4.1 Statistics Toolkit

Mark Schemes

Course	DP IB Maths
Section	4. Statistics & Probability
Topic	4.1 Statistics Toolkit
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1

a) Just take the first 6 weights :

$$\text{mean} = \frac{52+57+63+80+56+66}{6}$$

$$= \frac{187}{3} = 62.333333\ldots$$

$$\boxed{\text{mean} = 62.3\text{ kg } (1\text{ d.p.})}$$

Note : There are other possible convenience samples! The one used here is just an especially obvious one.

b) $\frac{18}{6} = 3$ take every 3^{rd} weight

$$\text{mean} = \frac{63+66+55+62+99+92}{6}$$

$$= \frac{437}{6} = 72.833333\ldots$$

$$\boxed{\text{mean} = 72.8\text{ kg } (1\text{ d.p.})}$$

c) The sum of the 18 weights is 1293.

So the true mean is:

$$\frac{1293}{18} = \frac{431}{6} = 71.8\dot{3} \text{ kg}$$

The systematic sample mean (72.8) is a lot closer to the true mean here than the convenience sample mean (62.3) is.

Because a systematic sample is generally more random than a convenience sample, it will often give more representative results.

This isn't guaranteed, though! Starting on the first data value in (b) would give a mean of 81, which is not much more accurate than the value in (a). Starting on the second value in (b) would give a mean of $61.\dot{6}$, which is less accurate than the value in (a).

Question 2

a) (i) Opportunity sampling

(ii) This survey will be quick, easy, and inexpensive to carry out.

But it is unlikely to be representative of the shop's customers as a whole – it will only be surveying people who shop during lunchtime on a particular day of the week.

b) Quota sampling. For example, divide the shop's opening hours over a week into a number of 'blocks' and then survey a fixed number of people during each block to create the sample.

Question 3

(i) Kids small

size with highest frequency

(ii) The mode is a kids size, but the majority of masks sold (51 out of 84) are adult sizes.

(iii) Number sold during week:

$$29 + 4 + 8 + 24 + 15 + 4 = 84$$

$84 \div 7 =$ 12 masks per day

Question 4

$$\text{standard deviation} = \sqrt{\frac{\Sigma x^2}{n} - \bar{x}^2}$$

$$\Sigma x = 696 \qquad \Sigma x^2 = 54998 \qquad n = 9$$

$$\sigma = \sqrt{\frac{54998}{9} - \left(\frac{696}{9}\right)^2} = 11.421227\ldots$$

