4.4 Probability Distributions

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Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.4 Probability Distributions
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1

H = 'heads' T = 'tails'

There are 8 possible outcomes:

a) {H, H, H}

{H, H, T} {H, T, H} {T, H, H}

{H, T, T} {T, H, T} {T, T, H}

{T, T, T}

x	0	1	2	3
$P(X = x)$	$\frac{1}{27}$	$\frac{6}{27}$	$\frac{12}{27}$	$\frac{8}{27}$

b) $\{H, H, H\}$ $\frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27} = P(X = 3)$

$\{H, H, T\}$ $\frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$

$\{H, T, H\}$ $\frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{27}$

$\{T, H, H\}$ $\frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{27}$

$\frac{4}{27} + \frac{4}{27} + \frac{4}{27} = \frac{12}{27} = P(X = 2)$

$\{H, T, T\}$ $\frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{2}{27}$

$\{T, H, T\}$ $\frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{2}{27}$

$\{T, T, H\}$ $\frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{27}$

$\frac{2}{27} + \frac{2}{27} + \frac{2}{27} = \frac{6}{27} = P(X = 1)$

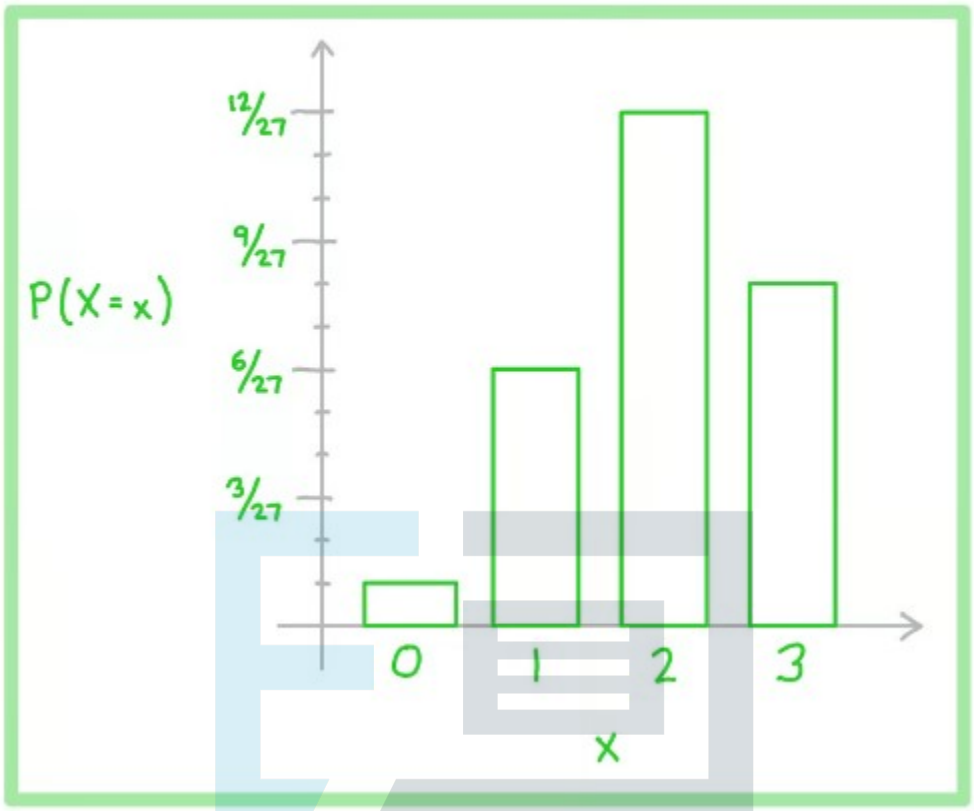
$\{T, T, T\}$ $\frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27} = P(X = 0)$

c)

$$P(X = x) = f(x) = \begin{cases} \frac{1}{27} & x = 0 \\ \frac{6}{27} & x = 1 \\ \frac{12}{27} & x = 2 \\ \frac{8}{27} & x = 3 \\ 0 & \text{otherwise} \end{cases}$$

