4.5 A model of computation

Name: ______________________

Class: ______________________

Date: ______________________

Time: **100 minutes**

Marks: **67 marks**

Comments:

Q1.

Why is it not possible to create a Turing machine that solves the Halting problem?

(Total 1 mark)

Q2.

To define a Turing machine the finite alphabet of symbols that it can use needs to be specified and there needs to be a tape.

State **two** other components of a Turing machine.

(Total 2 marks)

Q3.

Explain what a Universal Turing Machine is.

(Total 2 marks)

Q4.

Why can a Universal Turing Machine be considered to be more powerful than any computer that you can purchase?

(Total 1 mark)

Q5.

A Turing machine has been designed to recognise palindromic binary numbers, ie numbers such as `101` and `0110` that read the same from left to right as from right to left.

The machine has states S_B, S_0, S_1, S_{C0}, S_{C1}, S_L, S_Y and S_N.

S_B is the start state and S_Y and S_N are the stop states.

The machine stores data on a single tape which is infinitely long in one direction. The machine's alphabet is 0, 1 and ☐, where ☐ is the symbol used to indicate a blank cell on the tape. The machine will enter state S_Y if the value represented on the tape is a palindromic binary number, otherwise it will enter state S_N.

The transition rules for this Turing machine can be expressed as a transition function δ.

Rules are written in the form:

$$\delta(\text{Current State, Input Symbol}) = (\text{Next State, Output Symbol, Movement})$$

So, for example, the rule:

$$\delta(S_B, 0) = (S_0, \square, \longrightarrow)$$

means:

IF	the machine is currently in state S_B AND the input symbol read from the tape is 0
THEN	the machine should change to state S_0, write a blank symbol (☐) to the tape and move the read/write head one cell to the right

The machine's transition function, δ, is defined by:

$\delta(S_B, 0) = (S_0, \square, \longrightarrow)$
$\delta(S_B, 1) = (S_1, \square, \longrightarrow)$
$\delta(S_B, \square) = (S_Y, \square \longrightarrow)$

$\delta(S_0, 0) = (S_0, 0, \longrightarrow)$
$\delta(S_0, 1) = (S_0, 1, \longrightarrow)$
$\delta(S_0, \square) = (S_{C0}, \square, \longleftarrow)$

$\delta(S_1, 0) = (S_1, 0, \longrightarrow)$
$\delta(S_1, 1) = (S_1, 1, \longrightarrow)$
$\delta(S_1, \square) = (S_{C1}, \square, \longleftarrow)$

$\delta(S_{C0}, 0) = (S_L, \square, \longleftarrow)$
$\delta(S_{C0}, 1) = (S_N, 1, \longleftarrow)$
$\delta(S_{C0}, \square) = (S_Y, \square, \longrightarrow)$

$\delta(S_{C1}, 0) = (S_N, 0, \longleftarrow)$
$\delta(S_{C1}, 1) = (S_L, \square, \longleftarrow)$
$\delta(S_{C1}, \square) = (S_Y, \square, \longrightarrow)$

