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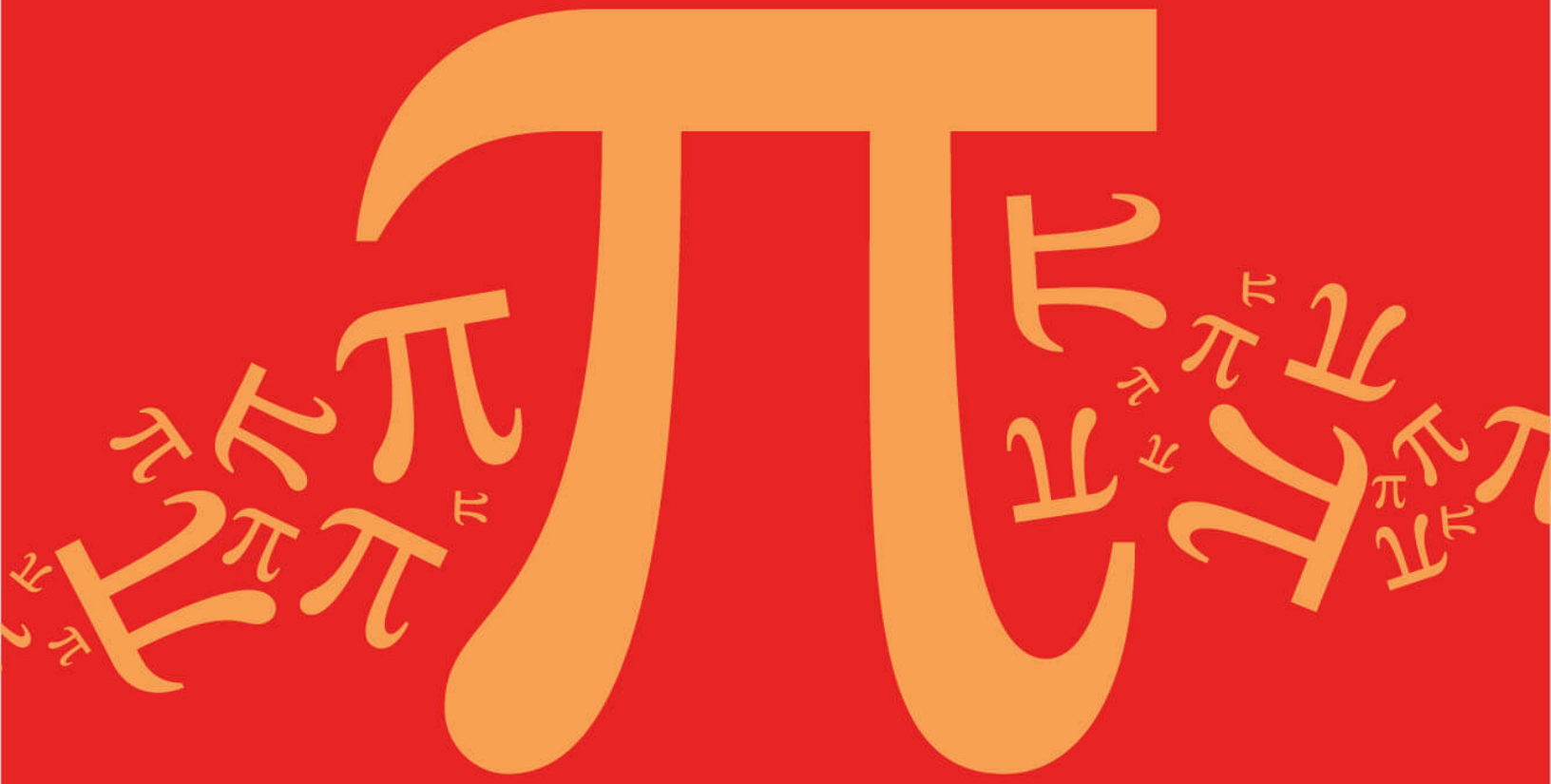
4.5 Binomial Distribution



IB Maths - Revision Notes

AA HL

4.5.1 The Binomial Distribution

Properties of Binomial Distribution

What is a binomial distribution?

