4.6 Normal Distributions

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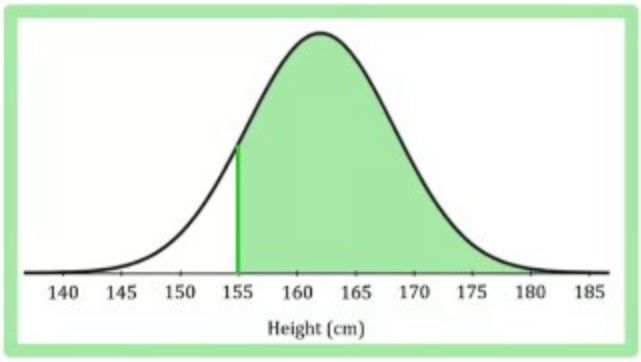
Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.6 Normal Distributions
Difficulty	Medium

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful

Question 1



(b)(i) Using the normal distribution on your calculator.

Lower = 155

Upper = 999...

$P(X > 155) = 0.866739...$

$P(X > 155) = 0.8667$ (4dp)

(ii) The normal distribution is symmetrical about its mean.

$P(X > \mu + k) = P(X < \mu - k)$

$P(X > 169) = P(X < 155)$

$= 1 - P(X > 155)$

$= 1 - 0.866739...$

$= 0.133260...$

$P(X > 169) = 0.1333$ (4dp)

(c)(i) 68% of data lies between $\mu \pm \sigma$

$\mu + \sigma = 162 + 6.3 = 168.3$

$\mu - \sigma = 162 - 6.3 = 155.7$

68% of heights lie in range 155.7cm to 168.3cm

95 % of data lies between $\mu \pm 2\sigma$

$\mu + 2\sigma = 162 + 2(6.3) = 174.6$

$\mu - 2\sigma = 162 - 2(6.3) = 149.4$

95% of heights lie in range 149.4 cm to 174.6 cm

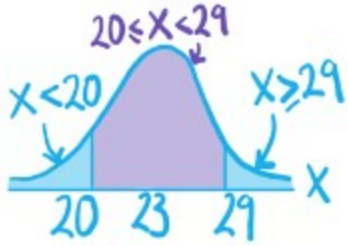
99.7 % of data lies between $\mu \pm 3\sigma$

$\mu + 3\sigma = 162 + 3(6.3) = 180.9$

$\mu - 3\sigma = 162 - 3(6.3) = 143.1$

99.7% of heights lie in range 143.1 cm to 180.9 cm

Question 2



$X \sim N(23, 4^2)$
$\mu = 23$
$\sigma^2 = 4^2$
$\sigma = 4$

a) For a normal distribution $P(X<k) = P(X \leq k)$

For probabilities use 4dp or 3sf (whichever is more accurate)

(i) Lower = -999...
Upper = 20
$P(X<20) = 0.226627...$

$P(X<20) = 0.2266$ (4dp)

(ii) Lower = 29
Upper = 999...
$P(X \geq 29) = 0.066807...$

$P(X \geq 29) = 0.0668$ (4dp)

(iii) Lower = 20
Upper = 29
$P(20 \leq X < 29) = 0.706565...$

$P(20 \leq X < 29) = 0.7066$ (4dp)

$Y \sim N(100, 225)$
$\mu = 100$
$\sigma^2 = 225$
$\sigma = 15$

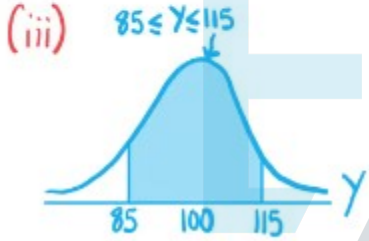
b) (i) Lower = -999...
Upper = 90 $P(Y \leq 90) = 0.252492...$

$P(Y \leq 90) = 0.2525$ (4dp)

(ii) Lower = 140
Upper = 999... $P(Y > 140) = 0.003830...$

$P(Y > 140) = 0.00383$ (3sf)

(iii)



Lower = 85
Upper = 115
$P(85 \leq Y \leq 115) = 0.682689...$

$P(85 \leq Y \leq 115) = 0.6827$ (4dp)

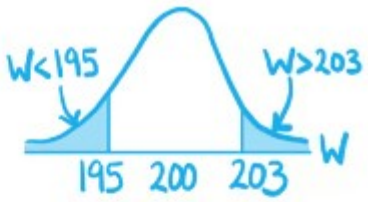
Question 3

(a) $W \sim N(200, 1.75^2)$

$\mu = 200$

$\sigma^2 = 1.75^2$

$\sigma = 1.75$



(i) Lower = -999...

