EXAM PAPERS PRACTICE

4.5 Linear Momentum & Conservation

PHYSICS

AQA A Level Revision Notes

EXAM PAPERS PRACTICE

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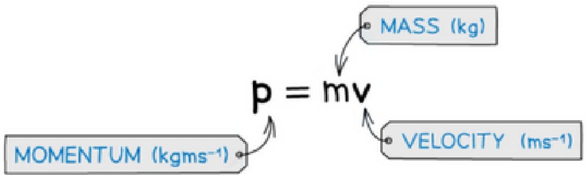
CONTENTS

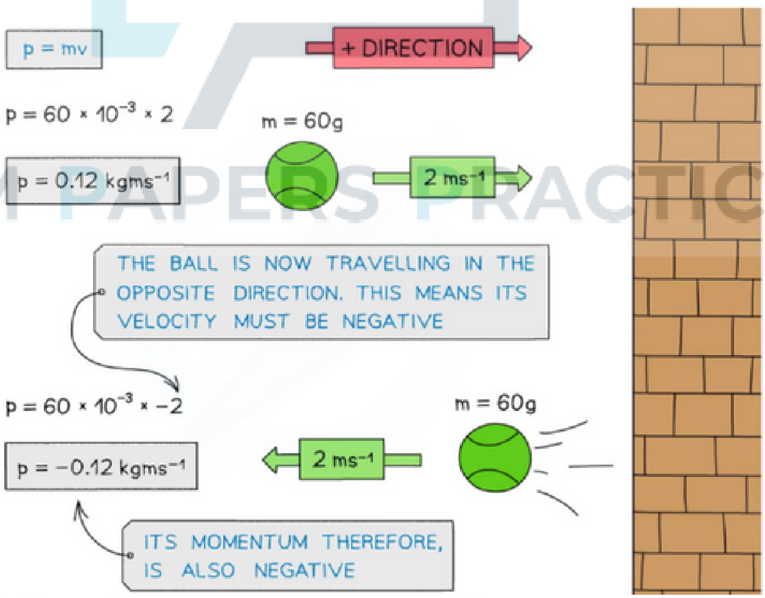
Linear Momentum

- Linear momentum (p) is defined as the product of mass and velocity



Momentum is the product of mass and velocity

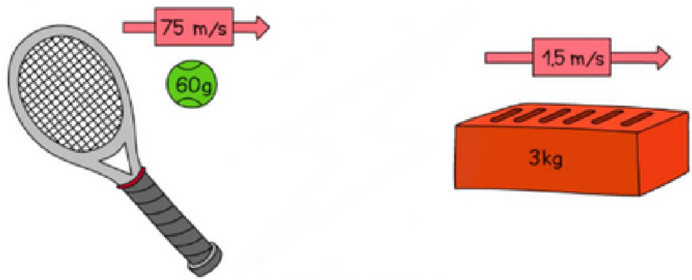
- Momentum is a vector quantity - it has both a magnitude and a direction
- This means it can have a negative or positive value
  - If an object travelling to the right has positive momentum, an object travelling to the left (in the opposite direction) has a negative momentum
- The SI unit for momentum is **kg m s**$^{-1}$



When the ball is travelling in the opposite direction, its velocity is negative. Since momentum = mass × velocity, its momentum is also negative

Worked Example

Which object has the most momentum?



MOMENTUM = MASS × VELOCITY
MOMENTUM = 0.06 kg × 75 m/s
= 4.5 kgm/s

MOMENTUM = MASS × VELOCITY
MOMENTUM = 3 kg × 1.5 m/s
= 4.5 kgm/s

- Both the tennis ball and the brick have the **same momentum**
- Even though the brick is much heavier than the ball, the ball is travelling much faster than the brick
- This means that on impact, they would both exert a similar force (depending on the time it takes for each to come to rest)

Exam Tip

Since momentum is in **kg m s^{-1}**:

- If the mass is given in grams, make sure to convert to kg by dividing the value by 1000.
  - If the velocity is given in km s^{-1}, make sure to convert to m s^{-1} by multiplying the value by 1000
- The direction you consider positive is your choice, as long the signs of the numbers (positive or negative) are consistent with this throughout the question