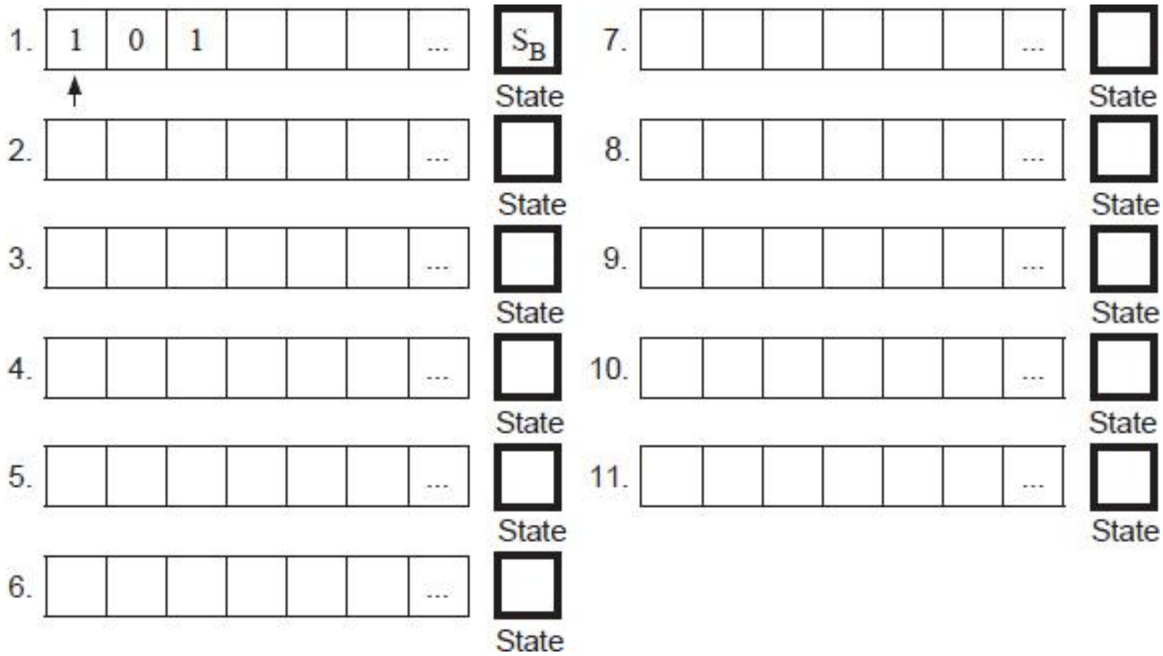
$\delta(S_L, 0) = (S_L, 0, \longleftarrow)$
$\delta(S_L, 1) = (S_L, 1, \longleftarrow)$
$\delta(S_L, \square) = (S_B, \square, \longrightarrow)$

(a) This Turing machine is carrying out a computation. The machine starts in state S_B with the string `101` on the tape. All other cells contain the blank symbol, ☐ (not shown). The read/write head is located at the left hand symbol of the string and is indicated with an upward arrow.

Trace the computation of the Turing machine, using the transition function δ.

Show the contents of the tape, the current position of the read/write head and the current state as the input symbols are processed.

The initial configuration of the machine has been completed for you in step 1.



(5)

(b) The three rules shown below are part of the machine's transition function.

$\delta\ (S_0, 0) = (S_0, 0, \longrightarrow)$

$\delta\ (S_0, 1) = (S_0, 1, \longrightarrow)$

$\delta\ (S_0, \square) = (S_{C0}, \square, \longleftarrow)$

Explain what effect these three rules, taken together, have on the tape, the read/write head and the state of the Turing machine.

(2)

(Total 7 marks)

Q6.

(a) Two important components of a Turing machine are the transition function and the tape.

Five components of a typical modern computer system are listed below:

1. DVD-ROM
2. Compiler

3. Main Memory
4. Processor
5. Program

Complete the table by writing into it the numbers of the modern computer system components from the list above that would fulfil the role most similar to the transition function and the tape in a Turing machine.

Turing machine component	**Number (1-5) of modern computer system component with most similar role**
Transition function	
Tape	

(2)

(b) Explain the relationship between the Turing machine, as a model of computation, and an algorithm.

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(1)

(c) A Turing machine has been designed to complete a task.

The machine has states S_B, S_F, S_0, S_1, S_R, S_E, and S_H. S_B is the start state and S_H is the stop state.

The machine stores data on a single tape which is infinitely long in one direction. The machine's alphabet is 0, 1, #, ☐, where ☐ is the symbol used to indicate a blank cell on the tape.

The transition rules for this Turing machine can be expressed as a transition function δ. Rules are written in the form:

δ(Current State, Input Symbol) = (Next State, Output Symbol, Movement)

So, for the example, the rule:

$$\delta(S_B , \square) = (S_F , \#, \longleftarrow)$$

means:

IF the machine is currently in state $_{SB}$ AND the input symbol read from the tape is a blank symbol (☐)

THEN the machine should change to state $_{SF}$, write a # to the tape and move the read/write head one cell to the left

The machine's transition function, δ, is defined by:

$\delta(S_B, 0) = (S_B, 0, \longrightarrow)$
$\delta(S_B, 1) = (S_B, 1, \longrightarrow)$
$\delta(S_B, \square) = (S_F, \#, \longleftarrow)$

