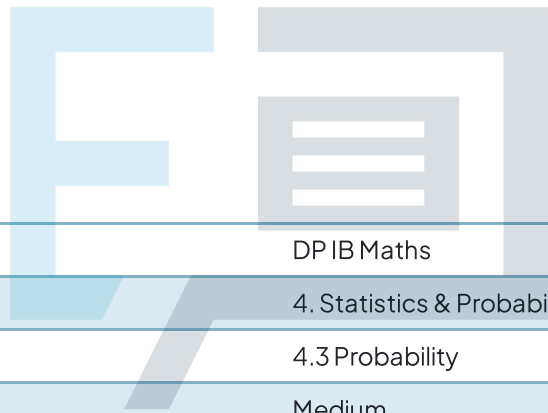




4.3 Probability

Mark Schemes



| | |
|------------|-----------------------------|
| Course | DP IB Maths |
| Section | 4. Statistics & Probability |
| Topic | 4.3 Probability |
| Difficulty | Medium |

Exam Papers Practice

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful



Question 1

(i) SUM OF MALES $7+21+19+5+2+1=55$

$$P(\text{MALE}) = \frac{55}{120} = \frac{11}{24}$$

$$P(\text{MALE}) = \frac{11}{24}$$

(ii) SUM < 51 (FIRST 4 ROWS)

$$14+29+12+6+7+21=89$$

$$P(< 51) = \frac{89}{120}$$

(iii) NO MALES ARE < 45 cm

$$P(< 45) = 0$$

(iv) SUM $45 \leq L < 54$ FOR FEMALES ONLY

$$12+6+3=21$$

$$P(45 \leq F < 54) = \frac{21}{120} = \frac{7}{40}$$

$$P(45 \leq F < 54) = \frac{7}{40}$$



Question 2

DRAW A SAMPLE SPACE DIAGRAM TO SHOW ALL POSSIBILITIES

| x | 1 | 2 | 3 |
|---|---|---|---|
| 1 | 1 | 2 | 3 |
| 2 | 2 | 4 | 6 |
| 3 | 3 | 6 | 9 |

USE THE SAMPLE SPACE DIAGRAM TO FIND REQUIRED PROBABILITIES

(i) $P(6) = \frac{2}{9}$

(ii) $P(<4) = \frac{5}{9}$

(iii) $P(000) = \frac{4}{9}$

Exam Papers Practice

Question 3

(a)

(i) IF $P(\text{CHEESE ROLLING}) = 0.44$

$$x = 0.44 - 0.21 = 0.23$$

$$x = 0.23$$

(ii) PROBABILITIES SHOULD TOTAL 1

$$y = 1 - (0.32 + 0.09 + 0.23 + 0.21 + 0.11)$$

$$= 1 - 0.96 = 0.04$$

$$y = 0.04$$

(b)

(i) ATLEAST ONE = EVERYTHING INSIDE CIRCLES

$$1 - 0.11 = 0.89 \quad (\text{COULD ADD ALL INSIDE})$$

$$P(\text{ATLEAST ONE}) = 0.89$$

(ii) EXACTLY ONE = INSIDE INDIVIDUAL CIRCLES (NOT OVERLAPS)

$$0.32 + 0.04 + 0.21 = 0.57$$

$$P(\text{EXACTLY ONE}) = 0.57$$



Question 4 (a) 'GIVEN THEY ENJOY AT LEAST ONE GENRE' IGNORE VALUES OUTSIDE CIRCLES (24)

$$15 + 3 + 16 + 27 + 4 + 2 + 9 = 76$$

(i) $P(\text{BOLLYWOOD MUSICALS}) = \frac{16 + 3 + 4 + 2}{76}$ ← VALUES IN B

$$P(\text{BOLLYWOOD}) = \frac{25}{76}$$

(ii) $P(\text{ONLY ONE GENRE}) = \frac{15 + 16 + 9}{76} = \frac{40}{76}$ ← VALUES NOT IN INTERSECTIONS

