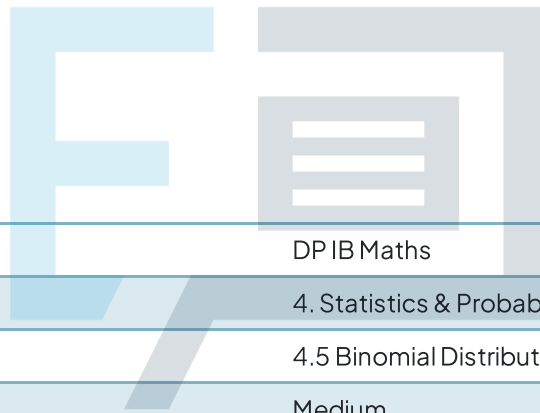




4.5 Binomial Distributions

Mark Schemes



Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.5 Binomial Distributions
Difficulty	Medium

Exam Papers Practice

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful

Question 1

- a)
- A trial has two outcomes (success or failure)
 - An experiment has a finite number of trials (n)
 - The probability of success is constant (p)
 - The trials are independent of each other.

If these four conditions are met then the number of successes, X , follows a binomial distribution $B(n,p)$.

(b)(i) S is not binomial as there are more than two outcomes.

(ii) W is not binomial as there is not a fixed number of trials.

(iii) Y is binomial with $B(12, \frac{1}{2})$ as it has 12 independent spins and landing on a prime is a success which has a constant probability of $\frac{1}{2}$.

(iv) L is not binomial as getting a win depends on the previous spin.

Question 2

- (a) Let X be the number of times the coin lands heads up when tossed 20 times.

$$X \sim B(20, \frac{1}{2})$$

$$E(X) = np \text{ (from formula booklet)}$$

$$E(X) = (20)(\frac{1}{2})$$

$$E(X) = 10$$

(b) $X \sim B(20, \frac{1}{2})$

Using the calculator (Probability Distribution)

$$P(X=15) = 0.0147857\dots$$

$$P(X=15) = 0.0148 \text{ (4dp)}$$

Question 3

- (a) Let X be the number of days in a week that Yussuf catches a taxi.

$$X \sim B(5, 0.6)$$

$$E(X) = np \text{ (from formula booklet)}$$

$$E(X) = (5)(0.6)$$

$$E(X) = 3$$

(b) $X \sim B(5, 0.6)$

Using the calculator (Probability Distribution)

$$P(X=0) = 0.01024$$

(c) $X \sim B(5, 0.6)$

At most once means $X \leq 1$

Using the calculator (Cumulative Distribution)

$$P(X \leq 1) = 0.08704$$

$$P(X \leq 1) = 0.08704$$

Question 4

a) Let X be the number of successful operations.

$$X \sim B(12, 0.78)$$

All successful $\Rightarrow X=12$

Using the calculator (Probability Distribution)

$$P(X=12) = 0.0507148\dots$$

$$P(X=12) = 0.0507 \text{ (4dp)}$$

b) $X \sim B(12, 0.78)$

All but two $\Rightarrow X=10$

Using the calculator (Probability Distribution)

$$P(X=10) = 0.266278\dots$$

$$P(X=10) = 0.2663 \text{ (4dp)}$$



Question 5

Let X be the number of jellyfish in the sample that contain microplastic particles.

$$X \sim B(40, 0.95)$$

(i) Using the calculator (Probability Distribution)

$$P(X=38) = 0.277671\dots$$

$$P(X=38) = 0.2777 \text{ (4dp)}$$

(ii) Using the calculator (Probability Distribution)

$$P(X=40) = 0.128512\dots$$

$$P(X=40) = 0.1285 \text{ (4dp)}$$

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Question 6

(i) Let X be the number of times the dice lands on a 2 when rolled 10 times.

$$X \sim B(10, 0.25)$$

Using the calculator (Probability Distribution)

$$P(X=4) = 0.145998\dots$$

$$P(X=4) = 0.1460 \text{ (4dp)}$$

(ii) Let Y be the number of times the dice lands on a 2 when rolled 9 times.

$$Y \sim B(9, 0.25)$$

[3 two's in the
first 9 rolls]

and

[A two on
the 10th roll]

$$P(Y=3)$$

$$0.233596\dots$$

x

$$0.25$$

x

$$0.25$$

$$= 0.058399\dots$$

$$0.0584 \text{ (4dp)}$$

Question 7

$$L \sim B(24, 0.0296)$$

(i) Using the calculator (Probability Distribution)

$$P(L=0) = 0.486204\dots$$

$$P(L=0) = 0.4862 \text{ (4dp)}$$

(ii) Using the calculator (Probability Distribution)

$$P(L=2) = 0.124856\dots$$

$$P(L=2) = 0.1249 \text{ (4dp)}$$

(iii) At least 2 $\Rightarrow L \geq 2$

Using the calculator (Cumulative Distribution)

$$P(L \geq 2) = 1 - P(L \leq 1)$$

$$= 1 - 0.842139\dots$$

$$= 0.157861\dots$$

$$P(L \geq 2) = 0.1579 \text{ (4dp)}$$

Question 8

(a)(i) $3 < X < 14$ is $X \leq 13$ without $X \leq 2$
Using the calculator (Cumulative Distribution)
$$P(3 < X < 14) = P(X \leq 13) - P(X \leq 2)$$
$$= 0.998596... - 0.048598...$$
$$= 0.94999...$$