Upper = 195

$P(W < 195) = 0.002137...$

$P(W < 195) = 0.00214$ (3sf)

(ii) Lower = 203

Upper = 999...

$P(W > 203) = 0.043238...$

$P(W > 203) = 0.0432$ (4 dp)

b) Let X be the number of chocolate bars in the sample that have a weight of at least 195g then
$X \sim B(12, p)$

$p = P(W \geq 195)$
$= 1 - P(W < 195)$
$= 1 - 0.002137...$
$= 0.997863...$

Use full answer to avoid rounding errors.

$\therefore X \sim B(12, 0.997863)$

If all of the sample exceed 195g then $X = 12$

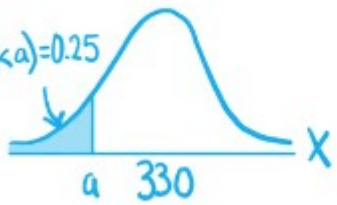
$P(X = 12) = (0.997863)^{12}$
$= 0.974655...$

$P(X = 12) = 0.9747$ (4dp)

Question 4

(a) $X \sim N(330, 10^2)$ $\mu = 330$ $\sigma^2 = 10^2$ $\sigma = 10$

(i) $P(X < a) = 0.25$

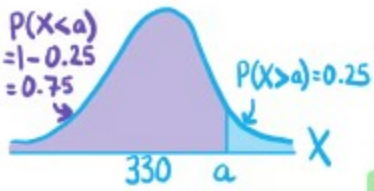


Inverse Normal
Area = 0.25

$a = 323.255...$

$a = 323.26$ (2dp)

(ii)

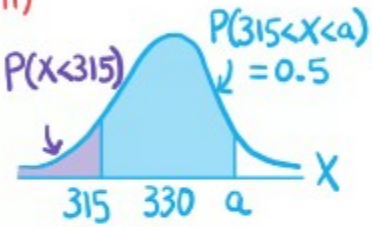


Inverse Normal
Area = 0.75 ← Area uses P(X<a)

$a = 336.744...$

$a = 336.74$ (2dp)

(iii)



$P(X < 315) = 0.066807...$

$P(X < a) = P(X < 315) + P(315 < X < a)$

$P(X < a) = 0.066807... + 0.5$

$= 0.566807...$

Inverse Normal
Area = 0.566807

$a = 331.682...$

$a = 331.68$ (2dp)

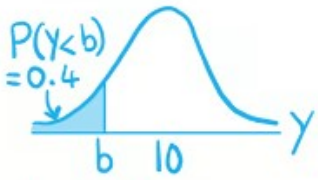
(b) $Y \sim N(10, 10)$

$\mu = 10$

$\sigma^2 = 10$

$\sigma = \sqrt{10}$

(i) $P(Y < b) = 0.4$



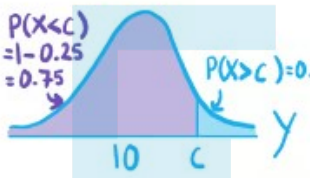
Inverse Normal
Area = 0.4

$b = 9.19884...$

$b = 9.20$ (2dp)

(ii) $P(X < c) = 1 - 0.25 = 0.75$

$P(X > c) = 0.25$

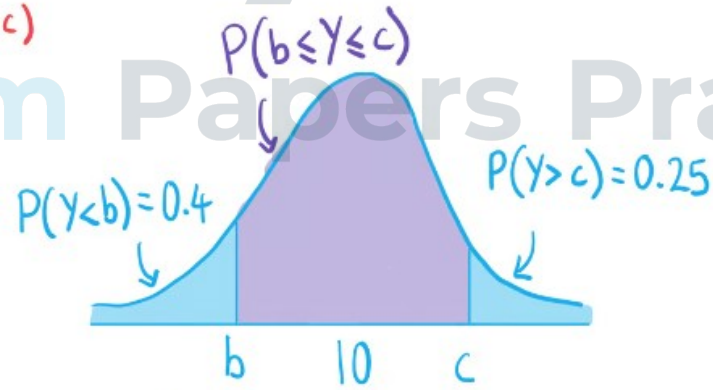


Inverse Normal
Area = 0.75

$c = 12.1329...$

$c = 12.13$ (2dp)

c)