$\frac{12}{27} = \frac{4}{9}$ $\frac{6}{27} = \frac{2}{9}$

d)



Sum of all probabilities must equal one:

Question 2

$$\frac{1}{3k} + \frac{2}{3k} + \frac{3}{3k} + \frac{4}{3k} + \frac{5}{3k} = 1$$

$$\frac{1+2+3+4+5}{3k} = 1$$

$$\frac{15}{3k} = 1$$

$$3k = 15$$

$$k = \frac{15}{3}$$

$$k = 5$$

Question 3

a) Sum of all probabilities must equal one:

$$k + 3k + 5k + 7k = 16k = 1$$

$$k = \frac{1}{16}$$

b) $P(X > 3) = P(X = 5 \text{ or } 7) = P(X = 5) + P(X = 7)$

$$P(X > 3) = \frac{5}{16} + \frac{7}{16} = \frac{12}{16}$$

$$P(X > 3) = \frac{3}{4}$$

c) There is a finite number of possible values that X can take, so X is a discrete random variable.

Question 4

a) Sum of all probabilities must equal one:

$$0.23 + 0.23 + k + k + 0.13 + 0.13 = 1$$

$$2k + 0.72 = 1$$

$$2k = 0.28$$

$$k = 0.14$$

b)

x	-1	0	1	2	3	4
P(X = x)	0.23	0.14	0.13	0.14	0.13	0.23

x	-1	0	1	2	3	4
P(X = x)	0.23	0.14	0.13	0.14	0.13	0.23

c) $P(0 \leq X < 3) = P(X = 0 \text{ or } 1 \text{ or } 2)$

$= P(X = 0) + P(X = 1) + P(X = 2)$

$P(0 \leq X < 3) = 0.14 + 0.13 + 0.14 = \boxed{0.41}$

Question 5

(i) $P(X<4) = P(X \neq 4) = 1 - P(X=4)$ *

$P(X<4) = 1 - \frac{1}{8} = \boxed{\frac{7}{8}}$

* This is easier than adding up the probabilities for 0, 1, 2, and 3!

(ii) $P(X>1) = P(X = 2 \text{ or } 3 \text{ or } 4)$

$P(X>1) = \frac{1}{4} + \frac{1}{12} + \frac{1}{8} = \boxed{\frac{11}{24}}$

(iii) $P(2<X\leq 4) = P(X = 3 \text{ or } 4)$

$P(2<X\leq 4) = \frac{1}{12} + \frac{1}{8} = \boxed{\frac{5}{24}}$

(iv) $P(0<X<4) = P(X = 1 \text{ or } 2 \text{ or } 3)$

$P(0<X<4) = \frac{1}{3} + \frac{1}{4} + \frac{1}{12} = \frac{8}{12} = \boxed{\frac{2}{3}}$

Question 6

a) Sum of all probabilities must equal one:

$\frac{6}{20} + p + \frac{3}{20} + \frac{5}{20} + \frac{3}{20} + \frac{1}{20} = 1$

$p + \frac{18}{20} = 1$

$\boxed{p = \frac{2}{20}} = \frac{1}{10}$

b) Possible outcomes that add up to 5 are:

$\{0, 5\}$ $\{5, 0\}$ $\{2, 3\}$ $\{3, 2\}$

$\{0, 5\}$ $\frac{6}{20} \times \frac{1}{20} = \frac{6}{400}$

$\{5, 0\}$ $\frac{1}{20} \times \frac{6}{20} = \frac{6}{400}$

$\{2, 3\}$ $\frac{5}{20} \times \frac{3}{20} = \frac{15}{400}$

$\{3, 2\}$ $\frac{3}{20} \times \frac{5}{20} = \frac{15}{400}$

$\frac{6}{400} + \frac{6}{400} + \frac{15}{400} + \frac{15}{400} = \frac{42}{400} = \boxed{\frac{21}{200}}$

$= 0.105$

c) $\frac{6}{20} + \left(\frac{2}{20} + \frac{3}{20}\right) = \frac{11}{20}$

Note: Both '1' sectors on the spinner are included in the event $X = 1$.