$$P(\text{ONLY ONE GENRE}) = \frac{10}{19}$$

(iii) $P(\text{EXACTLY TWO GENRES}) = \frac{3 + 27 + 2}{76} = \frac{32}{76}$ ← VALUES IN DUAL INTERSECTIONS

$$P(\text{EXACTLY TWO GENRES}) = \frac{8}{19}$$

Exam Papers Practice



(b) (i) INTERSECTION

$$P(A \cap C) = \frac{27+4}{100} = \frac{31}{100} = 0.31$$

$$P(A \cap C) = \frac{31}{100} \quad \text{OR } 0.31$$

(ii) $P(A \cup C)$ COULD ADD ALL VALUES IN UNION \rightarrow A OR C OR TAKE VALUES NOT IN EITHER

OR

$$\frac{15+3+4+27+9+2}{100} = \frac{60}{100} = \frac{3}{5} = 0.6$$

$$\frac{100 - (16+24)}{100} = \frac{60}{100} = \frac{3}{5} = 0.6$$

$$P(A \cup C) = \frac{3}{5} \quad \text{OR } 0.6$$

(iii) $P(C|B)$ = CONDITIONAL C GIVEN B

$$n(B) = 3+4+2+16 = 25 \quad \text{TOTAL IN B}$$

$$n(C \cap B) = 4+2 = 6 \quad \text{TOTAL IN INTERSECTION}$$

$$P(C|B) = \frac{6}{25} = 0.24$$

$$P(C|B) = \frac{6}{25} \quad \text{OR } 0.24$$

(iv) $P(B')$ = $1 - P(B)$ NOT IN B

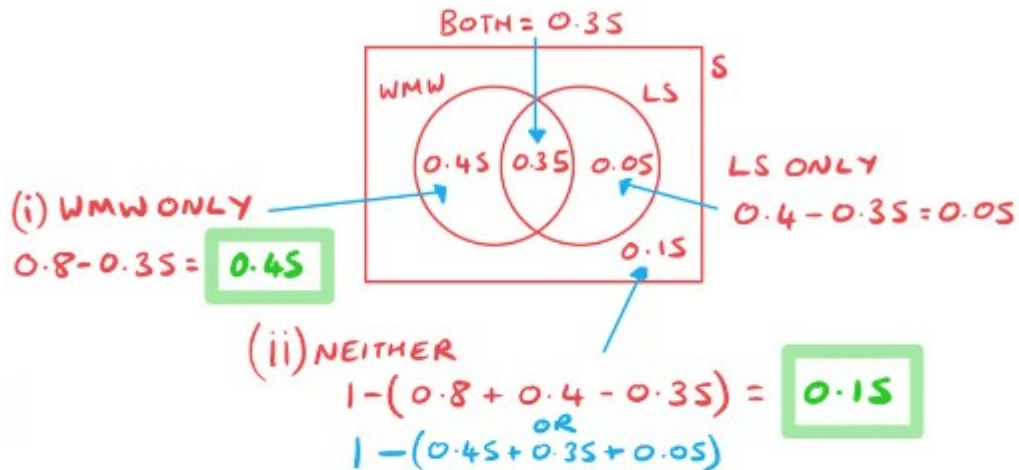
$$P(B) = \frac{3+4+2+16}{100} = \frac{25}{100}$$

$$P(B') = 1 - \frac{25}{100} = \frac{75}{100} = \frac{3}{4} = 0.75$$

$$P(B') = \frac{3}{4} \quad \text{OR } 0.75$$

Question 5

DRAW A VENN DIAGRAM TO ORGANISE OPTIONS



OR USE JUST CALCULATIONS

(i) WMW ONLY $0.8 - 0.35 = 0.45$

$$P(\text{WMW ONLY}) = 0.45$$

(ii) NEITHER $1 - (0.8 + 0.4 - 0.35) = 0.15$

$$P(\text{NEITHER}) = 0.15$$

Exam Papers Practice

Question 6

(a) 'U' = 'OR' = ADD PROBABILITIES

MUTUALLY EXCLUSIVE = NO COMMON OUTCOMES
(NO OVERLAP IN VENN DIAGRAM)