$\delta(S_F, \#) = (S_F, \#, \longleftarrow)$
$\delta(S_F, 0) = (S_0, \#, \longrightarrow)$
$\delta(S_F, 1) = (S_1, \#, \longrightarrow)$
$\delta(S_F, \square) = (S_E, \square, \longrightarrow)$
$\delta(S_0, \#) = (S_0, \#, \longrightarrow)$
$\delta(S_0, 0) = (S_0, 0, \longrightarrow)$
$\delta(S_0, 1) = (S_0, 1, \longrightarrow)$
$\delta(S_0, \square) = (S_R, 0, \longleftarrow)$

$\delta(S_1, \#) = (S_1, \#, \longrightarrow)$
$\delta(S_1, 0) = (S_1, 0, \longrightarrow)$
$\delta(S_1, 1) = (S_1, 1, \longrightarrow)$
$\delta(S_1, \square) = (S_R, 1, \longleftarrow)$

$\delta(S_R, \#) = (S_F, \#, \longleftarrow)$
$\delta(S_R, 0) = (S_R, 0, \longleftarrow)$
$\delta(S_R, 1) = (S_R, 1, \longleftarrow)$

$\delta(S_E, \#) = (S_E, \square, \longrightarrow)$
$\delta(S_E, 0) = (S_H, 0, \longrightarrow)$
$\delta(S_E, 1) = (S_H, 1, \longrightarrow)$
$\delta(S_E, \square) = (S_H, \square, \longrightarrow)$

This Turing machine is carrying out a computation. The machine starts in state S_B with the string `10` on the tape. All other cells contain the blank symbol, $\square$ (not shown). The read/write head is positioned at the left hand end of the string on the tape, as indicated by the arrow:

								State
	1	0				...		S_B
	↑							

After eight steps of the computation have been completed, the tape contents, state and position of the read/write head are:

								State
	1	#	#	0		...		S_F
	↑							

Trace the computation of the Turing machine, from step nine onwards, using the transition function δ. Show the contents of the tape, the current position of the read/write head and the current state as the input symbols are processed. Step eight has been repeated at the start of the trace.

Step									State
8.		1	#	#	0		...		S_F
		↑							
9.							...		
10.							...		
16.							...		
17.							...		
18.							...		

11.	...	State
12.	...	State
13.	...	State
14.	...	State
15.	...	State
19.	...	State
20.	...	State
21.	...	State
22.	...	State

(6)

(d) Explain the purpose of the Turing machine that you have traced the execution of.

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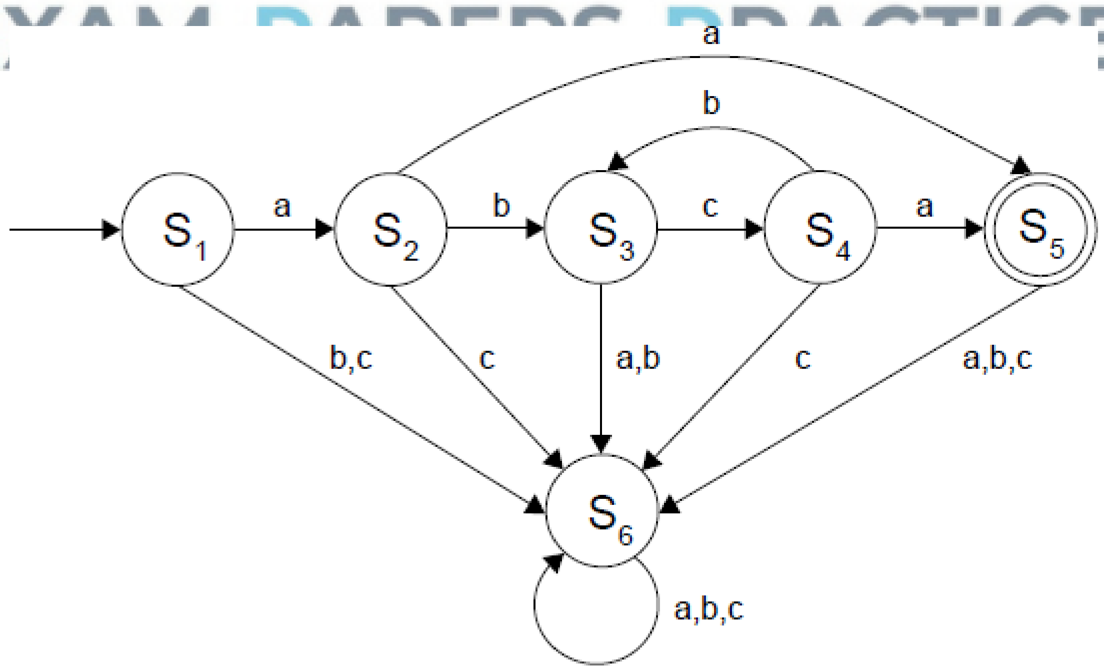
(1)
(Total 10 marks)

Q7.