$$\bar{x} = \frac{\Sigma x}{n} \quad \text{(mean)}$$

$$\boxed{\sigma = 11.4 \text{ cm } (3 \text{ s.f.})}$$

Question 5

a) (i) $\boxed{\text{median} = 115\text{ g}}$

(ii) $\boxed{Q_1 = 103\text{ g}}$

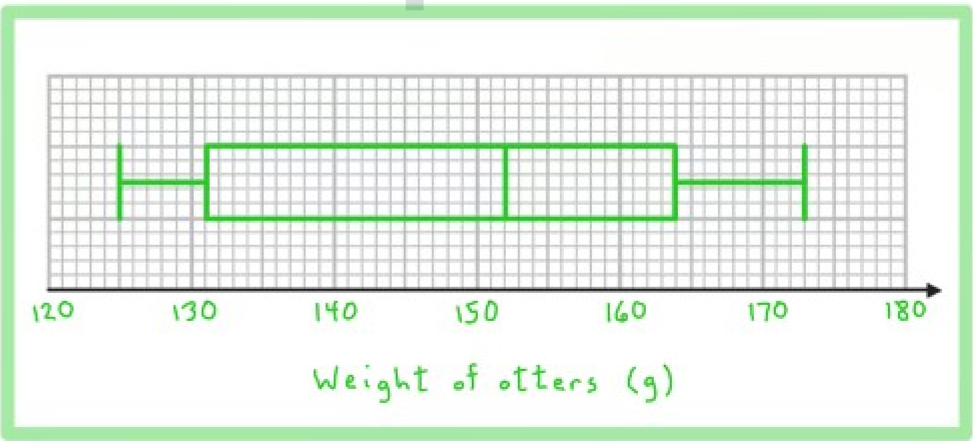
(iii) $Q_3 = 120\text{ g}$

$Q_3 - Q_1 = 120 - 103 = 17$

$\boxed{\text{IQR} = 17\text{ g}}$

b) largest weight $= \overset{\text{smallest}}{\overset{\downarrow}{125}} + \overset{\text{range}}{\overset{\downarrow}{48}} = 173\text{ g}$

$\text{LQ} = \underset{\text{UQ}}{\underset{\uparrow}{164}} - \underset{\text{IQR}}{\underset{\uparrow}{33}} = 131\text{ g}$



Question 6

a) (i) Median is $\frac{20+1}{2} = 10.5^{th}$ value:

$$\boxed{Q_2 = \frac{1.2+1.3}{2} = 1.25 \text{ m}}$$

$\frac{20}{4} = 5 \Rightarrow Q_1$ is 5.5^{th} value

$20 \times \frac{3}{4} = 15 \Rightarrow Q_3$ is 15.5^{th} value

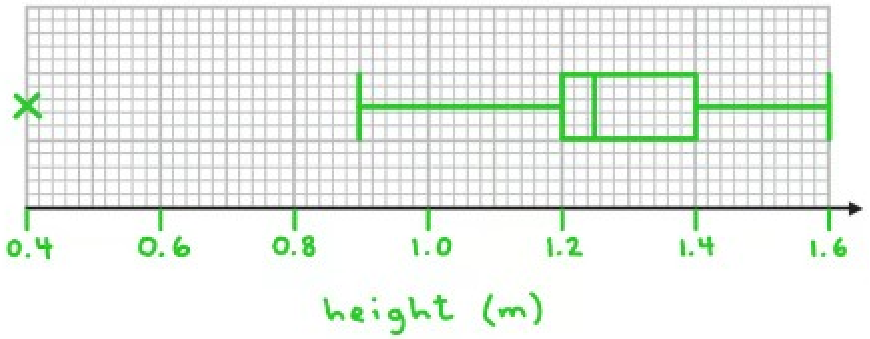
$$\boxed{Q_1 = \frac{1.2+1.2}{2} = 1.2 \text{ m} \qquad Q_3 = \frac{1.4+1.4}{2} = 1.4 \text{ m}}$$

(ii) $\boxed{IQR = 1.4 - 1.2 = 0.2 \text{ m}}$ $IQR = Q_3 - Q_1$

(iii) $1.2 - 1.5 \times 0.2 = 0.9$ lower boundary

$1.4 + 1.5 \times 0.2 = 1.7$ upper boundary

$\boxed{0.4 \text{ m is an outlier}}$



b)

a) (i) Median is $\frac{20+1}{2} = 10.5$th value:

$$\boxed{Q_2 = \frac{1.2 + 1.3}{2} = 1.25\text{ m}}$$

$\frac{20}{4} = 5 \Rightarrow Q_1$ is 5.5th value

$20 \times \frac{3}{4} = 15 \Rightarrow Q_3$ is 15.5th value

$$\boxed{Q_1 = \frac{1.2 + 1.2}{2} = 1.2\text{ m} \qquad Q_3 = \frac{1.4 + 1.4}{2} = 1.4\text{ m}}$$