$$P(A) + P(B) = 0.35 + 0.25 = 0.6 = P(A \cup B)$$

$$P(A) + P(B) = P(A \cup B)$$

A AND B ARE MUTUALLY EXCLUSIVE



(b) ' \cap ' = 'AND' = MULTIPLY

INDEPENDENT = NO EFFECT ON EACH OTHER

$$P(C) \times P(D) = 0.2 \times 0.4 = 0.08 \neq 0.18 = P(C \cap D)$$

$$P(C) \times P(D) \neq P(C \cap D)$$

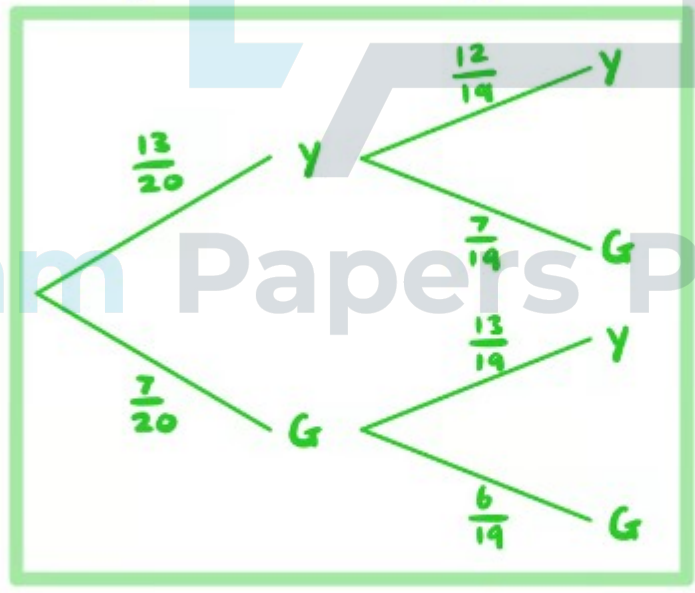
C AND D ARE NOT INDEPENDENT

Question 7

(a)

INITIAL TOTAL
 $7+13=20$

WITHOUT REPLACEMENT
TOTALS ARE NOW $20-1=19$



← YELLOW REDUCED BY 1
 $13-1=12$

← GREEN STAYS THE SAME

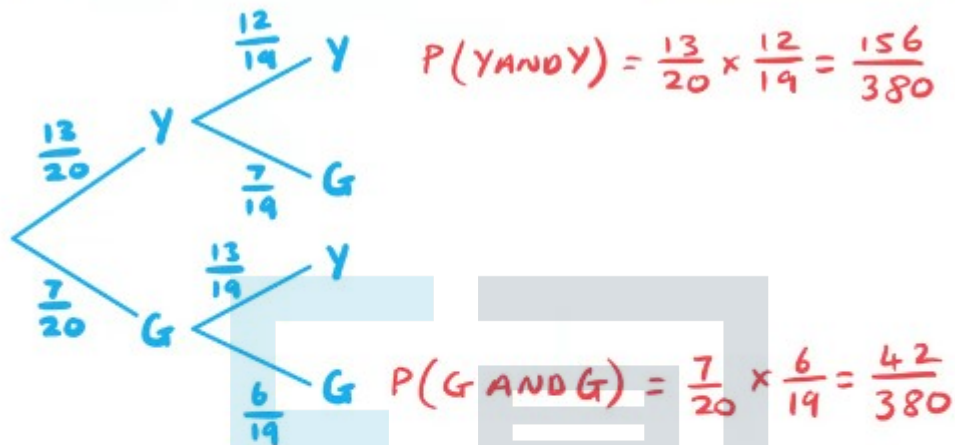
← YELLOW STAYS THE SAME

← GREEN REDUCED BY 1
 $7-1=6$

↑ CHECK EACH BRANCH PAIR = 1

YOU CAN THEN CALCULATE ALL POSSIBLE OUTCOMES BY MULTIPLYING ALONG EACH BRANCH

(b) USING TREE DIAGRAM FROM PART (a)
MULTIPLY ALONG BRANCHES WITH SAME COLOUR



NOT SIMPLIFYING MAKES PROBABILITIES
EASIER TO ADD

$$P(Y \text{ AND } Y) + P(G \text{ AND } G) = P(\text{SAME COLOUR})$$

$$\left(\frac{13}{20} \times \frac{12}{19}\right) + \left(\frac{7}{20} \times \frac{6}{19}\right) = \frac{156}{380} + \frac{42}{380} = \frac{198}{380} = \frac{99}{190}$$

