DP IB Maths: AI HL

4.2 Correlation & Regression

Contents

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4.2.1 Bivariate Data

Scatter Diagrams

What does bivariate data mean?

- **Bivariate data** is data which is collected on **two variables** and looks at how one of the factors affects the other
  - Each data value from one variable will be **paired** with a data value from the other variable
  - The two variables are often related, but do not have to be

What is a scatter diagram?

- A **scatter diagram** is a way of graphing bivariate data
  - One variable will be on the *x*-axis and the other will be on the y-axis
  - The variable that can be **controlled** in the data collection is known as the **independent** or **explanatory variable** and is plotted on the *x*-axis
  - The variable that is **measured** or discovered in the data collection is known as the **dependent** or **response variable** and is plotted on the y-axis
- Scatter diagrams can contain **outliers** that do not follow the trend of the data

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Correlation

What is correlation?

- **Correlation** is how the **two variables change in relation to each other**
  - Correlation could be the result of a **causal relationship** but this is not always the case
- **Linear correlation** is when the changes are proportional to each other
- **Perfect linear correlation** means that the bivariate data will all lie on a straight line on a scatter diagram
- When describing correlation mention
  - The type of the correlation
    - **Positive correlation** is when an **increase** in one variable results in the other variable **increasing**
    - **Negative correlation** is when an **increase** in one variable results in the other variable **decreasing**
    - **No linear correlation** is when the data points don't appear to follow a trend
  - The strength of the correlation
    - **Strong linear correlation** is when the data points lie **close** to a **straight line**
    - **Weak linear correlation** is when the data points are **not close** to a **straight line**
- If there is **strong linear correlation** you can draw a l**ine of best fit** (by eye)
  - The line of best fit will pass through the mean point $(\bar{x}, \bar{y})$
  - If you are asked to draw a line of best fit
    - Plot the mean point
    - Draw a line going through it that follows the trend of the data

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What is the difference between correlation and causation?

- It is important to be aware that just because correlation exists, it does not mean that the change in one of the variables is **causing** the change in the other variable
  - **Correlation does not imply causation!**
- If a change in one variable **causes** a change in the other then the two variables are said to have a **causal relationship**
  - Observing correlation between two variables does **not always** mean that there is a causal relationship
    - There could be **underlying factors** which is causing the correlation
  - Look at the two variables in question and consider the context of the question to decide if there could be a causal relationship
    - If the two variables are temperature and number of ice creams sold at a park then it is likely to be a causal relationship
    - Correlation may exist between global temperatures and the number of monkeys kept as pets in the UK but they are unlikely to have a causal relationship

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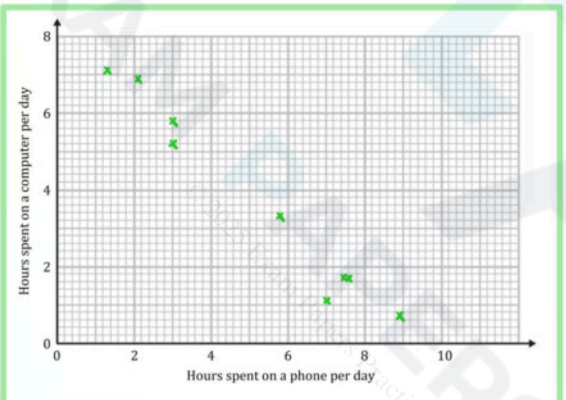
Worked example

A teacher is interested in the relationship between the number of hours her students spend on a phone per day and the number of hours they spend on a computer. She takes a sample of nine students and records the results in the table below.

Hours spent on a phone per day	7.6	7.0	8.9	3.0	3.0	7.5	2.1	1.3	5.8
Hours spent on a computer per day	1.7	1.1	0.7	5.8	5.2	1.7	6.9	7.1	3.3

a) Draw a scatter diagram for the data.



b) Describe the correlation.

Strong negative linear correlation

c) Draw a line of best fit.