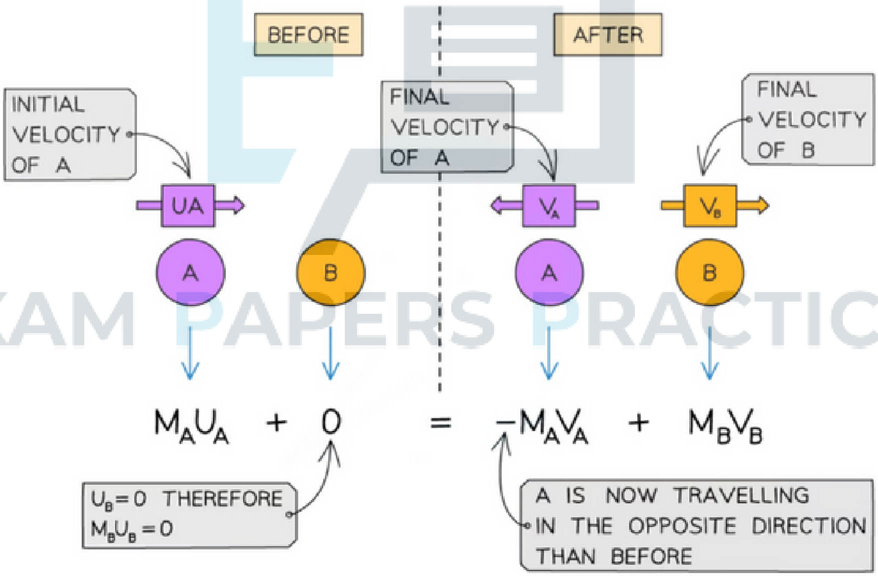
The Principle of Conservation of Momentum

- The principle of conservation of linear momentum states:

  The total momentum before a collision = the total momentum after a collision provided no external force acts

- Linear momentum is the momentum of an object that only moves in one dimension
- Momentum is a **vector** quantity
  - This means oppositely-directed vectors can cancel each other out resulting in a net momentum of zero
  - If after a collision an object starts to move in the opposite direction to which it was initially travelling, its velocity will now be **negative**

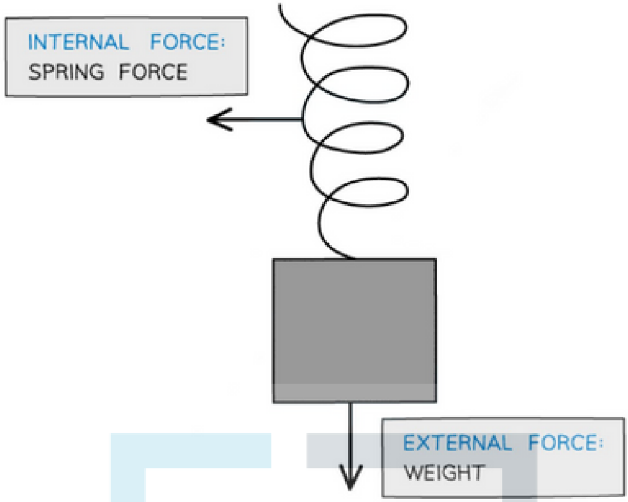
- Momentum, just like energy, is **always conserved**



The conservation of momentum for two objects A and B colliding then moving apart

External and Internal Forces

- **External forces** are forces that act on a structure from outside e.g. friction and weight
- **Internal forces** are forces exchanged by the particles in the system e.g. tension in a string
- Forces which are internal or external will depend on the system itself, as shown in the diagram below:



Internal and external forces on a mass on a spring

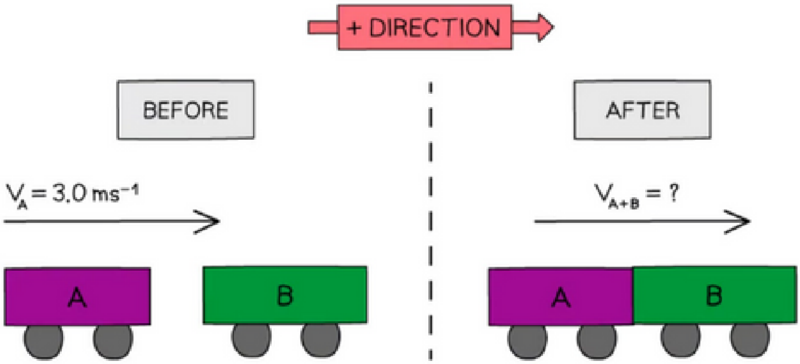
- Systems with no external forces may be described as '**closed**' or '**isolated**'
  - These are keywords that refer to a system that is not affected by external forces
- For example, a swimmer diving from a boat:
  - The diver will move **forwards**, and, to conserve momentum, the boat will move **backwards**
  - This is because the momentum beforehand was zero and no **external forces** were present to affect the motion of the diver or the boat

Worked Example

Trolley **A** of mass 0.80 kg collides head-on with stationary trolley **B** whilst travelling at

3.0 m s^{-1}. Trolley **B** has twice the mass of trolley **A**. On impact, the trolleys stick together.

