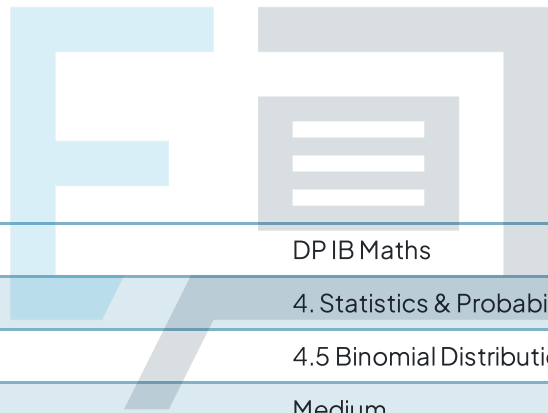




# 4.5 Binomial Distributions

## Mark Schemes



Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.5 Binomial Distributions
Difficulty	Medium

# Exam Papers Practice

To be used by all students preparing for DP IB Maths AA SL  
Students of other boards may also find this useful

## Question 1

- a)
- A trial has two outcomes (success or failure)
  - An experiment has a finite number of trials ( $n$ )
  - The probability of success is constant ( $p$ )
  - The trials are independent of each other.

If these four conditions are met then the number of successes,  $X$ , follows a binomial distribution  $B(n, p)$ .

(b)(i)  $S$  is not binomial as there are more than two outcomes.

(ii)  $W$  is not binomial as there is not a fixed number of trials.

(iii)  $Y$  is binomial with  $B(12, \frac{1}{2})$  as it has 12 independent spins and landing on a prime is a success which has a constant probability of  $\frac{1}{2}$ .

(iv)  $L$  is not binomial as getting a win depends on the previous spin.

Question 2

(a) Let  $X$  be the number of times the coin lands heads up when tossed 20 times.

$$X \sim B(20, \frac{1}{2})$$

$$E(X) = np \text{ (from formula booklet)}$$

$$E(X) = (20)(\frac{1}{2})$$

$$E(X) = 10$$

(b)  $X \sim B(20, \frac{1}{2})$

Using the calculator (Probability Distribution)

$$P(X=15) = 0.0147857\dots$$

$$P(X=15) = 0.0148 \text{ (4dp)}$$

Question 3

(a) Let  $X$  be the number of days in a week that Yussuf catches a taxi.

$$X \sim B(5, 0.6)$$

$$E(X) = np \text{ (from formula booklet)}$$

$$E(X) = (5)(0.6)$$

$$E(X) = 3$$

(b)  $X \sim B(5, 0.6)$

Using the calculator (Probability Distribution)

$$P(X=0) = 0.01024$$

(c)  $X \sim B(5, 0.6)$

At most once means  $X \leq 1$

Using the calculator (Cumulative Distribution)

$$P(X \leq 1) = 0.08704$$

$$P(X \leq 1) = 0.08704$$

#### Question 4

(a) Let  $X$  be the number of successful operations.

$$X \sim B(12, 0.78)$$

All successful  $\Rightarrow X=12$

Using the calculator (Probability Distribution)

$$P(X=12) = 0.0507148\dots$$

$$P(X=12) = 0.0507 \quad (4dp)$$

(b)  $X \sim B(12, 0.78)$

All but two  $\Rightarrow X=10$

Using the calculator (Probability Distribution)

$$P(X=10) = 0.266278\dots$$

$$P(X=10) = 0.2663 \quad (4dp)$$



## Question 5

Let  $X$  be the number of jellyfish in the sample that contain microplastic particles.

$$X \sim B(40, 0.95)$$

(i) Using the calculator (Probability Distribution)

$$P(X=38) = 0.277671\dots$$

$$P(X=38) = 0.2777 \text{ (4dp)}$$

(ii) Using the calculator (Probability Distribution)

$$P(X=40) = 0.128512\dots$$

$$P(X=40) = 0.1285 \text{ (4dp)}$$

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## Question 6

(i) Let  $X$  be the number of times the dice lands on a 2 when rolled 10 times.

$$X \sim B(10, 0.25)$$

Using the calculator (Probability Distribution)

$$P(X=4) = 0.145998\dots$$

$$P(X=4) = 0.1460 \text{ (4dp)}$$

(ii) Let  $Y$  be the number of times the dice lands on a 2 when rolled 9 times.

$$Y \sim B(9, 0.25)$$

[3 two's in the  
first 9 rolls]

and

[A two on  
the 10<sup>th</sup> roll]

$$P(Y=3)$$

$$0.25$$

$$0.233596\dots$$

$$\times 0.25$$

$$= 0.058399\dots$$

$$0.0584 \text{ (4dp)}$$

## Question 7

$$L \sim B(24, 0.0296)$$

(i) Using the calculator (Probability Distribution)

$$P(L=0) = 0.486204\dots$$

$$P(L=0) = 0.4862 \text{ (4dp)}$$

(ii) Using the calculator (Probability Distribution)

$$P(L=2) = 0.124856\dots$$

$$P(L=2) = 0.1249 \text{ (4dp)}$$

(iii) At least 2  $\Rightarrow L \geq 2$

Using the calculator (Cumulative Distribution)

$$\begin{aligned} P(L \geq 2) &= 1 - P(L \leq 1) \\ &= 1 - 0.842139\dots \\ &= 0.157861\dots \end{aligned}$$

