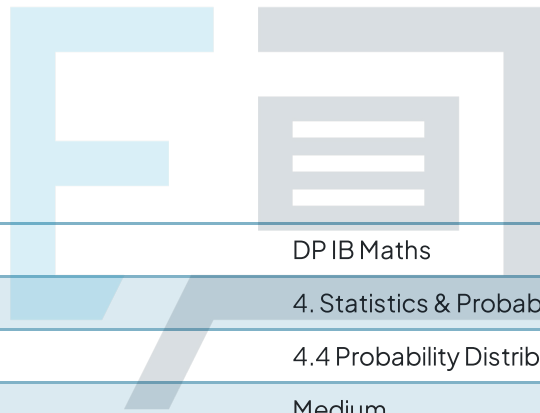




4.4 Probability Distributions

Mark Schemes



Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.4 Probability Distributions
Difficulty	Medium

Exam Papers Practice

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful



Question 1

$H = \text{'heads'}$ $T = \text{'tails'}$

There are 8 possible outcomes :

a)

$\{H, H, H\}$

$\{H, H, T\}$ $\{H, T, H\}$ $\{T, H, H\}$

$\{H, T, T\}$ $\{T, H, T\}$ $\{T, T, H\}$

$\{T, T, T\}$

x	0	1	2	3
$P(X=x)$	$\frac{1}{27}$	$\frac{6}{27}$	$\frac{12}{27}$	$\frac{8}{27}$

$$b) \{H, H, H\} \quad \frac{2}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{8}{27} = P(X=3)$$

$$\{H, H, T\} \quad \frac{2}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{4}{27}$$

$$\{H, T, H\} \quad \frac{2}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{4}{27}$$

$$\{T, H, H\} \quad \frac{1}{3} \times \frac{2}{3} \times \frac{2}{3} = \frac{4}{27}$$

$$\frac{4}{27} + \frac{4}{27} + \frac{4}{27} = \frac{12}{27} = P(X=2)$$

$$\{H, T, T\} \quad \frac{2}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{2}{27}$$

$$\{T, H, T\} \quad \frac{1}{3} \times \frac{2}{3} \times \frac{1}{3} = \frac{2}{27}$$

$$\{T, T, H\} \quad \frac{1}{3} \times \frac{1}{3} \times \frac{2}{3} = \frac{2}{27}$$

$$\frac{2}{27} + \frac{2}{27} + \frac{2}{27} = \frac{6}{27} = P(X=1)$$

$$\{T, T, T\} \quad \frac{1}{3} \times \frac{1}{3} \times \frac{1}{3} = \frac{1}{27} = P(X=0)$$

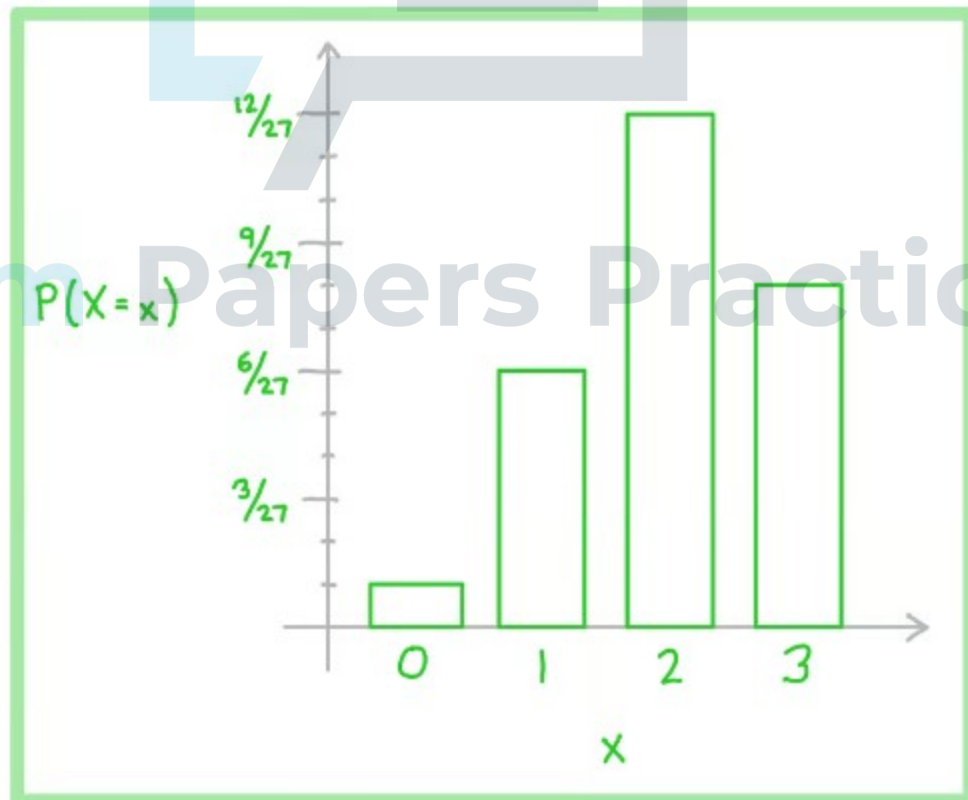
c)