Using the conservation of momentum, calculate the common velocity of both trolleys after the collision.



MOMENTUM BEFORE $= (M_A \times V_A) + (M_B \times V_B)$

$= (0.8\ kg \times 3.0\ ms^{-1}) + 0$

$= 2.4\ kgms^{-1}$

SINCE TROLLEY B IS STATIONARY, V = 0 THEREFORE ITS MOMENTUM IS 0

MOMENTUM AFTER $= (M_A + M_B) \times V_{A+B}$

$= (0.8\ kg + 1.60\ kg) \times V_{A+B}$

$= 2.4\ kg \times V_{A+B}$

TROLLEY B HAS TWICE THE MASS OF TROLLEY A

THE PRINCIPLE OF CONVERSATION OF MOMENTUM STATES THAT THE TOTAL MOMENTUM OF A SYSTEM REMAINS CONSTANT PROVIDED NO EXTERNAL FORCE ACTS ON IT

MOMENTUM BEFORE = MOMENTUM AFTER

$2.4\ kgms^{-1} = 2.4\ kg \times V_{A+B}$

$$V_{A+B} = \frac{2.4\ kgms^{-1}}{2.4\ kg}$$

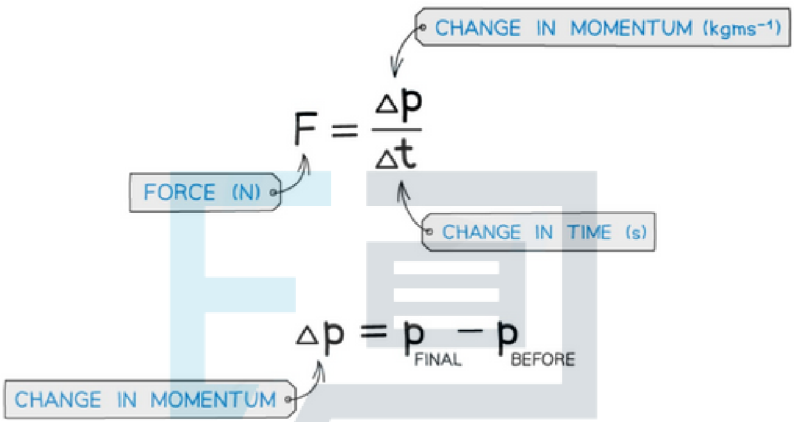
REARRANGING FOR V_{A+B}

$V_{A+B} = 1.0\ ms^{-1}$

4.5.3 Impulse

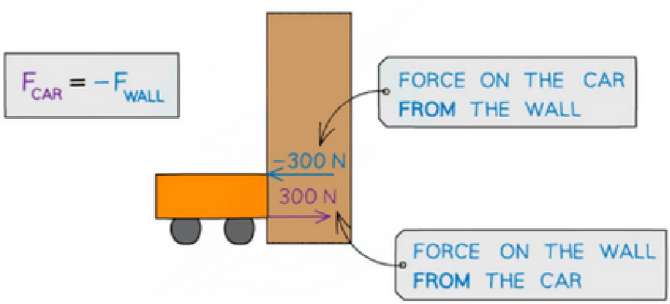
Force & Momentum

- Force is defined as the **rate of change of momentum** on a body
- The change in momentum is defined as the final momentum minus the initial momentum
- These can be expressed as follows:



Direction of Forces

- Force and momentum are **vectors** so they can take either positive or negative values
- The force that is equal to the rate of change of momentum is still the **resultant force**
- A force on an object will be negative if it is directed in the opposite motion to its initial velocity
  - This means that the force is **produced by** the object it has collided with



The wall produces a force of -300N on the car and (due to Newton's Third Law) the car also produces a force of 300 N back onto the wall

Worked Example

A car of mass 1500 kg hits a wall at an initial velocity of 15 m s^{-1}.

It then rebounds off the wall at 5 m s^{-1} and comes to rest after 3.0 s.

Calculate the average force experienced by the car.

