Exam Papers Practice

4.6 Normal Distributions

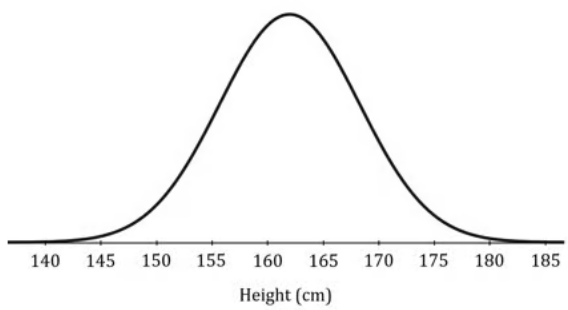
Question Paper

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.6 Normal Distributions
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1a

The random variable, X is seen on the following diagram which shows the distribution of heights, in cm, of adult women in the UK:



The distribution of heights follows a normal distribution, with a mean of 162 cm and a standard deviation of 6.3 cm.

On the diagram above, shade in the region representing P$(X > 155)$

[2 marks]

Question 1b

(i)
Find the probability that a randomly selected woman has a height of more than 155cm.

(ii)
Use your answer from part (b)(i) to find the probability that a randomly selected woman has a height of more than 169cm.

[4 marks]

Question 1c

Suggest a range of heights within which the height of approximately

(i)
68%

(ii)
95%

(iii)
99.7%

of adult women in the UK will fall.

[3 marks]

Question 2a

For the random variable $X \sim N(23, 4^2)$ find the following probabilities:

(i)
$P(X < 20)$

(ii)
$P(X \geq 29)$

(iii)
$P(20 \leq X < 29)$

[3 marks]

Question 2b

For the random variable $Y \sim N(100,225)$ find the following probabilities:

(i)
$P(Y \leq 90)$

(ii)
$P(Y > 140)$

(iii)
$P(85 \leq Y \leq 115)$

[3 marks]

Question 3a

The weight, W g, of a chocolate bar produced by a certain manufacturer is modelled as $W \sim N(200,\ 1.75^2)$.

Find:

(i)
$P(W < 195)$

(ii)
$P(W > 203)$

[2 marks]

Question 3b

Heledd buys a pack containing 12 of the chocolate bars. It may be assumed that the 12 bars in the pack represent a random sample.

Find the probability that all of the bars in the pack have a weight of at least 195 g.

[2 marks]

Question 4a

The random variable $X \sim N(330, 10^2)$.

Find the value of a, to 2 decimal places, such that:

(i)
$P(X < a) = 0.25$

(ii)
$P(X > a) = 0.25$

(iii)
$P(315 \leq X \leq a) = 0.5$

[4 marks]

Question 4b

The random variable $Y \sim N(10, 10)$.

Find the value of b and the value of c, each to 2 decimal places, such that:

(i)
$P(Y < b) = 0.4$

(ii)
$P(Y > c) = 0.25$

[2 marks]

Question 4c

Use a sketch of the distribution of Y to explain why $P(b \le Y \le c) = 0.35$.

[2 marks]

Question 5a

The test scores, X, of a group of RAF recruits in an aptitude test are modelled as a normal distribution with $X \sim N(210, 27.8^2)$.

(i)
Find the values of a and b such that $P(X < a) = 0.25$ and $P(X > b) = 0.25$.

(ii)
Hence find the interquartile range of the scores.

[3 marks]

Question 5b

Those who score in the top 30% on the test move on to the next stage of training.

One of the recruits, Amelia, achieves a score of 231. Determine whether Amelia will move on to the next stage of training.

[2 marks]

Question 6a

For the standard normal distribution $Z \sim N(0, 1^2)$, find:

(i)
$P(Z < 1.5)$

(ii)
$P(Z > -0.8)$

(iii)
$P(-2.1 < Z < -0.3)$

[4 marks]

Question 6b

The random variable $X \sim N(2, 0.1^2)$.

By using the coding relationship between and , re-express the probabilities from parts (a) (i), (ii) and (iii) in the forms $P(X < a)$, $P(X > b)$ and $P(c < X < d)$ respectively, where a, b, c, and d are constants to be found.

[3 marks]

Question 7a

The table below shows the percentage points of the normal distribution. The values z in the table are those which a random variable $Z \sim N(0,1)$ exceeds with probability p.

p	z	p	z
0.5000	0.0000	0.0500	1.6449
0.4000	0.2533	0.0250	1.9600
0.3000	0.5244	0.0100	2.3263
0.2000	0.8416	0.0050	2.5758
0.1500	1.0365	0.0010	3.0902
0.1000	1.2816	0.0005	3.2905

(i)
Use the percentage points table for the standard normal distribution to find the value of z for which $P(Z > z) = 0.2$.

(ii)
Use your answer to part (a)(i) along with the properties of the normal distribution to work out the values of a and b for which $P(Z < a) = 0.2$ and $P(Z < b) = 0.8$.

[3 marks]

Question 7b

The weights, W kg, of coconuts grown on the Coconutty As They Come coconut plantation are modelled as a normal distribution with mean 1.25 kg and standard deviation 0.38 kg. The plantation only considers coconuts to be exportable if their weight falls into the 20% to 80% interpercentile range.

Use your answer to part (a)(ii) to find the range of possible weights, to the nearest 0.01 kg, for an exportable coconut.

[2 marks]

Question 8a

A machine is used to fill cans of a particular brand of soft drink. The volume, V ml, of soft drink in the cans is normally distributed with mean 330 ml and standard deviation σ ml. Given that 15% of the cans contain more than 333.4 ml of soft drink, find:

the value of σ

[2 marks]

Question 8b

$P(320 \leq V \leq 340)$.

[1 mark]

Question 8c

Six cans of the soft drink are chosen at random.

Find the probability that all of the cans contain less than 329 ml of soft drink.

[3 marks]

Question 9a

The random variable $X \sim N(\mu, \sigma^2)$. It is known that $P(X > 36.88) = 0.025$ and $P(X < 27.16) = 0.1$

Find the values of a and b for which $P(Z > a) = 0.025$ and $P(Z < b) = 0.1$, where Z is the standard normal variable. Give your answers correct to 4 decimal places.

[2 marks]

Question 9b

Use your answers to part (a), along with the relationship between Z and X, to show that the following simultaneous equations must be true:

$$\mu + 1.96\sigma = 36.88$$

$$\mu - 1.2816\sigma = 27.16$$

[2 marks]

Question 9c

By solving the simultaneous equations in (b), determine the values of μ and σ. Give your answers correct to 2 decimal places.

[2 marks]

Question 10

The ages, A in years, that Liverpool players have made their debuts over the past 20 years are normally distributed with a mean of 22.5 years and a standard deviation of σ years.

Given that 10% of Liverpool players make their debuts before turning 20 years old, find:

(i)
the value of σ,

(ii)
the probability that a randomly selected player made his debut before his 18th birthday.

[5 marks]