$$P(X=x) = f(x) = \begin{cases} \frac{1}{27} & x=0 \\ \frac{6}{27} & x=1 \\ \frac{12}{27} & x=2 \\ \frac{8}{27} & x=3 \\ 0 & \text{otherwise} \end{cases}$$

$$\frac{12}{27} = \frac{4}{9}$$

$$\frac{6}{27} = \frac{2}{9}$$

d)





Question 2

Sum of all probabilities must equal one :

$$\frac{1}{3k} + \frac{2}{3k} + \frac{3}{3k} + \frac{4}{3k} + \frac{5}{3k} = 1$$

$$\frac{1+2+3+4+5}{3k} = 1$$

$$\frac{15}{3k} = 1$$

$$3k = 15$$

$$k = \frac{15}{3}$$

$$k = 5$$

Question 3

a) Sum of all probabilities must equal one :

$$k + 3k + 5k + 7k = 16k = 1$$

$$k = \frac{1}{16}$$

$$b) P(X > 3) = P(X = 5 \text{ or } 7) = P(X = 5) + P(X = 7)$$

$$P(X > 3) = \frac{5}{16} + \frac{7}{16} = \frac{12}{16}$$

$$P(X > 3) = \frac{3}{4}$$

c) There is a finite number of possible values that X can take, so X is a discrete random variable.

Question 4

a) Sum of all probabilities must equal one:

$$0.23 + 0.23 + k + k + 0.13 + 0.13 = 1$$

$$2k + 0.72 = 1$$

$$2k = 0.28$$

$$k = 0.14$$

b)

x	-1	0	1	2	3	4
$P(X=x)$	0.23	0.14	0.13	0.14	0.13	0.23

x	-1	0	1	2	3	4
$P(X=x)$	0.23	0.14	0.13	0.14	0.13	0.23

$$\begin{aligned} c) P(0 \leq X < 3) &= P(X=0 \text{ or } 1 \text{ or } 2) \\ &= P(X=0) + P(X=1) + P(X=2) \end{aligned}$$

$$P(0 \leq X < 3) = 0.14 + 0.13 + 0.14 = 0.41$$

Question 5

$$(i) P(X < 4) = P(X \neq 4) = 1 - P(X = 4)^*$$

$$P(X < 4) = 1 - \frac{1}{8} = \boxed{\frac{7}{8}}$$

* This is easier than adding up the probabilities for 0, 1, 2, and 3!

$$(ii) P(X > 1) = P(X = 2 \text{ or } 3 \text{ or } 4)$$

$$P(X > 1) = \frac{1}{4} + \frac{1}{12} + \frac{1}{8} = \boxed{\frac{11}{24}}$$

$$(iii) P(2 < X \leq 4) = P(X = 3 \text{ or } 4)$$

$$P(2 < X \leq 4) = \frac{1}{12} + \frac{1}{8} = \boxed{\frac{5}{24}}$$

$$(iv) P(0 < X < 4) = P(X = 1 \text{ or } 2 \text{ or } 3)$$

$$P(0 < X < 4) = \frac{1}{3} + \frac{1}{4} + \frac{1}{12} = \frac{8}{12} = \boxed{\frac{2}{3}}$$

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Question 6

a) Sum of all probabilities must equal one:

$$\frac{6}{20} + p + \frac{3}{20} + \frac{5}{20} + \frac{3}{20} + \frac{1}{20} = 1$$

$$p + \frac{18}{20} = 1$$

$$\boxed{p = \frac{2}{20}} = \frac{1}{10}$$



b) Possible outcomes that add up to 5 are:

$$\{0, 5\} \quad \{5, 0\} \quad \{2, 3\} \quad \{3, 2\}$$

$$\{0, 5\} \quad \frac{6}{20} \times \frac{1}{20} = \frac{6}{400}$$

$$\{5, 0\} \quad \frac{1}{20} \times \frac{6}{20} = \frac{6}{400}$$

$$\{2, 3\} \quad \frac{5}{20} \times \frac{3}{20} = \frac{15}{400}$$

$$\{3, 2\} \quad \frac{3}{20} \times \frac{5}{20} = \frac{15}{400}$$

$$\frac{6}{400} + \frac{6}{400} + \frac{15}{400} + \frac{15}{400} = \frac{42}{400} = \frac{21}{200} = 0.105$$