STEP 1

FORCE IS EQUAL TO THE RATE OF CHANGE IN MOMENTUM

$$F = \frac{\Delta p}{\Delta t}$$

STEP 2

CHANGE IN MOMENTUM

Δp = FINAL MOMENTUM − INITIAL MOMENTUM

STEP 3

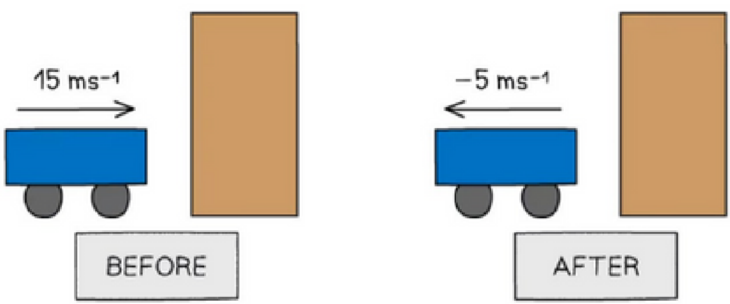
INITIAL MOMENTUM

INITIAL MOMENTUM = MASS × INITIAL VELOCITY

$$p_i = m \times v_i$$

$$= 1500 \text{ kg} \times 15 \text{ ms}^{-1}$$

$$p_i = 22500 \text{ kgms}^{-1}$$



STEP 4

FINAL MOMENTUM

FINAL MOMENTUM = MASS × FINAL VELOCITY

$$p_f = m \times v_f$$

$$= 1500 \text{ kg} \times -5 \text{ ms}^{-1}$$

$$p_f = -7500 \text{ kgms}^{-1}$$

STEP 5

CALCULATE CHANGE IN MOMENTUM Δp

$$\Delta p = -7500 - 22500 = -30000 \text{ kgms}^{-1}$$

STEP 6

SUBSTITUTE THIS VALUE BACK INTO THE FORCE EQUATION

$$F = \frac{\Delta p}{\Delta t} = \frac{-30000}{3} = -10000 \text{N}$$

Exam Tip

In an exam question, carefully consider what produces the force(s) acting. Look out for words such as '**from**' or '**acting on**' to determine this and don't be afraid to draw a force diagram to figure out what is going on.

Impulse

- The force and momentum equation can be rearranged to find the impulse
- Impulse, I, is equal to the **change in momentum**:

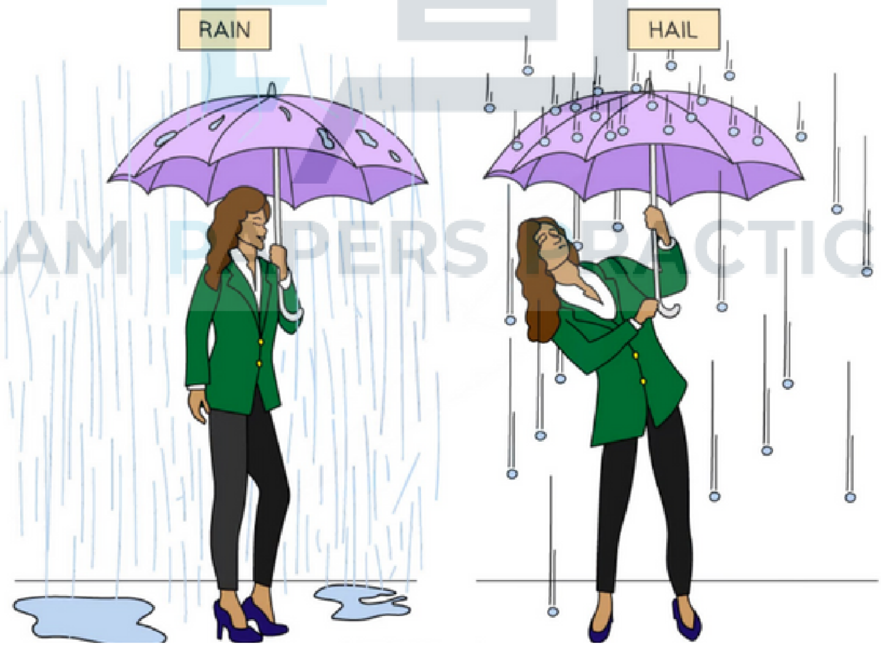
$$I = F\Delta t = \Delta p = mv - mu$$

- Where:
  - I = impulse (N s)
  - F = force (N)
  - t = time (s)
  - p = momentum (kg m s^{-1})
  - m = mass (kg)
  - v = final velocity (m s^{-1})
  - u = initial velocity (m s^{-1})

- This equation is only used when the force is **constant**
  - Since the impulse is proportional to the force, it is also a vector
  - The impulse is in the same direction as the force
- The unit of impulse is **N s**
- The impulse quantifies the effect of a force acting over a time interval
  - This means a **small force acting over a long time** has the same effect as a **large force acting over a short time**

