis. At time t seconds, $t \geqslant 0$, the velocity, v m/s, of P is given by $v = 2t^2 - 16t + 30$

(a) Find the acceleration, in m/s^2, of P when $t = 5$

(2)

P comes to instantaneous rest at the points M and N at times t_1 seconds and t_2 seconds where $t_2 > t_1$

(b) Find the exact distance MN

(8)

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Question 4 continued

(Total for Question 4 is 10 marks)

5 A solid cuboid has width x cm, length $4x$ cm and height h cm.
The volume of the cuboid is 75 cm^3 and the surface area of the cuboid is $S\text{ cm}^2$

(a) Show that $S = 8x^2 + \dfrac{375}{2x}$

(4)

Given that x can vary, using calculus,

(b) (i) find to 3 significant figures, the value of x for which S is a minimum,

(ii) justify that this value of x gives a minimum value of S

(5)

(c) Find, to 3 significant figures, the minimum value of S

(2)

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Question 5 continued

Question 5 continued

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Question 5 continued

(Total for Question 5 is 11 marks)

6 Solve the equation

$$\log_2 x^3 + \log_4 x^2 - 3\log_x 2 = 0$$

giving your answers to 3 significant figures.

(8)

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Question 6 continued

(Total for Question 6 is 8 marks)

7 The equation of a curve is $y = \sqrt{\dfrac{e^{4x}}{2x - 3}}$

When x is increased to $(x + \delta x)$, y increases to $(y + \delta y)$ where δx and δy are small.

(a) Show that $\delta y \approx \dfrac{e^{2x}(4x - 7)}{(2x - 3)^{\frac{3}{2}}}\,\delta x$

(7)

Given that $x = 2.5$

(b) find an estimate, to 2 significant figures, of the value of δy when the value of x increases by 0.2%

(3)

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Question 7 continued

Question 7 continued

Question 7 continued

(Total for Question 7 is 10 marks)

8 $$f'(x) = 18x^2 - 2x + 13$$

Given that $(2x - 1)$ is a factor of $f(x)$

show that the curve with equation $y = f(x)$ has only one intersection with the x-axis.

(9)

Question 8 continued

(Total for Question 8 is 9 marks)

9 (a) Using the formulae on page 2, show that

(i) $\cos^2 A = \dfrac{\cos 2A + 1}{2}$

(ii) $\sin^2 A = \dfrac{1 - \cos 2A}{2}$

(4)

(b) Show that

$$(2\sin x - \cos x)(\sin x - 3\cos x) = \frac{1}{2}(\cos 2x - 7\sin 2x + 5)$$

(5)

$$y = (2\sin x - \cos x)(\sin x - 3\cos x)$$

(c) Solve, for $0° \leqslant x \leqslant 180°$ the equation, $\dfrac{dy}{dx} = 0$

Give your answers to the nearest whole number.

(4)

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Question 9 continued

Question 9 continued

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Question 9 continued

(Total for Question 9 is 13 marks)

10 O, A and B are fixed points such that

$$\overrightarrow{OA} = (b + 1)\mathbf{i} + b\mathbf{j}$$
$$\overrightarrow{AB} = 3\mathbf{i}$$

The unit vector parallel to $\overrightarrow{OB}$ is $\dfrac{\sqrt{17}}{34}\left[(3a + 2)\mathbf{i} + b\mathbf{j}\right]$

Given that a and b are constants where $a > 0$ and $b > 0$

find the exact value of

(i) a

(ii) b

(10)

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Question 10 continued

Question 10 continued

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Question 10 continued

(Total for Question 10 is 10 marks)

P 7 2 8 6 4 A 0 2 9 3 2

11 $$f(x) = 10 + 6x - x^2$$

Given that $f(x)$ can be written in the form $A(x + B)^2 + C$ where A, B and C are constants,

(a) find the value of A, the value of B and the value of C

(4)

(b) Hence, or otherwise, find

(i) the value of x for which $f(x)$ has its greatest value

(ii) the greatest value of $f(x)$

(2)

The curve C has equation $y = f(x)$

The curve S with equation $y = x^2 - x + 13$ intersects curve C at two points.

(c) Find the x coordinate of each of these two points.

(3)

(d) Use algebraic integration to find the exact area of the finite region bounded by the curve C and the curve S.

(5)

Question 11 continued

Question 11 continued

(Total for Question 11 is 14 marks)

TOTAL FOR PAPER IS 100 MARKS