(ii) $\boxed{IQR = 1.4 - 1.2 = 0.2\text{ m}}$ $IQR = Q_3 - Q_1$

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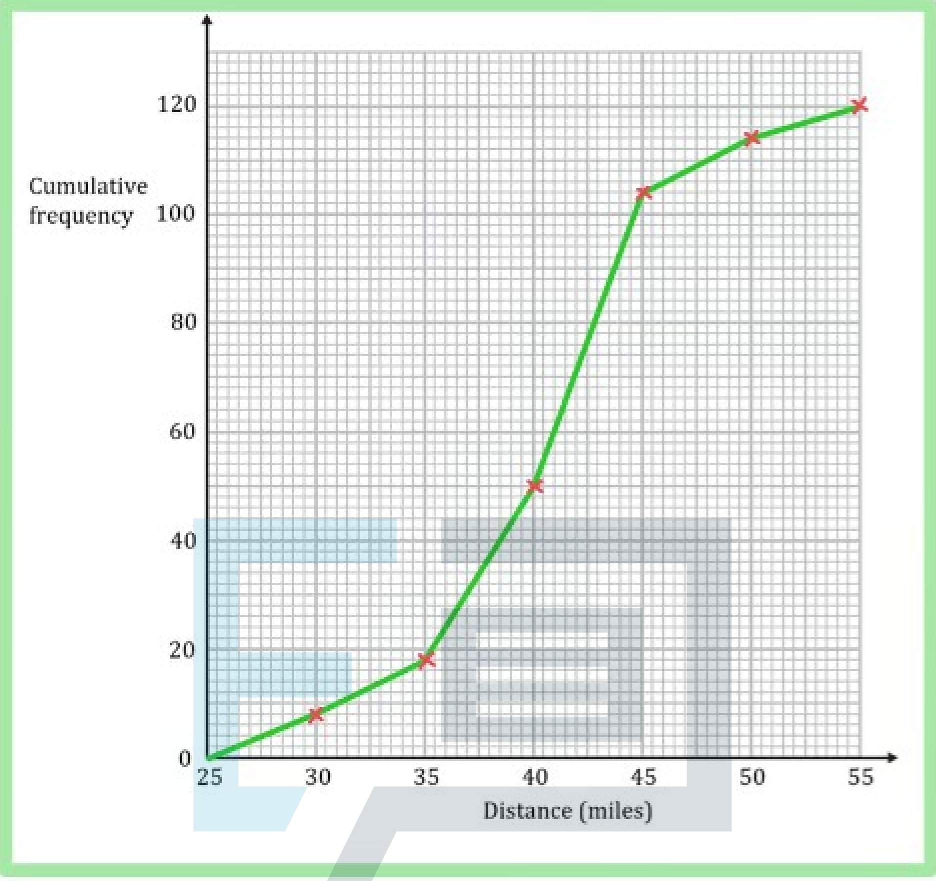
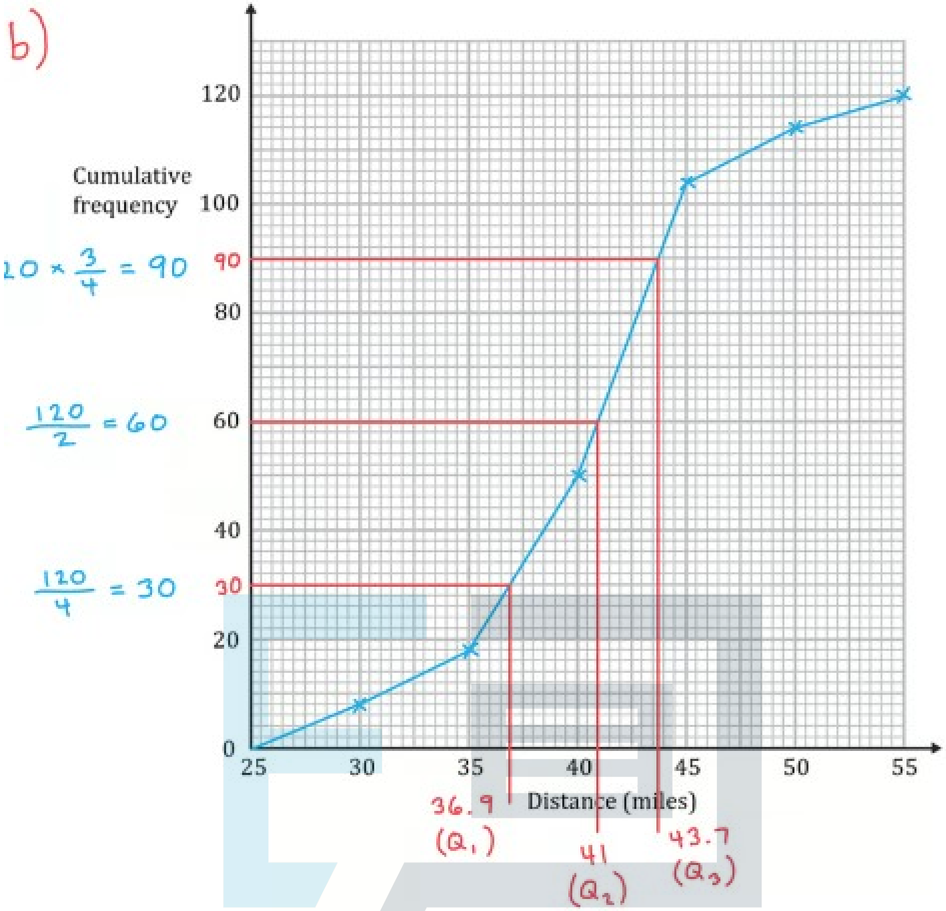
Question 7 a)