Examples of Impulse

- An example in everyday life of impulse is when standing under an umbrella when it is raining, compared to hail (frozen water droplets)
  - When rain hits an umbrella, the water droplets tend to splatter and fall off it and there is only a very **small** change in momentum
  - However, hailstones have a **larger mass** and tend to bounce back off the umbrella, creating a **greater** change in momentum
  - Therefore, the impulse on an umbrella is **greater** in hail than in rain
  - This means that **more force** is required to hold an umbrella upright in hail compared to rain



Since hailstones bounce back off an umbrella, compared to water droplets from rain, there is a greater impulse on an umbrella in hail than in rain

Worked Example

A 58 g tennis ball moving horizontally to the left at a speed of 30 m s^{-1} is struck by a tennis racket which returns the ball back to the right at 20 m s^{-1}.

(i) Calculate the impulse delivered to the ball by the racket
(ii) State which direction the impulse is in

(i) Step 1: Write the known quantities

- Taking the initial direction of the ball as positive (the left)
- Initial velocity, $u = 30$ m s^{-1}
- Final velocity, $v = -20$ m s^{-1}
- Mass, $m = 58$ g $= 58 \times 10^{-3}$ kg

Step 2: Write down the impulse equation

$$\text{Impulse } I = \Delta p = m(v - u)$$

Step 3: Substitute in the values

$$I = (58 \times 10^{-3}) \times (-20 - 30) = -2.9 \text{ N s}$$

(ii) Direction of the impulse

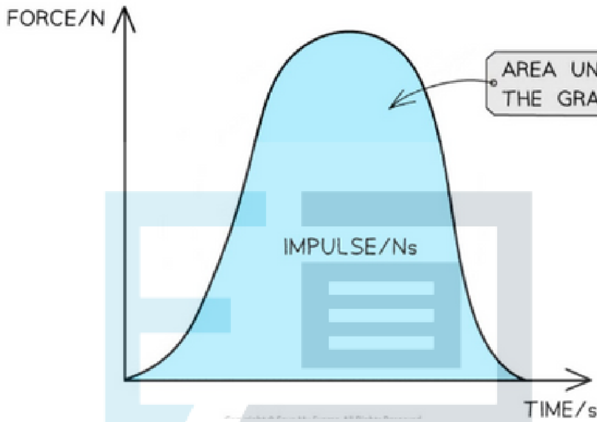
- Since the impulse is negative, it must be in the opposite direction to which the tennis ball was initial travelling (since the left is taken as positive)
- Therefore, the direction of the impulse is to the right

Exam Tip

Remember that if an object changes direction, then this must be reflected by the change in sign of the velocity. As long as the magnitude is correct, the final sign for the impulse doesn't matter as long as it is consistent with which way you have considered positive (and negative). For example, if the left is taken as positive and therefore the right as negative, an impulse of 20 N s to the right is equal to -20 N s

Impulse of a Force-Time Graph

- In real life, forces are often not constant and will vary over time
- If the force is plotted against time, ***the impulse is equal to the area under the force-time graph***



When the force is not constant, the impulse is the area under a force-time graph

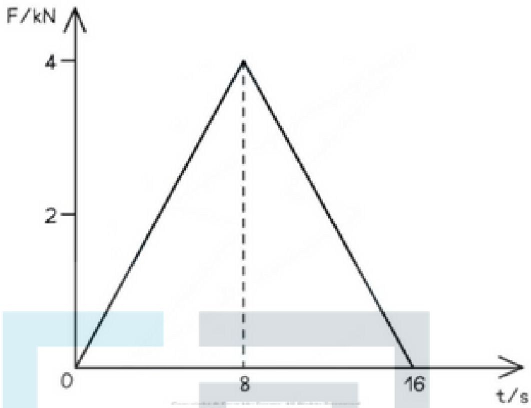
- This is because

Impulse = Force × Change in time

- The impulse is therefore equal whether there is a small force over a long period of time or a large force over a small period of time
- The force-time graph may be a **curve** or a **straight line**
  - If the graph is a curve, the area can be found by counting the squares underneath
  - If the graph is made up of straight lines, split the graph into sections. The total area is the **sum of the areas** of each section

Worked Example

A ball of mass 3.0 kg, initially at rest, is acted on by a force F which varies with t as shown by the graph.



Calculate the velocity of the ball after 16 s.