The diagram below shows a Finite State Automaton (FSA). The FSA has input alphabet {a, b, c} and six states, S_1, S_2, S_3, S_4, S_5 and S_6.



(a) Complete the empty cells in the **part of the transition table shown** below for the

FSA in the diagram.

Current State	S_1	S_1	S_1	S_2	S_2	S_2	S_3	S_3	S_3
Input Symbol	a	b	c	a	b	c			
Next State	S_2	S_6	S_6	S_5	S_3	S_6			

(1)

(b) Give the name of the state that the FSA will end up in when processing the string `abcb`.

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(1)

(c) Describe the purpose of state S_6.

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(1)

(d) The FSA in the diagram above accepts strings that consist of:

- a letter `a`
- followed by zero or more occurrences of the string `bc` and
- ending with a second letter `a`.

For any FSA, it is possible to write a regular expression that will match the same language (set of strings) as the FSA.

Write a regular expression that will match the same language that is accepted by the FSA above.

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(2)

(e) The Turing Machine is a more powerful abstract model of computation than the FSA.

Explain why the Turing Machine model can be used to recognise a greater range of languages than an FSA could.

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(1)

(Total 6 marks)

Q8.

A Turing machine has been designed to recognise palindromic binary numbers, ie numbers such as `101` and `0110` that read the same from left to right as from right to left.

The machine has states S_B, S_0, S_1, S_{C0}, S_{C1}, S_L, S_Y and S_N.

S_B is the start state and S_Y and S_N are the stop states.

The machine stores data on a single tape which is infinitely long in one direction. The machine's alphabet is 0, 1 and □, where □ is the symbol used to indicate a blank cell on the tape. The machine will enter state S_Y if the value represented on the tape is a palindromic binary number, otherwise it will enter state S_N.

The transition rules for this Turing machine can be expressed as a transition function δ. Rules are written in the form:

δ(Current State, Input Symbol) = (Next State, Output Symbol, Movement)

So, for example, the rule:

$$\delta\ (S_B, 0) = (S_0\ , \square, \longrightarrow)$$

means:

IF the machine is currently in state S_B AND the input symbol read from the tape is 0

THEN the machine should change to state S_0, write a blank symbol (□) to the tape and move the read / write head one cell to the right

The machine's transition function, δ, is defined by:

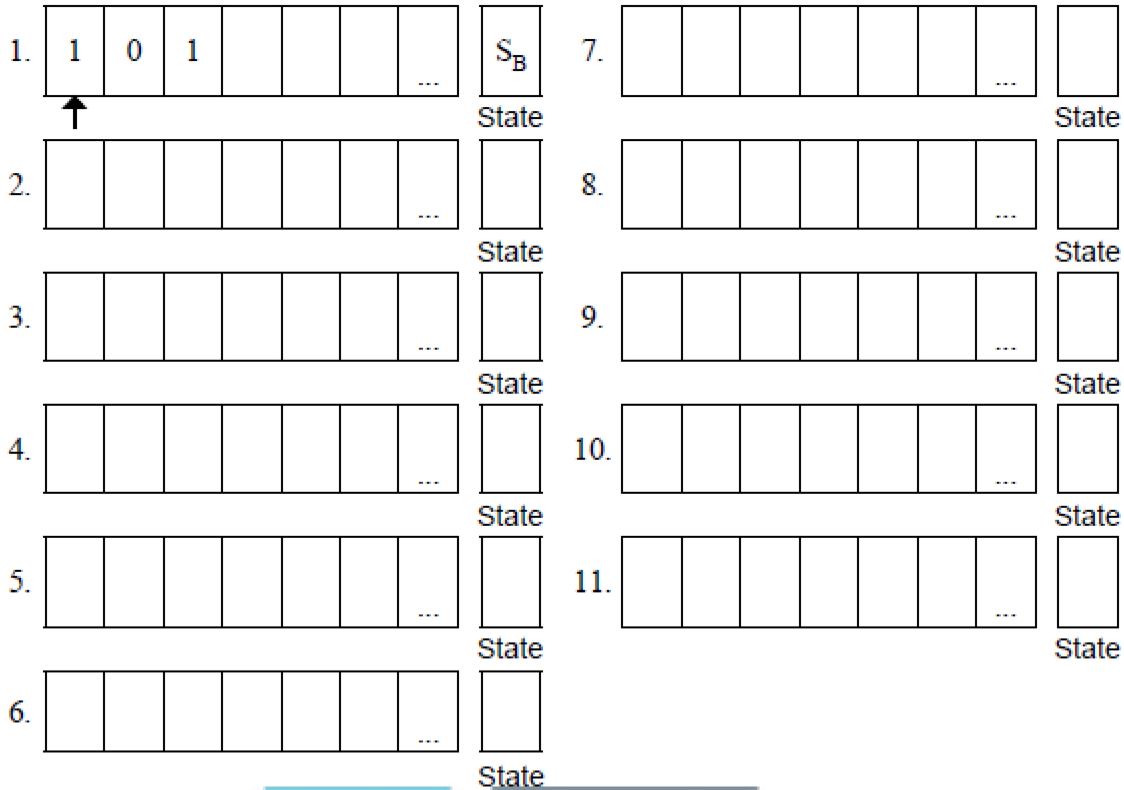
$\delta\ (S_B, 0) = (S_0, \square, \rightarrow)$	$\delta\ (S_{C0}, 0) = (S_L, \square, \leftarrow)$
$\delta\ (S_B, 1) = (S_1, \square, \rightarrow)$	$\delta\ (S_{C0}, 1) = (S_N, 1, \leftarrow)$
$\delta\ (S_B, \square) = (S_Y, \square, \rightarrow)$	$\delta\ (S_{C0}, \square) = (S_Y, \square, \rightarrow)$
$\delta\ (S_0, 0) = (S_0, 0, \rightarrow)$	$\delta\ (S_{C1}, 0) = (S_N, 0, \leftarrow)$
$\delta\ (S_0, 1) = (S_0, 1, \rightarrow)$	$\delta\ (S_{C1}, 1) = (S_L, \square, \leftarrow)$
$\delta\ (S_0, \square) = (S_{C0}, \square, \leftarrow)$	$\delta\ (S_{C1}, \square) = (S_Y, \square, \rightarrow)$
$\delta\ (S_1, 0) = (S_1, 0, \rightarrow)$	$\delta\ (S_L, 0) = (S_L, 0, \leftarrow)$
$\delta\ (S_1, 1) = (S_1, 1, \rightarrow)$	$\delta\ (S_L, 1) = (S_L, 1, \leftarrow)$
$\delta\ (S_1, \square) = (S_{C1}, \square, \leftarrow)$	$\delta\ (S_L, \square) = (S_B, \square, \rightarrow)$

(a) This Turing machine is carrying out a computation. The machine starts in state S_B with the string 101 on the tape. All other cells contain the blank symbol, □ (not shown).
The read / write head is located at the left hand symbol of the string and is indicated with an upward arrow.

Trace the computation of the Turing machine, using the transition function δ.

Show the contents of the tape, the current position of the read/write head and the current state as the input symbols are processed.

The initial configuration of the machine has been completed for you in step 1.



(5)

(b) The three rules shown below are part of the machine's transition function.

Explain what effect these three rules, taken together, have on the tape, the read / write head and the state of the Turing machine:

$$\delta(S_0, 0) = (S_0, 0, \rightarrow)$$
$$\delta(S_0, 1) = (S_0, 1, \rightarrow)$$
$$\delta(S_0, \square) = (S_{C0}, \square, \leftarrow)$$

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(2)

(c) A Universal Turing machine (UTM) is a special type of Turing machine that can be considered to act like an interpreter.

Explain how a UTM can be considered to be an interpreter.

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(2)
(Total 9 marks)

Q9.