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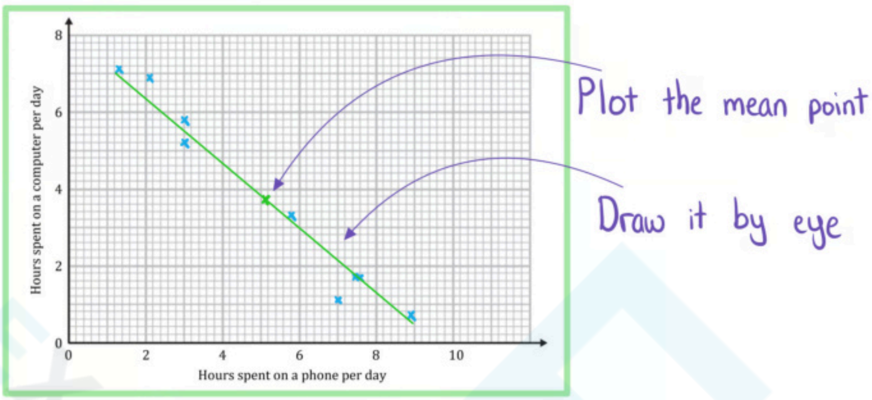
Mean point $(\bar{x}, \bar{y}) = (5.133..., 3.722...)$



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4.2.2 Correlation Coefficients

PMCC

What is Pearson's product-moment correlation coefficient?

- Pearson's product-moment correlation coefficient (PMCC) is a way of giving a numerical value to a **linear relationship** of bivariate data
- The PMCC of a sample is denoted by the letter r
  - r can take any value such that $-1 \leq r \leq 1$
  - A **positive value** of r describes **positive correlation**
  - A **negative value** of r describes **negative correlation**
  - $r = 0$ means there is **no linear correlation**
  - $r = 1$ means **perfect positive linear** correlation
  - $r = -1$ means **perfect negative linear** correlation
  - The closer to 1 or -1 the stronger the correlation

How do I calculate Pearson's product-moment correlation coefficient (PMCC)?

- You will be expected to use the statistics mode on your GDC to calculate the PMCC
- The formula can be useful to deepen your understanding

$$r = \frac{S_{xy}}{S_x S_y}$$

  - $S_{xy} = \sum_{i=1}^{n} x_i y_i - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)\left(\sum_{i=1}^{n} y_i\right)$ is linked to the **covariance**
  - $S_x = \sqrt{\sum_{i=1}^{n} x_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} x_i\right)^2}$ and $S_y = \sqrt{\sum_{i=1}^{n} y_i^2 - \frac{1}{n}\left(\sum_{i=1}^{n} y_i\right)^2}$ are linked to the **variances**
- You **do not need to learn this** as using your GDC will be expected

When does the PMCC suggest there is a linear relationship?

- **Critical values** of r indicate when the PMCC would suggest there is a linear relationship
  - In your exam you will be given critical values where appropriate
  - Critical values will depend on the size of the sample
- If the **absolute value** of the **PMCC** is **bigger** than the **critical value** then this suggests a linear model is appropriate

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Spearman's Rank

What is Spearman's rank correlation coefficient?

- Spearman's rank correlation coefficient is a measure of how well the relationship between two variables can be described using a **monotonic** function
  - **Monotonic** means the points are either always increasing or always decreasing
  - This can be used as a way to **measure correlation in linear models**
  - Though Spearman's Rank correlation coefficient can also be used to assess a non-linear relationship
- Each data is ranked, from biggest to smallest or from smallest to biggest
  - For n data values, they are ranked from 1 to n
  - It doesn't matter whether variables are ranked from biggest to smallest or smallest to biggest, but they must be ranked in the **same order for both variables**
- Spearman's rank of a sample is denoted by r_s
  - r_s can take any value such that $-1 \leq r_s \leq 1$
  - A **positive value** of r_s describes a **degree of agreement** between the rankings
  - A **negative value** of r_s describes a **degree of disagreement** between the rankings
  - $r_s = 0$ means the data shows **no monotonic behaviour**
  - $r_s = 1$ means the rankings are in complete agreement: the data is **strictly increasing**
    - An increase in one variable means an increase in the other
  - $r_s = -1$ means the rankings are in complete disagreement: the data is **strictly decreasing**
    - An increase in one variable means a decrease in the other
  - The **closer to 1 or -1** the **stronger the correlation** of the rankings

How do I calculate Spearman's rank correlation coefficient (PMCC)?

- Rank each set of data independently
  - 1 to n for the x-values
  - 1 to n for the y-values
- If some values are equal then give each the average of the ranks they would occupy
  - For example: if the 3rd, 4th and 5th highest values are equal then give each the ranking of 4
    - $\frac{3+4+5}{3} = 4$
- Calculate the PMCC of the **rankings** using your GDC
  - This value is **Spearman's rank correlation coefficient**

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Appropriateness & Limitations

Which correlation coefficient should I use?

- **Pearson's PMCC** tests for a **linear relationship** between two variables
  - It will not tell you if the variables have a non-linear relationship
    - Such as exponential growth
  - Use this if you are interested in a linear relationship
- **Spearman's rank** tests for a **monotonic relationship** (always increasing or always decreasing) between two variables
  - It will not tell you what function can be used to model the relationship
    - Both linear relationships and exponential relationships can be monotonic
  - Use this if you think there is a non-linear monotonic relationship

How are Pearson's and Spearman's correlation coefficients connected?