Step 1: List the known quantities

$m = 3.0$ kg

$u = 0$ m s^{-1} (since it is initially at rest)

Step 2: Calculate the impulse

The impulse is the area under the graph. The graph can be split up into two right-angled triangles with a base of 8 s and a height of 4 kN

$$\text{Area} = \left(\frac{1}{2} \times 8 \times (4 \times 10^3)\right) + \left(\frac{1}{2} \times (16 - 8) \times (4 \times 10^3)\right)$$

Area = impulse = 32×10^3 Ns

Step 3: Write the equation for impulse

$$\text{Impulse, } I = \Delta p = m(v - u)$$

Step 4: Substitute in the values

$$I = mv$$

$$32 \times 10^3 = 3.0 \times v$$

$$v = (32 \times 10^3) \div 3.0$$

$$v = 10666 \text{ m s}^{-1} = 11 \text{ km s}^{-1}$$

Exam Tip

Some maths tips for this section:**Rate of Change**

- 'Rate of change' describes how one variable changes with respect to another
- In maths, how fast something changes with **time** is represented as dividing by **Δt** (e.g. acceleration is the rate of change in velocity)
- More specifically, **Δt** is used for finite and quantifiable changes such as the difference in time between two events

Areas

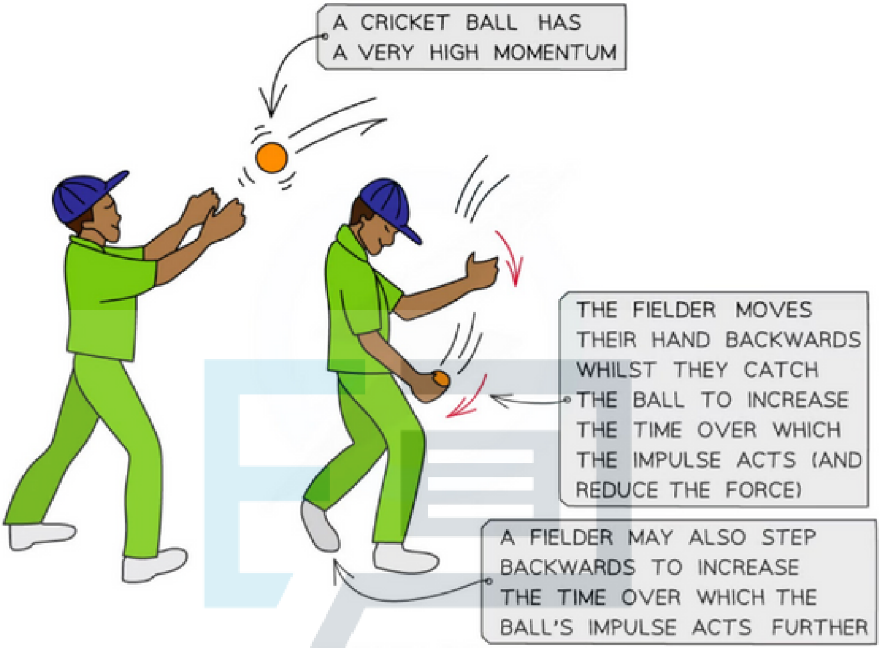
- The area under a graph may be split up into different shapes, so make sure you're comfortable with calculating the area of squares, rectangles, right-angled triangles and trapeziums!

Impact Forces

- Impact forces are reduced by increasing the contact time
- This fact is used in everyday life to lower the risk of injury
- Some example of where reducing impact force is important:
  - In sport
  - In packaging

In Sports

- For example, in cricket:
  - When a cricket fielder relaxes their hands and pulls them back when catching a ball
  - A cricket ball travels at very high speeds and therefore has a **high momentum**
  - When a fielder catches the ball, it exerts a force onto their hands
  - Stopping a ball with high momentum at once will cause a large force onto their hands
  - This is because a change in momentum (impulse) acts over a **short period of time** which creates a **large force** on the fielder's hands and could cause serious injury
  - A fielder moves their hands back when they catch the ball, which **increases the time** for its change in momentum to reduce
  - This means there will be **less force** exerted on the fielder's hands and therefore less chance of injury



A cricket fielder moves their hands backwards when catching a cricket ball to reduce the force it will exert on their hands

- In football:
    - Increasing the contact time is sometimes used to advantage, as the longer the contact time, the larger change in momentum
    - When kicking a football, after a strong kick the motion is followed through
    - The momentum from the foot is transferred to the ball
    - This creates a **large impulse** and the ball then has a **higher velocity**