$$P(\text{SAME COLOUR}) = \frac{99}{190}$$

Question 8

(a) A AND C MUTUALLY EXCLUSIVE \Rightarrow DO NOT OVERLAP

A AND B INDEPENDENT \Rightarrow

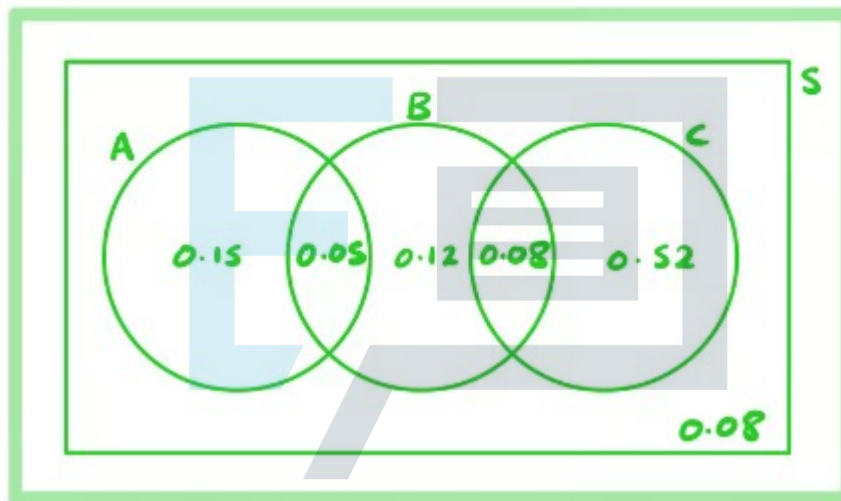
$$P(A \cap B) = P(A)P(B) = 0.2 \times 0.25 = 0.05$$

$$A \text{ ONLY} = 0.2 - 0.05 = 0.15$$

$$B \text{ ONLY} = 0.25 - (0.05 + 0.08) = 0.12$$

$$C \text{ ONLY} = 0.6 - 0.08 = 0.52$$

$$\text{OUTSIDE} = 1 - (0.15 + 0.05 + 0.12 + 0.08 + 0.52) = 0.08$$



Exam Papers Practice



(b) $A' = \underline{\text{NOT IN A}}$ $\cup = \underline{\text{OR}}$ $\cap = \underline{\text{AND}}$

(i) $\text{NOT IN A} \underline{\text{AND}} \text{NOT IN C}$

$0.12 + 0.08 = 0.2$

$P(A' \cap C') = 0.2$

(ii) $(\text{IN A} \underline{\text{AND}} \text{NOT IN B}) \underline{\text{OR}} \text{IN C}$

$0.15 + 0.6 = 0.75$

$P((A \cap B') \cup C) = 0.75$

(iii) $\text{NOT IN A} \underline{\text{OR}} (\text{NOT IN INTERSECTION OF B AND C})$