- A binomial distribution is a **discrete probability distribution**
- A **discrete random variable** X follows a **binomial distribution** if it **counts the number of successes** when an experiment satisfies the following conditions:
  - There are a **fixed finite number of trials (*n*)**
  - The outcome of each trial is **independent** of the outcomes of the other trials
  - There are **exactly two outcomes** of each trial (**success or failure**)
  - The **probability of success is constant (*p*)**
- If X follows a binomial distribution then it is denoted $X \sim \mathrm{B}(n, p)$
  - n is the **number of trials**
  - p is the **probability of success**
- The **probability of failure is 1 – *p*** which is sometimes denoted as ***q***
- The formula for the probability of ***r* successful trials** is given by:
  - $\mathrm{P}(X = r) = {}^{n}C_{r} \times p^{r}(1-p)^{n-r}$ for $r = 0, 1, 2, \ldots, n$
    - ${}^{n}C_{r} = \dfrac{n!}{r!(n-r)!}$ where $n! = n \times (n-1) \times (n-2) \times \ldots \times 3 \times 2 \times 1$
  - You will be expected to use the distribution function on your **GDC to calculate probabilities** with the binomial distribution

What are the important properties of a binomial distribution?

- The **expected number (mean)** of successful trials is

$$\mathrm{E}(X) = np$$

  - You are given this in the **formula booklet**
- The **variance** of the number of successful trials is

$$\mathrm{Var}(X) = np(1-p)$$

  - You are given this in the **formula booklet**
  - Square root to get the **standard deviation**
- The distribution can be represented visually using a vertical line graph
  - If p is **close to 0** then the graph has a **tail to the right**
  - If p is **close to 1** then the graph has a **tail to the left**
  - If p is **close to 0.5** then the graph is **roughly symmetrical**
  - **If $p = 0.5$** then the graph is **symmetrical**

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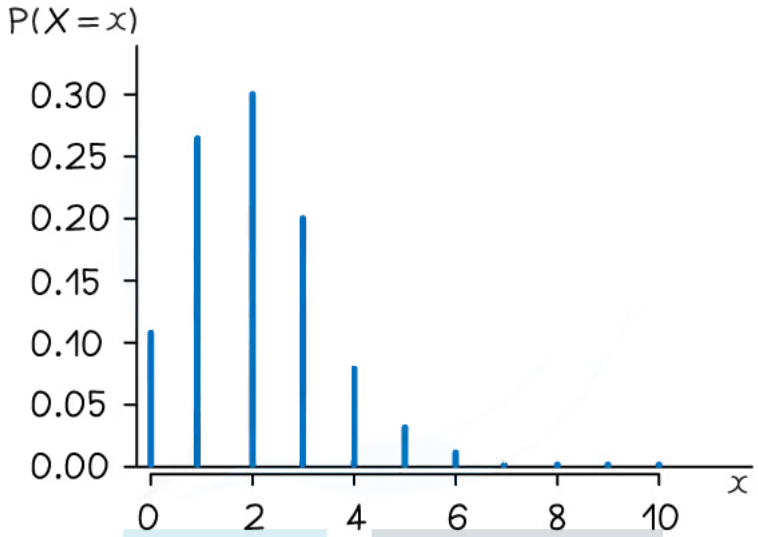
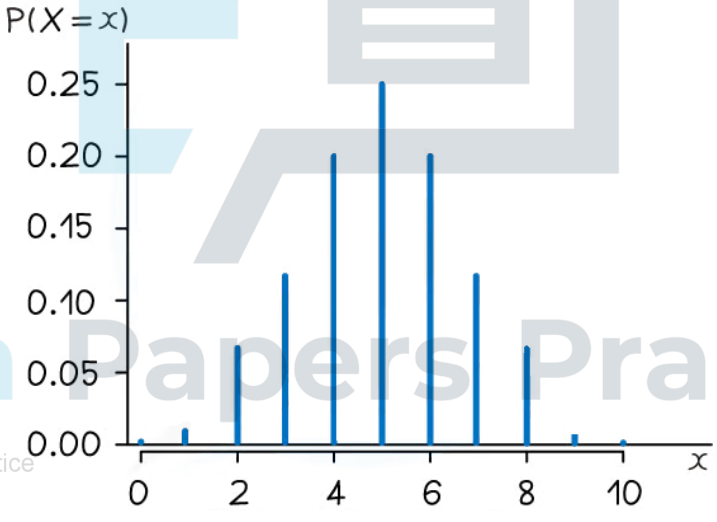
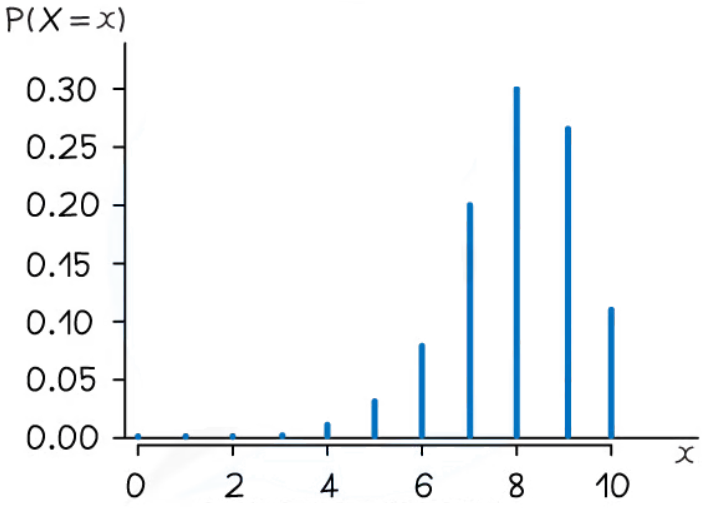
$X \sim B(10, 0.2)$



$X \sim B(10, 0.5)$



$X \sim B(10, 0.8)$



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Modelling with Binomial Distribution

How do I set up a binomial model?