The total area is 1

$P(Y < b) + P(b \leq Y \leq c) + P(Y > c) = 1$

$P(b \leq Y \leq c) = 1 - P(Y < b) - P(Y > c)$

$= 1 - 0.4 - 0.25$

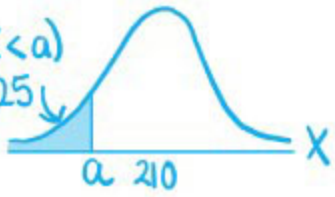
$= 0.35$

Question 5

(a) $X \sim N(210\ 27.8^2)$

$\mu = 210$
$\sigma^2 = 27.8^2$
$\sigma = 27.8$

(i) $P(X<a) = 0.25$

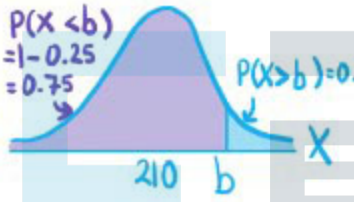


Inverse Normal
Area = 0.25
$a = 191.249...$

$a = 191$ (3sf)

$P(X<b) = 1 - 0.25 = 0.75$

$P(X>b) = 0.25$



Inverse Normal
Area = 0.75
$a = 228.750...$

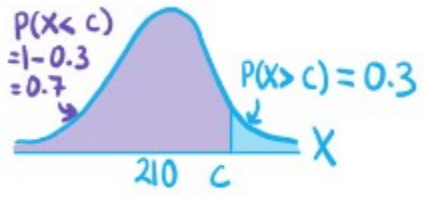
$a = 229$ (3sf)

(ii) $P(X<LQ) = 0.25$ and $P(X>UQ) = 0.25$

$IQR = 228.750... - 191.249...$
$= 37.501...$

$IQR = 37.5$ (3sf)

(b) In the top 30% means $X \geq c$ where $P(X \geq c) = 0.3$



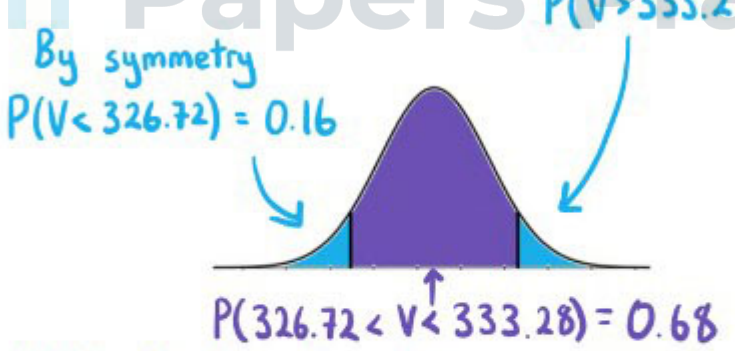
Inverse Normal
Area = 0.7

$c = 224.578...$

$231 > 224.578...$

Amelia is in the top 30% and will move on to the next stage of training.

Question 6 (a) $V \sim N(330, \sigma^2)$



68% of data lies between $\mu \pm \sigma$

$\mu + \sigma = 333.28$

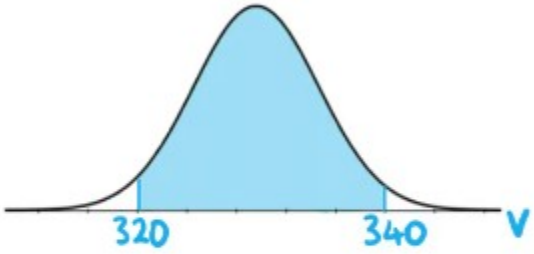
$330 + \sigma = 333.28$

$\sigma = 3.28$ as $\mu + \sigma = 333.28$

(b) Using the normal distribution on your calculator.

Lower = 320

Upper = 340



$P(320 \leq V \leq 340) = 0.99770227...$

$P(320 \leq V \leq 340) = 0.9977$ (4dp)

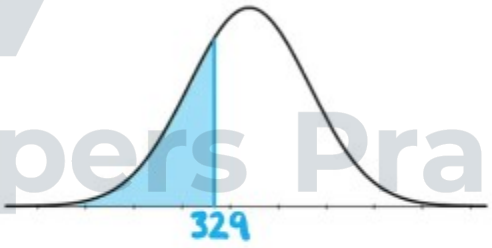
(c) Let X be the number of the 6 cans that contain less than 329ml.

$X \sim B(6, p)$

$p = P(V < 329)$

Lower = -999...

Upper = 329



$p = 0.38022951...$

$P(X = 6) = p^6$

$P(X = 6) = (0.38022951...)^6$

$= 0.00302186...$

$P(X = 6) = 0.00302$ (3sf)