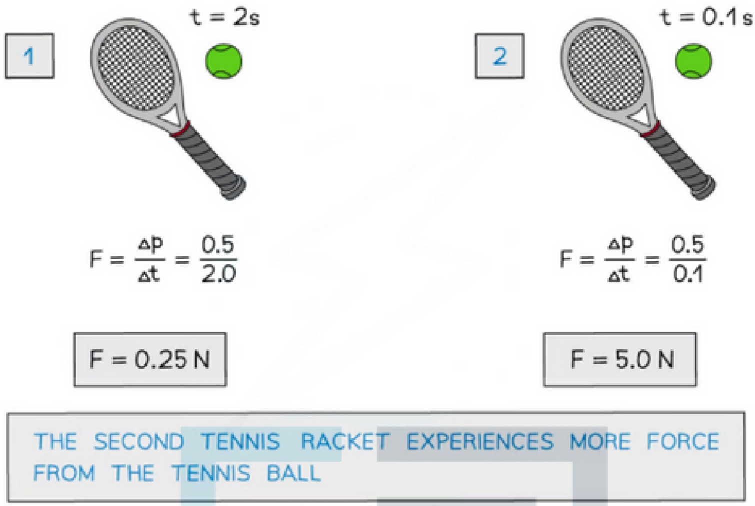
The follow through action of a football kick increases the change in momentum of the ball

In Packaging

- Packaging, especially for fragile items, uses bubble wrap or polyester packaging to reduce the impact forces that items experience in transit
- These help cushion the items by increasing the time over which they experience a force, which reduces the risk of damage

Worked Example

A tennis ball hits a racket with a change in momentum of 0.5 kg m s^{-1}. For the different contact times, which tennis racket experiences more force from the tennis ball?

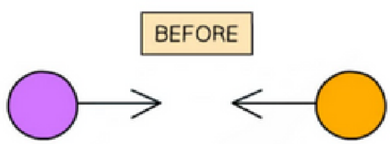


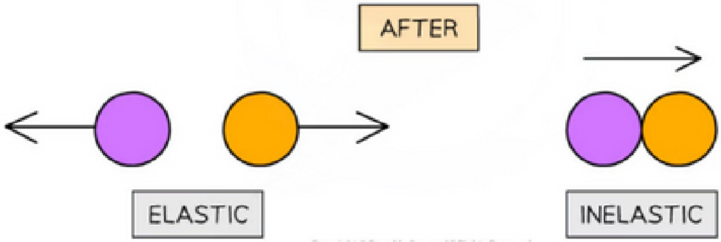
4.5.5 Collisions

Elastic & Inelastic Collisions

- In both collisions and explosions, momentum is always conserved
- However, **kinetic energy** might not always be
- A collision (or explosion) is either:
  - **Elastic** – if the kinetic energy **is** conserved
  - **Inelastic** – if the kinetic energy is **not** conserved

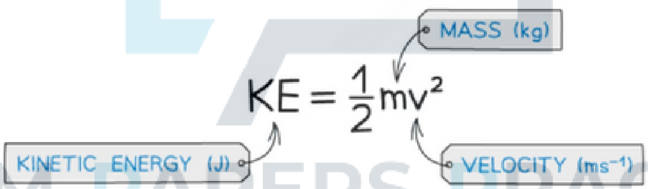
- Collisions are when objects striking against each other
  - **Elastic** collisions are commonly those where objects colliding do not stick together and then **move in opposite directions**
  - **Inelastic** collision are commonly those where objects collide and **stick together** after the collision





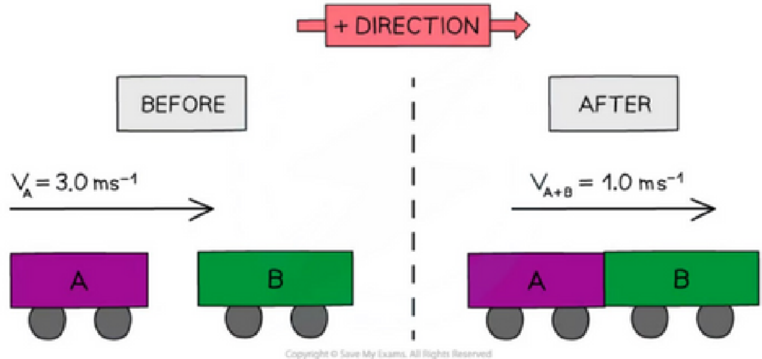
Elastic collisions are where two objects move in opposite directions. Inelastic collisions are where two objects stick together

- An explosion is commonly to do with **recoil**
  - For example, a gun recoiling after shooting a bullet or an unstable nucleus emitting an alpha particle and a daughter nucleus
- To find out whether a collision is elastic or inelastic, **compare the kinetic energy before and after the collision**
- The equation for kinetic energy is:



Worked Example

Trolley A of mass 0.80 kg collides head-on with stationary trolley B at speed 3.0 m s^{-1}. Trolley B has twice the mass of trolley A. The trolleys stick together and travel at a velocity of 1.0 m s^{-1}. Determine whether this is an elastic or inelastic collision.