- **Identify** what a **trial** is in the scenario
  - For example: rolling a dice, flipping a coin, checking hair colour
- **Identify** what the **successful outcome** is in the scenario
  - For example: rolling a 6, landing on tails, having black hair
- **Identify** the **parameters**
  - n is the number of trials and p is the probability of success in each trial
- Make sure you **clearly state** what your **random variable** is
  - For example, let X be the number of students in a class of 30 with black hair

What can be modelled using a binomial distribution?

- Anything that satisfies the **four conditions**
- For example: let T be the number of times a fair coin lands on tails when flipped 20 times:
  - A trial is flipping a coin: There are 20 trials so $n = 20$
  - We can assume each coin flip does not affect subsequent coin flips: they are **independent**
  - A success is when the coin lands on tails: **Two outcomes** - tails or not tails (heads)
  - The coin is fair: The probability of tails is constant with $p = 0.5$
- Sometimes it might **seem like there are more than two outcomes**
  - For example: let Y be the number of yellow cars that are in a car park full of 100 cars
    - Although there are more than two possible colours of cars, here the trial is whether a car is yellow so there are two outcomes (yellow or not yellow)
    - Y would still need to fulfil the other conditions in order to follow a binomial distribution
- Sometimes a **sample may be taken from a population**
  - For example: 30% of people in a city have blue eyes, a sample of 30 people from the city is taken and X is the number of them with blue eyes
    - As long as the population is large and the sample is random then it can be assumed that each person has a 30% chance of having blue eyes

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What can not be modelled using a binomial distribution?

- Anything where the number of trials is **not fixed** or is **infinite**
  - The number of emails received in an hour
  - The number of times a coin is flipped until it lands on heads
- Anything where the outcome of **one trial affects** the outcome of the **other trials**
  - The number of caramels that a person eats when they eat 5 sweets from a bag containing 6 caramels and 4 marshmallows
    - If you eat a caramel for your first sweet then there are less caramels left in the bag when you choose your second sweet
  - Anything where there are **more than two outcomes** of a trial

- A person's shoe size
- The number a dice lands on when rolled

- Anything where the **probability of success changes**
  - The number of times that a person can swim a length of a swimming pool in under a minute when swimming 50 lengths
    - The probability of swimming a lap in under a minute will decrease as the person gets tired
    - The probability is **not constant**

Exam Tip

- An exam question might involve different types of distributions so make it clear which distribution is being used for each variable

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Worked example

It is known that 8% of a large population are immune to a particular virus. Mark takes a sample of 50 people from this population. Mark uses a binomial model for the number of people in his sample that are immune to the virus.

a) State the distribution that Mark uses.

A trial is checking if a person is immune to the virus

A success is if the person is immune.

Let X be the number of people in the sample immune to the virus

$X \sim B(50, 0.08)$

Number of people in sample

Probability of being immune to the virus

b) State two assumptions that Mark must make in order to use a binomial model.

Mark needs to assume that:

- each person in the population has an 8% chance of being immune
- the sample is random and the people are independent a person being immune does not affect the immunity of others

For example:
If all 50 came from the same family then they would not be independent

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c) Calculated the expected number of people in the sample that are immune to the virus.

Formula booklet

Binomial distribution $X \sim \mathrm{B}(n, p)$	
Mean	$\mathrm{E}(X) = np$

$E(X) = 50 \times 0.08$

4 people

4.5.2 Calculating Binomial Probabilities

Calculating Binomial Probabilities

Throughout this section we will use the random variable $X \sim \mathrm{B}(n, p)$. For binomial, the probability of X taking a non-integer or negative value is always zero. Therefore any values of X mentioned in this section will be assumed to be non-negative integers.

How do I calculate P($X = x$): the probability of a single value for a binomial distribution?