$$P(L \geq 2) = 0.1579 \text{ (4dp)}$$

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## Question 8

(a)(i)  $3 < X < 14$  is  $X \leq 13$  without  $X \leq 2$   
Using the calculator (Cumulative Distribution)  
$$P(3 < X < 14) = P(X \leq 13) - P(X \leq 2)$$
$$= 0.998596... - 0.048598...$$
$$= 0.94999...$$

$$P(3 < X < 14) = 0.9500 \text{ (4dp)}$$

(ii)  $5 < X < 12$  is  $X \leq 11$  without  $X \leq 5$   
Using the calculator (Cumulative Distribution)  
$$P(5 < X < 12) = P(X \leq 11) - P(X \leq 5)$$
$$= 0.988030... - 0.432500...$$
$$= 0.55553...$$

$$P(5 < X < 12) = 0.5555 \text{ (4dp)}$$

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(b)  $\text{Var}(X) = np(1-p)$  (from formula booklet)  
$$\text{Var}(X) = (40)(0.15)(1-0.15)$$

$$\text{Var}(X) = 5.1$$



$$c) P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (\text{from formula booklet})$$

$$P(X \leq 3 | X \leq 9) = \frac{P(X \leq 3 \cap X \leq 9)}{P(X \leq 9)}$$

$$= \frac{P(X \leq 3)}{P(X \leq 9)} \quad \leftarrow (X \leq 3) \cap (X \leq 9) = (X \leq 3)$$

$$= \frac{0.130168...}{0.932779...} \quad \leftarrow \text{Using the calculator}$$

$$= 0.13954...$$

$$P(X \leq 3 | X \leq 9) = 0.1395 \quad (4dp)$$

Question 9 (a) Let  $X$  be the number of the 10 routines containing a mistake.

$$X \sim B(10, 0.2)$$

(i)  $E(X) = np$  (from formula booklet)

$$E(X) = (10)(0.2)$$

$$\text{Expected number of routines} = 2$$

(ii)  $\text{Var}(X) = np(1-p)$  (from formula booklet)

$$\text{Var}(X) = (10)(0.2)(1-0.2)$$

$$\text{Var}(X) = 1.6$$

$$\text{Standard Deviation of } X = \sqrt{\text{Var}(X)}$$

$$\text{Standard Deviation of } X = \sqrt{1.6}$$

$$= 1.264911...$$

$$\text{Standard Deviation} = 1.26 \quad (3sf)$$



(b)  $X \sim B(10, 0.2)$

(i) Using the calculator (Probability Distribution)

$$P(X=0) = 0.107374\dots$$

$$P(X=0) = 0.1074 \text{ (4dp)}$$

(ii) Using the calculator (Probability Distribution)

$$P(X=2) = 0.301989\dots$$

$$P(X=2) = 0.3020 \text{ (4dp)}$$

(iii) None more than 2  $\Rightarrow X \leq 2$

Using the calculator (Cumulative Distribution)

$$P(X \leq 2) = 0.677799\dots$$

$$P(X \leq 2) = 0.6778 \text{ (4dp)}$$

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$$c) P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (\text{from formula booklet})$$

$$P(X=3 | X \geq 2) = \frac{P(X=3 \cap X \geq 2)}{P(X \geq 2)}$$

$$= \frac{P(X=3)}{P(X \geq 2)} \quad \leftarrow (X=3) \cap (X \geq 2) = (X=3)$$

$$= \frac{P(X=3)}{1 - P(X \leq 1)}$$

$$= \frac{0.201326...}{1 - 0.375809...} \quad \leftarrow \text{Using the calculator}$$

$$= 0.32254...$$

$$P(X=3 | X \geq 2) = 0.3225 \quad (4dp)$$

d) From (a):

$$\text{Mean} = 2$$

$$\text{Standard Deviation} = \sqrt{1.6}$$

Less than one standard deviation away from the mean.

$$2 - \sqrt{1.6} < X < 2 + \sqrt{1.6}$$

$$0.735... < X < 3.264...$$

$X$  can only be positive integers  $\Rightarrow 1 \leq X \leq 3$

$1 \leq X \leq 3$  is  $X \leq 3$  without  $X=0$

$$P(1 \leq X \leq 3) = P(X \leq 3) - P(X=0)$$

$$= 0.879126... - 0.107374...$$

$$= 0.77175...$$

$$0.7718 \quad (4dp)$$

## Question 10

$$a) X \sim B(30, 0.9)$$

$$(i) E(X) = np \text{ (from formula booklet)}$$

$$E(X) = (30)(0.9)$$

$$E(X) = 27$$

$$(ii) \text{Var}(X) = np(1-p) \text{ (from formula booklet)}$$

$$\text{Var}(X) = (30)(0.9)(1-0.9)$$

$$\text{Var}(X) = 2.7$$

$$(b)(i) 90\% \text{ of } 30 = 27$$

$$90\% \text{ or more} \Rightarrow X \geq 27$$

Using the calculator (Cumulative Distribution)

$$P(X \geq 27) = 1 - P(X \leq 26)$$

$$= 1 - 0.352560\dots$$

$$= 0.647439\dots$$

$$P(X \geq 27) = 0.6474 \text{ (4dp)}$$

(ii) None prefer other brand  $\Rightarrow$  All prefer Woostershire

$$\therefore X = 30$$

Using the calculator (Probability Distribution)

$$P(X = 30) = 0.042391\dots$$

$$P(X = 30) = 0.0424 \text{ (4dp)}$$