A particular Turing machine has states S_B, S_0, S_1, S_R and S_T. S_B is the start state and S_T is the stop state. The machine stores data on a single tape which is infinitely long in one direction. The machine's alphabet is 0, 1, #, x, y and □ where □ is the symbol used to indicate a blank cell on the tape.

The transition rules for this Turing machine can be expressed as a transition function δ. Rules are written in the form:

δ(Current State, Input Symbol) = (Next State, Output Symbol, Movement)

So, for example, the rule:

$\delta(S_B, 1) = (S_1, y, \longrightarrow)$

means:

IF the machine is currently in state S_B AND the input symbol read from the tape is 1

THEN the machine should change to state S_1, write a y to the tape and move the read/write head one cell to the right

The machine's transition function, δ, is defined by:

$\delta(S_B, 0) = (S_0, x, \rightarrow)$
$\delta(S_B, 1) = (S_1, y, \rightarrow)$
$\delta(S_B, \#) = (S_T, \#, \rightarrow)$

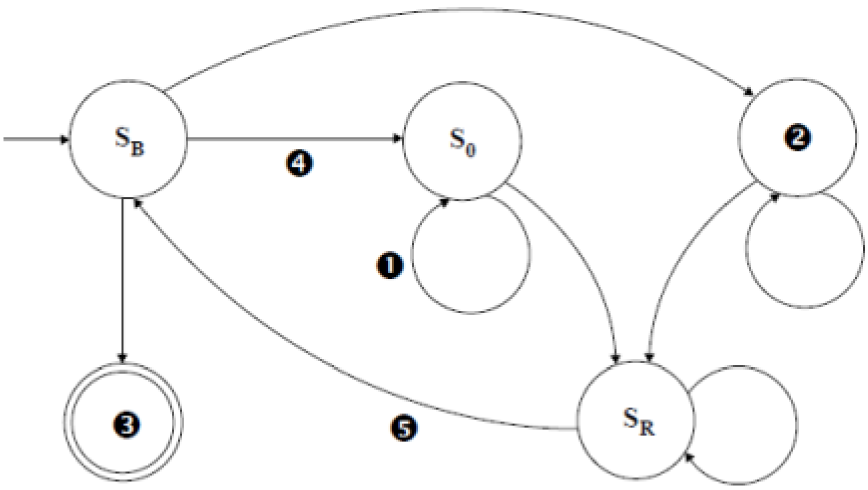
$\delta(S_0, 0) = (S_0, 0, \rightarrow)$
$\delta(S_0, 1) = (S_0, 1, \rightarrow)$
$\delta(S_0, \#) = (S_0, \#, \rightarrow)$
$\delta(S_0, \square) = (S_R, 0, \leftarrow)$

$\delta(S_1, 0) = (S_1, 0, \rightarrow)$
$\delta(S_1, 1) = (S_1, 1, \rightarrow)$
$\delta(S_1, \#) = (S_1, \#, \rightarrow)$
$\delta(S_1, \square) = (S_R, 1, \leftarrow)$

$\delta(S_R, 0) = (S_R, 0, \leftarrow)$
$\delta(S_R, 1) = (S_R, 1, \leftarrow)$
$\delta(S_R, \#) = (S_R, \#, \leftarrow)$
$\delta(S_R, x) = (S_B, 0, \rightarrow)$
$\delta(S_R, y) = (S_B, 1, \rightarrow)$

Figure 1 shows an unlabelled finite state transition diagram for this machine. Some of the state transition arrows represent more than one of the machine's transition rules. For example, the arrow labeled ❶ represents the three rules: $\delta(S_0, 0) = (S_0, 0, \rightarrow)$, $\delta(S_0, 1) = (S_0, 1, \rightarrow)$ and $\delta(S_0, \#) = (S_0, \#, \rightarrow)$.

Figure 1



(a) (i) Which states are represented by the labels ❷ and ❸ in **Figure 1**?

❷ ______________________ ❸ ______________________

(1)

(ii) Which of the machine's transition rule(s) is / are represented by the arrow labelled ❹ in **Figure 1**?

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(1)

(iii) Which of the machine's transition rule(s) is / are represented by the arrow labelled ❺ in **Figure 1**?

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(1)

The machine's transition rule, δ, is repeated here so that you can answer part **(b)** without having to turn back in the question paper booklet.