- You should have a **GDC** that can calculate **binomial probabilities**
- You want to use the "**Binomial Probability Distribution**" function
  - This is sometimes shortened to BPD, Binomial PD or Binomial Pdf
- You will need to enter:
  - The *'x'* value - the value of *x* for which you want to find $\mathrm{P}(X = x)$
  - The '*n*' value - the **number of trials**
  - The *'p'* value - the **probability of success**
- Some calculators will give you the option of **listing the probabilities** for **multiple values of *x* at once**
- There is a formula that you can use but you are expected to be able to use the distribution function on your GDC
  - $\mathrm{P}(X = x) = {}^{n}\mathrm{C}_{x} \times p^{x}(1-p)^{n-x}$
  - ${}^{n}\mathrm{C}_{x} = \dfrac{n!}{r!(n-r)!}$

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How do I calculate P($a \leq X \leq b$): the cumulative probabilities for a binomial distribution?

- You should have a **GDC** that can calculate **cumulative binomial probabilities**
  - Most calculators will find $\mathrm{P}(a \leq X \leq b)$
  - Some calculators can only find $\mathrm{P}(X \leq b)$
    - The identities below will help in this case
- You should use the "**Binomial Cumulative Distribution**" function
  - This is sometimes shortened to BCD, Binomial CD or Binomial Cdf
- You will need to enter:
  - The lower value - this is the **value *a***
    - This can be zero in the case $\mathrm{P}(X \leq b)$
  - The upper value - this is the **value *b***
    - This can be *n* in the case $\mathrm{P}(X \geq a)$

- The 'n' value - the **number of trials**
- The 'p' value - the **probability of success**

How do I find probabilities if my GDC only calculates P($X \leq x$)?

- To calculate P($X \leq x$) just enter x into the cumulative distribution function

- To calculate P($X < x$) use:
  - $P(X < x) = P(X \leq x - 1)$ which works when X is a binomial random variable
    - P($X < 5$) = P($X \leq 4$)

- To calculate P($X > x$) use:
  - $P(X > x) = 1 - P(X \leq x)$ which works for any random variable X
    - P($X > 5$) = 1 - P($X \leq 5$)

- To calculate P($X \geq x$) use:
  - $P(X \geq x) = 1 - P(X \leq x - 1)$ which works when X is a binomial random variable
    - P($X \geq 5$) = 1 - P($X \leq 4$)

- To calculate P($a \leq X \leq b$) use:
  - $P(a \leq X \leq b) = P(X \leq b) - P(X \leq a - 1)$ which works when X is a binomial random variable
    - P($5 \leq X \leq 9$) = P($X \leq 9$) - P($X \leq 4$)

What if an inequality does not have the equals sign (strict inequality)?

- For a binomial distribution (as it is discrete) you could **rewrite all strict inequalities** (< and >) as **weak inequalities** ($\leq$ and $\geq$) by using the identities for a binomial distribution
  - $P(X < x) = P(X \leq x - 1)$ and $P(X > x) = P(X \geq x + 1)$
  - For example: P($X < 5$) = P($X \leq 4$) and P($X > 5$) = P($X \geq 6$)
- It helps to think about the **range of integers** you want
  - Identify the smallest and biggest integers in the range
- If your range has no minimum or maximum then use 0 or n
  - $P(X \leq b) = P(0 \leq X \leq b)$
  - $P(X \geq a) = P(a \leq X \leq n)$

- $P(a < X \leq b) = P(a + 1 \leq X \leq b)$
  - P($5 < X \leq 9$) = P($6 \leq X \leq 9$)

- $P(a \leq X < b) = P(a \leq X \leq b - 1)$
  - P($5 \leq X < 9$) = P($5 \leq X \leq 8$)

- $P(a < X < b) = P(a + 1 \leq X \leq b - 1)$
  - P($5 < X < 9$) = P($6 \leq X \leq 8$)

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Exam Tip

- If the question is in context then write down the inequality as well as the final answer
  - This means you still might gain a mark even if you accidentally type the wrong numbers into your GDC

Worked example

The random variable $X \sim \mathrm{B}(40, 0.35)$. Find:

i) $\mathrm{P}(X = 10)$.

Identify n and p n = 40 p = 0.35

Use binomial probability distribution on GDC

P(X = 10) = 0.057056...

P(X = 10) = 0.057 (3sf)

ii) $\mathrm{P}(X \leq 10)$.

Identify upper and lower values

P(X ≤ 10) = P(0 ≤ X ≤ 10)

Use binomial cumulative distribution on GDC

P(X ≤ 10) = 0.121491...

P(X ≤ 10) = 0.121 (3sf)

iii) $\mathrm{P}(8 < X < 15)$.

Identify upper and lower values

P(8 < X < 15) = P(9 ≤ X ≤ 14)

Use binomial cumulative distribution on GDC

P(9 ≤ X ≤ 14) = 0.541827...

P(8 < X < 15) = 0.542 (3sf)

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