$\text{EVERYTHING} = 1$

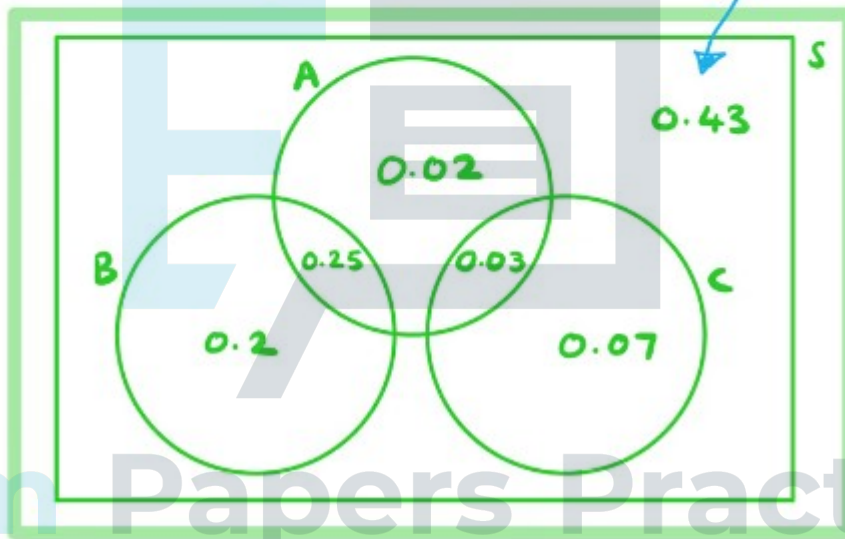
$P(A' \cup (B \cap C)') = 1$

Exam Papers Practice



Question 9

(a) $P((A \cup B \cup C)') = 0.43$ NOT IN A OR B OR C
B AND C MUTUALLY EXCLUSIVE \Rightarrow DO NOT OVERLAP
A AND C INDEPENDENT \Rightarrow
 $P(A \cap C) = P(A)P(C) = 0.3 \times 0.1 = 0.03$
 $ONLY C = 0.1 - 0.03 = 0.07$
 $A \cup B = 1 - 0.43 - 0.07 = 0.5$
 $A + B = 0.3 + 0.45 = 0.75$
 $A \cap B = 0.75 - 0.5 = 0.25$
 $ONLY B = 0.45 - 0.25 = 0.2$
 $ONLY A = 0.3 - 0.25 - 0.03 = 0.02$
CHECK ALL PROBABILITIES SUM TO 1



Exam Papers Practice

(b) (i) $P(B|A)$ = IN B GIVEN IN A

$$P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{0.25}{0.3} = \frac{5}{6}$$

$P(B|A) = \frac{5}{6}$

(ii) $P(A|B')$ = IN A GIVEN NOT IN B

$$P(A|B') = \frac{P(A \cap B')}{P(B')} = \frac{0.02 + 0.03}{1 - 0.45} = \frac{0.05}{0.55} = \frac{1}{11}$$

$P(A|B') = \frac{1}{11}$

(iii) $P(A|(B \cup C))$ = IN A GIVEN IN B OR C

$$P(A|(B \cup C)) = \frac{P(A \cap B) + P(A \cap C)}{P(B \cup C)} = \frac{0.25 + 0.03}{0.45 + 0.1} = \frac{0.28}{0.55} = \frac{28}{55}$$

$P(A|(B \cup C)) = \frac{28}{55}$

Exam Papers Practice

Question 10

(a)

$$(i) P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$P(A \cup B) = 0.27 + 0.39 - 0.21 = 0.45$$