- If there is **linear correlation** then the relationship is also **monotonic**
  - $r = 1 \Rightarrow r_s = 1$
  - $r = -1 \Rightarrow r_s = -1$
  - However the **converse is not true**
- It is possible for Spearman's rank to be 1 (or -1) but for the PMCC to be different
  - For example: data that follows an **exponential growth model**
    - $r_s = 1$ as the points are always increasing
    - $r < 1$ as the points do not lie on a straight line

Are Pearson's and Spearman's correlation coefficients affected by outliers?

- Pearson's PMCC **is** affected by outliers
  - as it uses the numerical value of each data point
- Spearman's rank is **not usually** affected by outliers
  - as it only uses the ranks of each data point

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Worked example

The table below shows the scores of eight students for a maths test and an English test.

Maths (x)	7	18	37	52	61	68	75	82
English (y)	5	3	9	12	17	41	49	97

a) Write down the value of Pearson's product-moment correlation coefficient, r.

Enter data into GDC.

$r = 0.79433...$

$r = 0.794$ (3sf)

b) Find the value of Spearman's rank correlation coefficient, r_s.

Rank the data

x rank	8	7	6	5	4	3	2	1
y rank	7	8	6	5	4	3	2	1

Find PMCC of ranks

$r_s = 0.97619...$

$r_s = 0.976$ (3sf)

c) Comment on the values of the two correlation coefficients.

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The value of r suggests there is strong positive linear correlation. The value of r_s suggests strong positive correlation, which is not necessarily linear.

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4.2.3 Linear Regression

Linear Regression

What is linear regression?

- If **strong linear correlation** exists on a scatter diagram then the data can be modelled by a **linear model**
  - Drawing lines of best fit by eye is not the best method as it can be difficult to judge the best position for the line
- The **least squares regression line** is the line of best fit that minimises the **sum of the squares** of the gap between the line and each data value
  - This is usually called the **regression line of *y* on *x***
  - It can be calculated by looking at the vertical distances between the line and the data values
- The **regression line of y on x** is written in the form $y = ax + b$
- *a* is the **gradient** of the line
  - It represents the change in *y* for each individual unit change in *x*
    - If *a* is **positive** this means *y* **increases** by *a* for a unit increase in *x*
    - If *a* is **negative** this means *y* **decreases** by |a| for a unit increase in *x*
- *b* is the ***y* – intercept**
  - It shows the value of *y* when *x* is zero
- You are expected to use your **GDC** to find the equation of the regression line
  - Enter the bivariate data and choose the **model "*ax* + *b*"**
  - Remember the **mean point** $(\bar{x}, \bar{y})$ will lie on the regression line

How do I use a regression line?

- The equation of the regression line can be used to decide what type of correlation there is if there is no scatter diagram
  - If *a* is **positive** then the data set has **positive correlation**
  - If *a* is **negative** then the data set has **negative correlation**
- The equation of the regression line can also be used to **predict** the value of a **dependent variable (*y*)** from an **independent variable (*x*)**
  - The equation should **only be used** to make **predictions for *y***
    - Using a *y* on *x* line to **predict *x* is not always reliable**
  - Making a prediction **within the range** of the given data is called **interpolation**
    - This is usually reliable
    - The stronger the correlation the more reliable the prediction
  - Making a prediction **outside of the range** of the given data is called **extrapolation**
    - This is much less reliable
  - The prediction will be more reliable if the number of data values in the original sample set is bigger

Worked example

Barry is a music teacher. For 7 students, he records the time they spend practising per week (x hours) and their score in a test (y %).

Time (x)	2	5	6	7	10	11	12
Score (y)	11	49	55	75	63	68	82

a) Write down the equation of the regression line of y on x, giving your answer in the form $y = ax + b$ where a and b are constants to be found.

Enter data into GDC

a is the coefficient of x $a = 5.5680...$

b is the constant term $b = 15.4136...$

$y = 5.57x + 15.4$

b) Give an interpretation of the value of a.

$a = 5.57$ means that the model suggests that the score increases by 5.57 % for every extra hour of practice.

c) Another of Barry's students practises for 15 hours a week, estimate their score. Comment on the validity of this prediction.

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Substitute $x = 15$

$y = (5.5680...) \times 15 + (15.4136...) = 98.93..$

The model predicts a score of 98.9% but this is unreliable as $x = 15$ is outside the range of data. Therefore extrapolation is being used.

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