Score x	0	1	2	3	5
$P(X \leq x)$	$\frac{6}{20}$	$\frac{11}{20}$	$\frac{16}{20}$	$\frac{19}{20}$	1

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$$c) \frac{6}{20} + \left(\frac{2}{20} + \frac{3}{20} \right) = \frac{11}{20}$$

Note: Both '1' sectors on the spinner are included in the event $X=1$.

$$\frac{11}{20} + \frac{5}{20} = \frac{16}{20}$$

$$\frac{16}{20} + \frac{3}{20} = \frac{19}{20}$$



$$d) (i) P(X \text{ is no more than } 1) = P(X \leq 1)$$

$$\frac{11}{20} = 0.55$$

$$(ii) P(X \text{ is at least } 3) = P(X \geq 3) = 1 - P(X \leq 2)$$

$$1 - \frac{16}{20} = \frac{4}{20} = \frac{1}{5} = 0.2$$

Question 7

a) The sum of all probabilities must equal one, so:

$$0.11 + k^2 + 0.1 + 2k + 0.1 = 1$$

$$k^2 + 2k + 0.31 = 1$$

$$k^2 + 2k + 0.31 - 1 = 0$$

$$k^2 + 2k - 0.69 = 0$$

b) Solve the quadratic:

$$k^2 + 2k - 0.69 = 0$$

$$(k - 0.3)(k + 2.3) = 0$$

$$k = 0.3 \text{ or } -2.3$$
 You can also use your GDC to solve this

But k is positive, so

$$k = 0.3$$

c) $k^2 = 0.3^2 = 0.09$ $2k = 2(0.3) = 0.6$

$$E(X) = \sum x P(X=x) \quad \left. \vphantom{\sum} \right\} \text{Expected value of a discrete random variable } X$$

$$\begin{aligned} E(X) &= (-3)(0.11) + (-1)(0.09) + (0)(0.1) + (1)(0.6) + (3)(0.1) \\ &= -0.33 - 0.09 + 0 + 0.6 + 0.3 \end{aligned}$$

$$E(X) = 0.48$$



Question 8

If X is a person's score, then :

$$P(X=-5) = 0.55 \quad P(X=2) = 0.15 \quad P(X=3) = 0.15$$

$$P(X=10) = 0.1 \quad P(X=k) = 0.05$$

And $E(X) = 0$, so :

$$E(X) = \sum x P(X=x) \quad \left. \vphantom{E(X)} \right\} \begin{array}{l} \text{Expected value of a} \\ \text{discrete random variable } X \end{array}$$

$$(-5)(0.55) + (2)(0.15) + (3)(0.15) + (10)(0.1) + k(0.05) = 0$$

$$-2.75 + 0.3 + 0.45 + 1 + 0.05k = 0$$

$$0.05k - 1 = 0$$

$$0.05k = 1$$

$$k = \frac{1}{0.05}$$

$$k = 20$$

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Question 9

a)

The sum of all probabilities must equal one, so:

$$0.1 + 0.05 + a + b + 0.1 = 1$$

$$a + b + 0.25 = 1$$

$$a + b = 1 - 0.25$$

$$a + b = 0.75$$

And $E(X) = 2.3$, so:

$$(0)(0.1) + (1)(0.05) + (2)(a) + (3)(b) + (4)(0.1) = 2.3$$

$$0 + 0.05 + 2a + 3b + 0.4 = 2.3$$

$$2a + 3b + 0.45 = 2.3$$

$$2a + 3b = 2.3 - 0.45$$

$$2a + 3b = 1.85$$

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b) Solve the simultaneous equations

$$\begin{array}{r} \textcircled{2} - 2 \times \textcircled{1} : \quad 2a + 3b = 1.85 \\ \quad \quad \quad \quad - (2a + 2b = 1.5) \\ \hline \quad \quad \quad \quad \quad \quad b = 0.35 \end{array}$$

Substitute into ①:

$$a + 0.35 = 0.75$$

$$a = 0.75 - 0.35 = 0.4$$

$$a = 0.4 \quad b = 0.35$$

You can also use your GDC to solve this

c) $P(1 \leq X < 4) = P(X = 1, 2, \text{ or } 3)$

$$P(1 \leq X < 4) = 0.05 + a + b$$

$$= 0.05 + 0.4 + 0.35$$

$$P(1 \leq X < 4) = 0.8$$