$\frac{11}{20} + \frac{5}{20} = \frac{16}{20}$

$\frac{16}{20} + \frac{3}{20} = \frac{19}{20}$

d) (i) $P(X \text{ is no more than } 1) = P(X \leq 1)$

$\boxed{\frac{11}{20}} = 0.55$

(ii) $P(X \text{ is at least } 3) = P(X \geq 3) = 1 - P(X \leq 2)$

$1 - \frac{16}{20} = \frac{4}{20} = \boxed{\frac{1}{5}} = 0.2$

Question 7

a) The sum of all probabilities must equal one, so:

$$0.11 + k^2 + 0.1 + 2k + 0.1 = 1$$

$$k^2 + 2k + 0.31 = 1$$

$$k^2 + 2k + 0.31 - 1 = 0$$

$$k^2 + 2k - 0.69 = 0$$

b) Solve the quadratic:

$$k^2 + 2k - 0.69 = 0$$

$$(k - 0.3)(k + 2.3) = 0$$

$$k = 0.3 \text{ or } -2.3$$

You can also use your GDC to solve this

But k is positive, so

$$k = 0.3$$

c) $k^2 = 0.3^2 = 0.09$ $\quad 2k = 2(0.3) = 0.6$

$E(X) = \sum x\,P(X=x)$ } Expected value of a discrete random variable X

$E(X) = (-3)(0.11) + (-1)(0.09) + (0)(0.1) + (1)(0.6) + (3)(0.1)$

$= -0.33 - 0.09 + 0 + 0.6 + 0.3$

$E(X) = 0.48$

Question 8 If X is a person's score, then:

$P(X=-5) = 0.55$ $\quad P(X=2) = 0.15$ $\quad P(X=3) = 0.15$

$P(X=10) = 0.1$ $\quad P(X=k) = 0.05$

And $E(X) = 0$, so:

$E(X) = \sum x\,P(X=x)$ } Expected value of a discrete random variable X

$(-5)(0.55) + (2)(0.15) + (3)(0.15) + (10)(0.1) + k(0.05) = 0$

$-2.75 + 0.3 + 0.45 + 1 + 0.05k = 0$

$0.05k - 1 = 0$

$0.05k = 1$

$k = \frac{1}{0.05}$

$k = 20$

Question 9

a) The sum of all probabilities must equal one, so:

$$0.1 + 0.05 + a + b + 0.1 = 1$$

$$a + b + 0.25 = 1$$

$$a + b = 1 - 0.25$$

$$a + b = 0.75$$

And $E(X) = 2.3$, so:

$$(0)(0.1) + (1)(0.05) + (2)(a) + (3)(b) + (4)(0.1) = 2.3$$

$$0 + 0.05 + 2a + 3b + 0.4 = 2.3$$

$$2a + 3b + 0.45 = 2.3$$

$$2a + 3b = 2.3 - 0.45$$

$$2a + 3b = 1.85$$

b) Solve the simultaneous equations

② − 2 × ①: $2a + 3b = 1.85$

$-(2a + 2b = 1.5)$

$b = 0.35$

Substitute into ①:

$a + 0.35 = 0.75$

$a = 0.75 - 0.35 = 0.4$

$a = 0.4 \quad b = 0.35$

You can also use your GDC to solve this

c) $P(1 \leq X < 4) = P(X = 1, 2, \text{or } 3)$

$P(1 \leq X < 4) = 0.05 + a + b$

$= 0.05 + 0.4 + 0.35$

$P(1 \leq X < 4) = 0.8$