KINETIC ENERGY = 1/2 mv^2

KINETIC ENERGY BEFORE $= \frac{1}{2} \times M_A \times (V_A)^2 + \frac{1}{2} \times M_B \times (V_B^2)$

$= \frac{1}{2} \times 0.8 \times (3.0)^2 + 0$ ← $V_B = 0$

$= 3.6 \text{ J}$

KINETIC ENERGY AFTER $= \frac{1}{2} \times M_{A+B} \times (V_{A+B})^2$

$= \frac{1}{2} \times 2.4 \times (1.0)^2$

$= 1.2 \text{ J}$

THIS IS AN INELASTIC COLLISION SINCE KINETIC ENERGY IS **NOT** CONSERVED

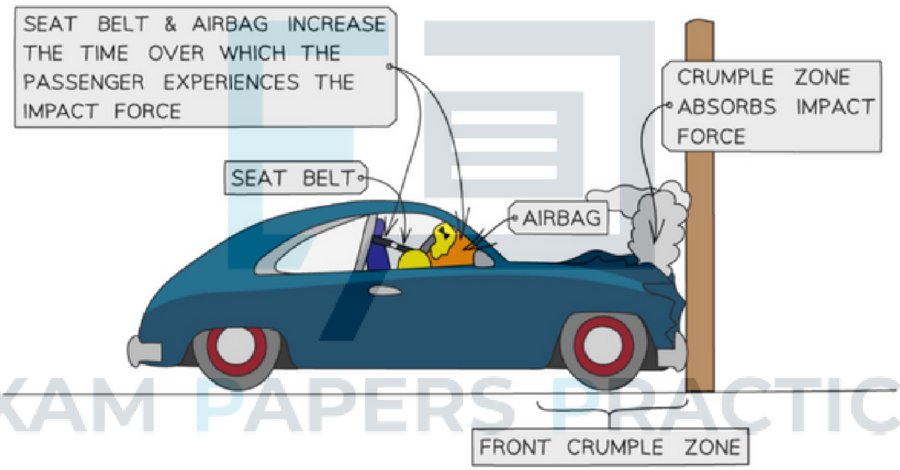
Exam Tip

If an object is stationary or at rest, it's velocity equals **0**, therefore, the momentum and kinetic energy are also equal to 0. When a collision occurs in which two objects are stuck together, treat the final object as a single object with a mass equal to the **sum** of the two individual objects.

Momentum Conservation Issues

- The force of an impact in a vehicle collision can be **decreased** by **increasing** the contact time over which the collision occurs
  - The contact time is the time in which the vehicle or the passenger is in contact with what it has collided with

- Vehicles have safety features such as **crumple zones**, **seat belts** and **airbags** to account for this
  - For a given force upon impact, these absorb the energy from the impact and increase the time over which the force takes place
  - This, in turn, increases the time taken for the change in momentum of the passenger and the vehicle to come to rest
  - The increased time reduces the force and risk of injury on a passenger



Designing Safety Features

- Vehicle safety features are designed to absorb energy upon an impact by changing **shape**
- **Seat belts**
  - These are designed to stop a passenger from colliding with the interior of a vehicle by keeping them fixed to their seat in an abrupt stop
  - They are designed to stretch slightly to increase the time for the passenger's momentum to reach zero and reduce the force on them in a collision

- **Airbags**
  - These are deployed at the front on the dashboard and steering wheel when a collision occurs
  - They act as a soft cushion to prevent injury on the passenger when they are thrown forward upon impact

- **Crumple zones**
  - These are designed into the exterior of vehicles
  - They are at the front and back and are designed to crush or crumple in a controlled way in a collision
  - This is why vehicles after a collision look more heavily damaged than expected, even for relatively small collisions
  - The crumple zones increase the time over which the vehicle comes to rest, **lowering the impact force** on the passengers

- The effect of the increase in time and force can be shown on a force-time graph
  - For the same change in momentum, which depends on the mass and speed of a vehicle, the increase in contact time will result in a **decrease** in the maximum force exerted on the vehicle and passenger
  - This is demonstrated by a lower peak and wider base on a force-time graph

- The effect of the increase in time and force can be shown on a force-time graph
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