$$P(3 < X < 14) = 0.9500 \text{ (4dp)}$$

(ii) $5 < X < 12$ is $X \leq 11$ without $X \leq 5$
Using the calculator (Cumulative Distribution)
$$P(5 < X < 12) = P(X \leq 11) - P(X \leq 5)$$
$$= 0.988030... - 0.432500...$$
$$= 0.55553...$$

$$P(5 < X < 12) = 0.5555 \text{ (4dp)}$$

(b) $\text{Var}(X) = np(1-p)$ (from formula booklet)
$$\text{Var}(X) = (40)(0.15)(1-0.15)$$

$$\text{Var}(X) = 5.1$$

$$(c) P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (\text{from formula booklet})$$

$$P(X \leq 3 | X \leq 9) = \frac{P(X \leq 3 \cap X \leq 9)}{P(X \leq 9)}$$

$$= \frac{P(X \leq 3)}{P(X \leq 9)} \quad \leftarrow (X \leq 3) \cap (X \leq 9) = (X \leq 3)$$

$$= \frac{0.130168...}{0.932779...} \quad \leftarrow \text{Using the calculator}$$

$$= 0.13954...$$

$$P(X \leq 3 | X \leq 9) = 0.1395 \quad (4dp)$$

Question 9 (a) Let X be the number of the 10 routines containing a mistake.

$$X \sim B(10, 0.2)$$

(i) $E(X) = np$ (from formula booklet)

$$E(X) = (10)(0.2)$$

$$\text{Expected number of routines} = 2$$

(ii) $\text{Var}(X) = np(1-p)$ (from formula booklet)

$$\text{Var}(X) = (10)(0.2)(1-0.2)$$

$$\text{Var}(X) = 1.6$$

$$\text{Standard Deviation of } X = \sqrt{\text{Var}(X)}$$

$$\text{Standard Deviation of } X = \sqrt{1.6}$$

$$= 1.264911...$$

$$\text{Standard Deviation} = 1.26 \quad (3sf)$$



(b) $X \sim B(10, 0.2)$

(i) Using the calculator (Probability Distribution)

$$P(X=0) = 0.107374\dots$$

$$P(X=0) = 0.1074 \text{ (4dp)}$$

(ii) Using the calculator (Probability Distribution)

$$P(X=2) = 0.301989\dots$$

$$P(X=2) = 0.3020 \text{ (4dp)}$$

(iii) None more than 2 $\Rightarrow X \leq 2$

Using the calculator (Cumulative Distribution)

$$P(X \leq 2) = 0.677799\dots$$

$$P(X \leq 2) = 0.6778 \text{ (4dp)}$$

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$$c) P(A|B) = \frac{P(A \cap B)}{P(B)} \quad (\text{from formula booklet})$$

$$P(X=3 | X \geq 2) = \frac{P(X=3 \cap X \geq 2)}{P(X \geq 2)}$$

$$= \frac{P(X=3)}{P(X \geq 2)} \quad \leftarrow (X=3) \cap (X \geq 2) = (X=3)$$

$$= \frac{P(X=3)}{1 - P(X \leq 1)}$$

$$= \frac{0.201326...}{1 - 0.375809...} \quad \leftarrow \text{Using the calculator}$$

$$= 0.32254...$$

$$P(X=3 | X \geq 2) = 0.3225 \quad (4dp)$$

d) From (a):

$$\text{Mean} = 2$$

$$\text{Standard Deviation} = \sqrt{1.6}$$

Less than one standard deviation away from the mean.

$$2 - \sqrt{1.6} < X < 2 + \sqrt{1.6}$$

$$0.735... < X < 3.264...$$

X can only be positive integers $\Rightarrow 1 \leq X \leq 3$

$1 \leq X \leq 3$ is $X \leq 3$ without $X=0$

$$P(1 \leq X \leq 3) = P(X \leq 3) - P(X=0)$$

$$= 0.879126... - 0.107374...$$

$$= 0.77175...$$

$$0.7718 \quad (4dp)$$

Question 10

$$a) X \sim B(30, 0.9)$$

$$(i) E(X) = np \text{ (from formula booklet)}$$

$$E(X) = (30)(0.9)$$

$$E(X) = 27$$

$$(ii) \text{Var}(X) = np(1-p) \text{ (from formula booklet)}$$

$$\text{Var}(X) = (30)(0.9)(1-0.9)$$

$$\text{Var}(X) = 2.7$$

$$(b)(i) 90\% \text{ of } 30 = 27$$

$$90\% \text{ or more} \Rightarrow X \geq 27$$

Using the calculator (Cumulative Distribution)

$$P(X \geq 27) = 1 - P(X \leq 26)$$

$$= 1 - 0.352560\dots$$

$$= 0.647439\dots$$

$$P(X \geq 27) = 0.6474 \text{ (4dp)}$$

$$(ii) \text{None prefer other brand} \Rightarrow \text{All prefer Woostershire}$$

$$\therefore X = 30$$

Using the calculator (Probability Distribution)

$$P(X = 30) = 0.042391\dots$$

$$P(X = 30) = 0.0424 \text{ (4dp)}$$