$\delta(S_B, 0) = (S_0, x, \rightarrow)$
$\delta(S_B, 1) = (S_1, y, \rightarrow)$
$\delta(S_B, \#) = (S_T, \#, \rightarrow)$

$\delta(S_0, 0) = (S_0, 0, \rightarrow)$
$\delta(S_0, 1) = (S_0, 1, \rightarrow)$
$\delta(S_0, \#) = (S_0, \#, \rightarrow)$
$\delta(S_0, \square) = (S_R, 0, \leftarrow)$

$\delta(S_1, 0) = (S_1, 0, \rightarrow)$
$\delta(S_1, 1) = (S_1, 1, \rightarrow)$
$\delta(S_1, \#) = (S_1, \#, \rightarrow)$
$\delta(S_1, \square) = (S_R, 1, \leftarrow)$

$\delta(S_R, 0) = (S_R, 0, \leftarrow)$
$\delta(S_R, 1) = (S_R, 1, \leftarrow)$
$\delta(S_R, \#) = (S_R, \#, \leftarrow)$
$\delta(S_R, x) = (S_B, 0, \rightarrow)$
$\delta(S_R, y) = (S_B, 1, \rightarrow)$

(b) This Turing machine is carrying out a computation. The machine starts in state S_B with the string 01# on the tape. All other cells contain the blank symbol, □ (not shown).

Trace the computation of the Turing machine, using the transition function δ. Show the contents of the tape, the current position of the read / write head and the current state as the input symbols are processed. The first three steps and final state have been completed for you.

								State
1.	0	1	#				...	S_B
2.	x	1	#				...	S_0
3.	x	1	#				...	S_0
4.							...	
5.							...	
6.							...	
7.							...	
8.							...	
9.							...	
10.							...	
11.							...	
12.							...	
13.							...	
14.							...	
15.							...	
16.							...	S_T

(6)

(i) Describe the purpose of the symbols x and y in this Turing machine's alphabet.

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(1)

(ii) What does the Turing machine do?

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(1)

(Total 11 marks)

Q10.

A particular Turing machine has states S_1, S_2, S_3 and S_4. S_1 is the start state and S_4 is the stop state. The machine uses one tape which is infinitely long in one direction to store data. The machine's alphabet is 1, □. The symbol □ is used to indicate a blank cell on the tape.

The transition rules for this Turing machine can be expressed as a transition function δ. Rules are written in the form:

δ(Current State, Input Symbol) = (Next State, Output Symbol, Movement)

So, for example, the rule:

$\delta\ (S_1, 1) = (S_1, 1, \rightarrow)$

means:

IF the machine is currently in state S_1 AND the input symbol read from the tape is 1
THEN the machine should remain in state S_1, write a 1 to the tape and move the read/write head one cell to the right

The machine's transition function, δ, is defined by:

$\delta\ (S_1, 1) = (S_1, 1, \rightarrow)$

$\delta\ (S_1, \square) = (S_2, \square, \leftarrow)$

$\delta\ (S_2, 1) = (S_3, \square, \leftarrow)$

$\delta\ (S_3, 1) = (S_4, \square, \leftarrow)$

(a) The Turing machine is carrying out a computation. The machine starts in state S_1 with the string `1111` on the tape. All other cells contain the blank symbol, □. The read/write head is positioned at the leftmost `1`, as indicated by the arrow.

			1	1	1	1			

.... Current State: S_1

Trace the computation of the Turing machine, using the transition function δ. Show the contents of the tape, the current position of the read / write head and the current state as the input symbols are processed.

.... Current State:

.... Current State:

.... Current State:

.... Current State:

.... Current State: ☐

.... Current State: ☐

.... Current State: ☐

(6)

(b) Explain what this Turing machine does.

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(1)

(c) Explain what a *Universal Turing machine* is.

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(2)

(Total 9 marks)

Q11.

A particular Turing machine has states S_1, S_2 and S_3.
S_1 is the start state and S_3 is the stop state.
The machine uses one tape which is infinitely long in one direction to store data.
The machine's alphabet is 0, 1, o, e and □, where □ is the symbol used to indicate a blank cell on the tape.