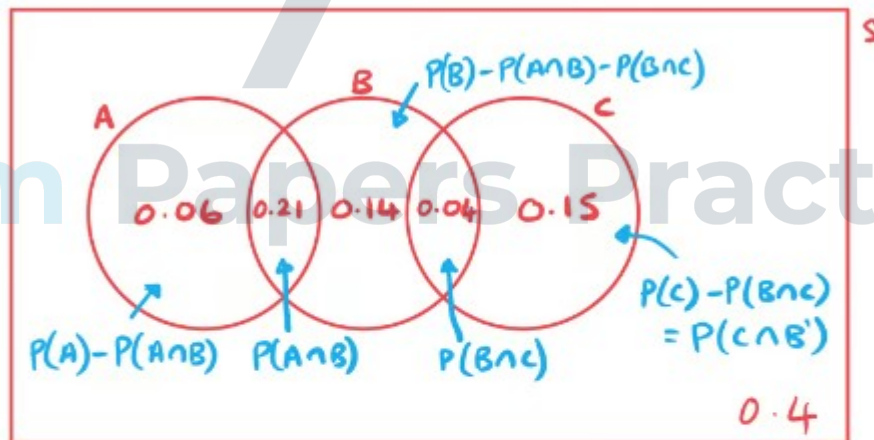
$$P(A \cup B) = 0.45$$

$$(ii) P(B|A) = \frac{P(B \cap A)}{P(A)} = \frac{P(A \cap B)}{P(A)}$$

$$P(B|A) = \frac{0.21}{0.27} = \frac{7}{9}$$

$$P(B|A) = \frac{7}{9}$$

(b) DRAWING A VENN DIAGRAM MAY HELP

 A AND C = MUTUALLY EXCLUSIVE \Rightarrow DONT OVERLAP


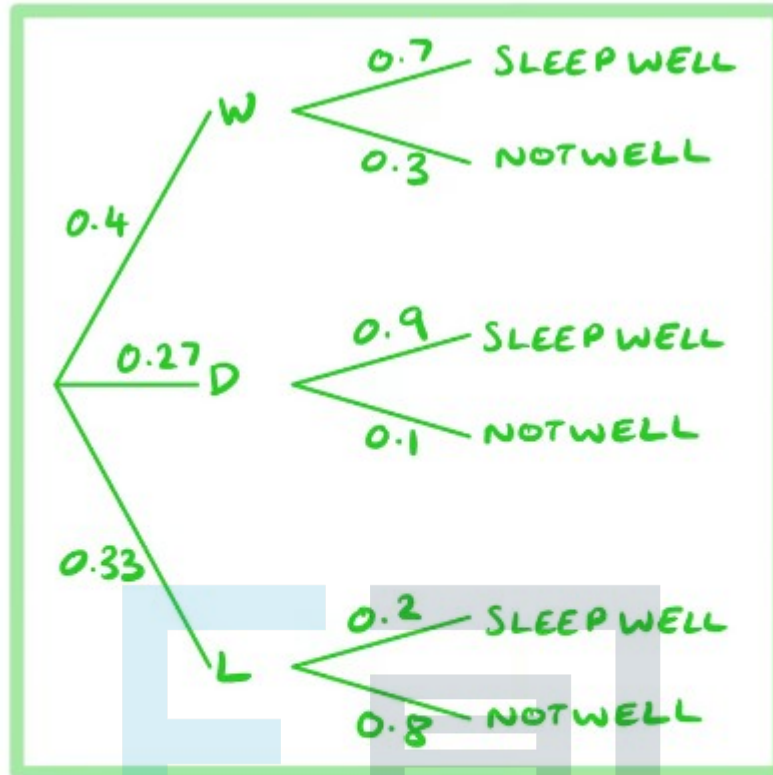
$$P(A \cup B \cup C) = P(A \cup B) + P(C \cap B')$$

\uparrow
 A OR B (FROM PART (a)) + C AND NOT IN B

$$P(A \cup B \cup C) = 0.45 + 0.15 = 0.6$$

$$P(A \cup B \cup C) = 0.6$$

Question 11 (a)



CHECK EACH
BRANCH PAIR = 1

YOU CAN THEN CALCULATE ALL POSSIBLE
OUTCOMES BY MULTIPLYING ALONG EACH
BRANCH

Exam Papers Practice

(b)

$$(i) P(\text{LOSE AND SLEEP WELL}) = 0.33 \times 0.2 = 0.066$$

$$P(\text{LOSE AND SLEEP WELL}) = 0.066$$

$$(ii) P(\text{SLEEPS WELL}) = \text{SLEEP WELL AFTER WIN, DRAW AND LOSE}$$

$$0.4 \times 0.7 + 0.27 \times 0.9 + 0.33 \times 0.2$$

$$0.28 + 0.243 + 0.066$$

$$P(\text{SLEEPS WELL}) = 0.589$$



$$(c) \quad P(\text{DRAW} | \text{SLEEPWELL}) = \frac{P(\text{WIN}_{\text{SLEEPWELL}}) + P(\text{LOSE}_{\text{SLEEPWELL}})}{P(\text{SLEEPWELL})}$$
$$\frac{(0.4 \times 0.7) + (0.33 \times 0.2)}{0.589} = \frac{0.346}{0.589}$$

↑ FROM (b)(i)

$$P(\text{DRAW} | \text{SLEEPWELL}) = \frac{346}{589}$$



Exam Papers Practice