The transition rules for this Turing machine can be expressed as a transition function δ.
Rules are written in the form:

δ(Current State, Input Symbol) = (Next State, Output Symbol, Movement)

So, for example, the rule:

$\delta(S_1, 0) = (S_1, 0, \rightarrow)$

means

IF the machine is currently in state S_1 AND the input symbol read from the tape is 0
THEN the machine should remain in state S_1, write a 0 to the tape and move the read/write head one cell to the right

The machine's transition function, δ, is defined by:

$\delta\ (S_1, 0) = (S_1, 0, \rightarrow)$

$\delta\ (S_1, 1) = (S_2, 1, \rightarrow)$

$\delta\ (S_1, \square) = (S_3, e, \rightarrow)$

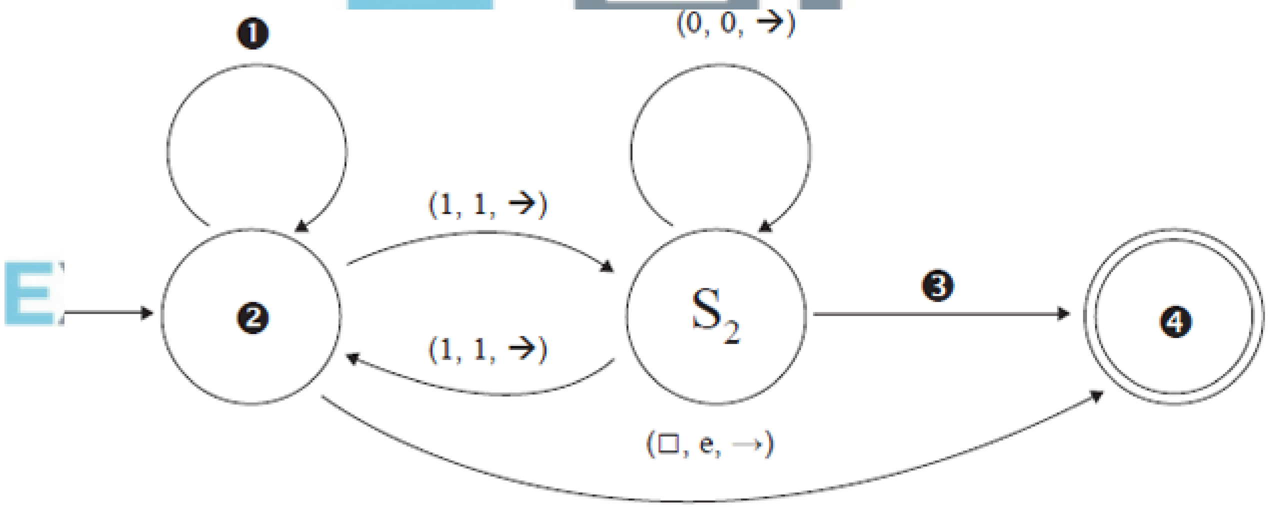
$\delta\ (S_2, 0) = (S_2, 0, \rightarrow)$

$\delta\ (S_2, 1) = (S_1, 1, \rightarrow)$

$\delta\ (S_2, \square) = (S_3, 0, \rightarrow)$

The diagram below shows a partially labelled finite state transition diagram for this machine.

Some labels are missing and have been replaced by numbers such as ❶. Each state transition arrow is labelled with the input symbol, the output symbol and the direction of movement, in that order. For example (□, e, →) means that if the input symbol is □, an e is written to the tape and the read/write head moves right one cell.



(a) Four labels are missing from the diagram above.

Write the missing labels in the table below.

Number	Correct Label
❶	
❷	
❸	

❹	

(2)

(b) The Turing machine is carrying out a computation using one tape which is infinitely long in one direction. The machine starts in state S_1 with the string `01100` on the tape.
All other cells contain the blank symbol, □. The read/write head is positioned at the leftmost zero, as indicated by the arrow.

			0 ↑	1	1	0	0		

Current State: S_1

Trace the computation of the Turing machine, using the transition function δ. Show the contents of the tape, the current position of the read/write head and the current state as the input symbols are processed.

Current State:

Current State:

Current State:

Current State:

Current State:

Current State:

(4)

(c) What is the purpose of the algorithm represented by this Turing machine?

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(1)

(d) Explain the importance of the theory of Turing machines to the subject of computation.

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(2)
(Total 9 marks)