Cumulative frequency
120
100
80
60
40
20
0
25
30
35
40
45
50
55
Distance (miles)

b)

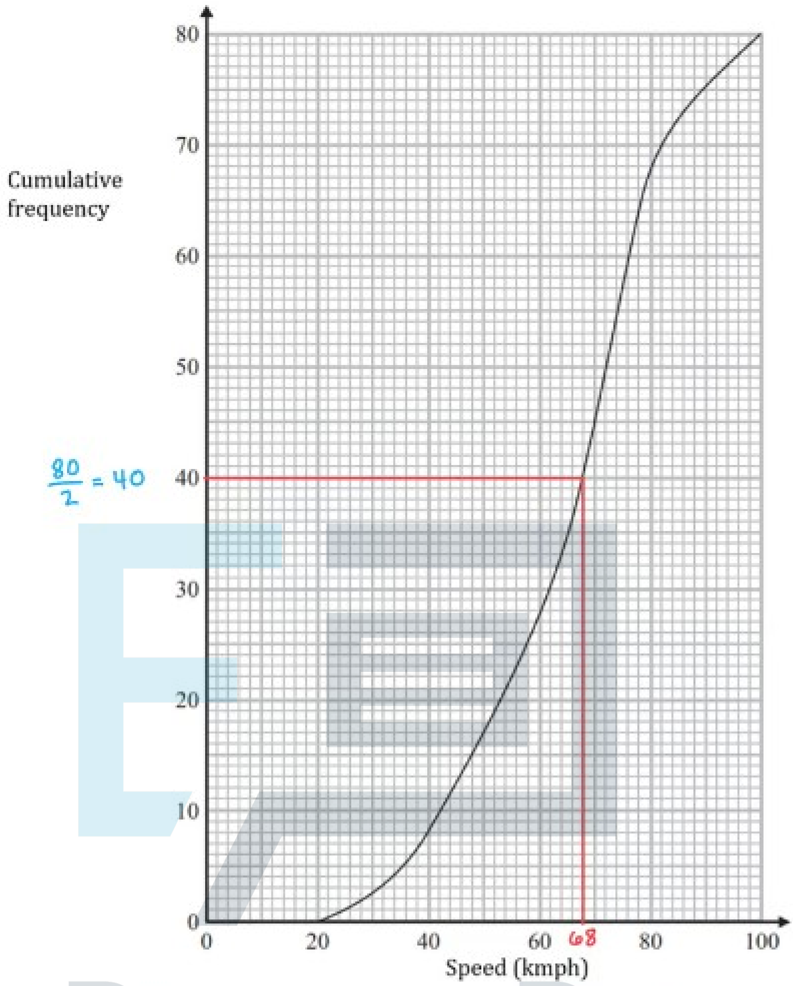


median = 41 miles

IQR = 43.7 − 36.9 = 6.8 miles

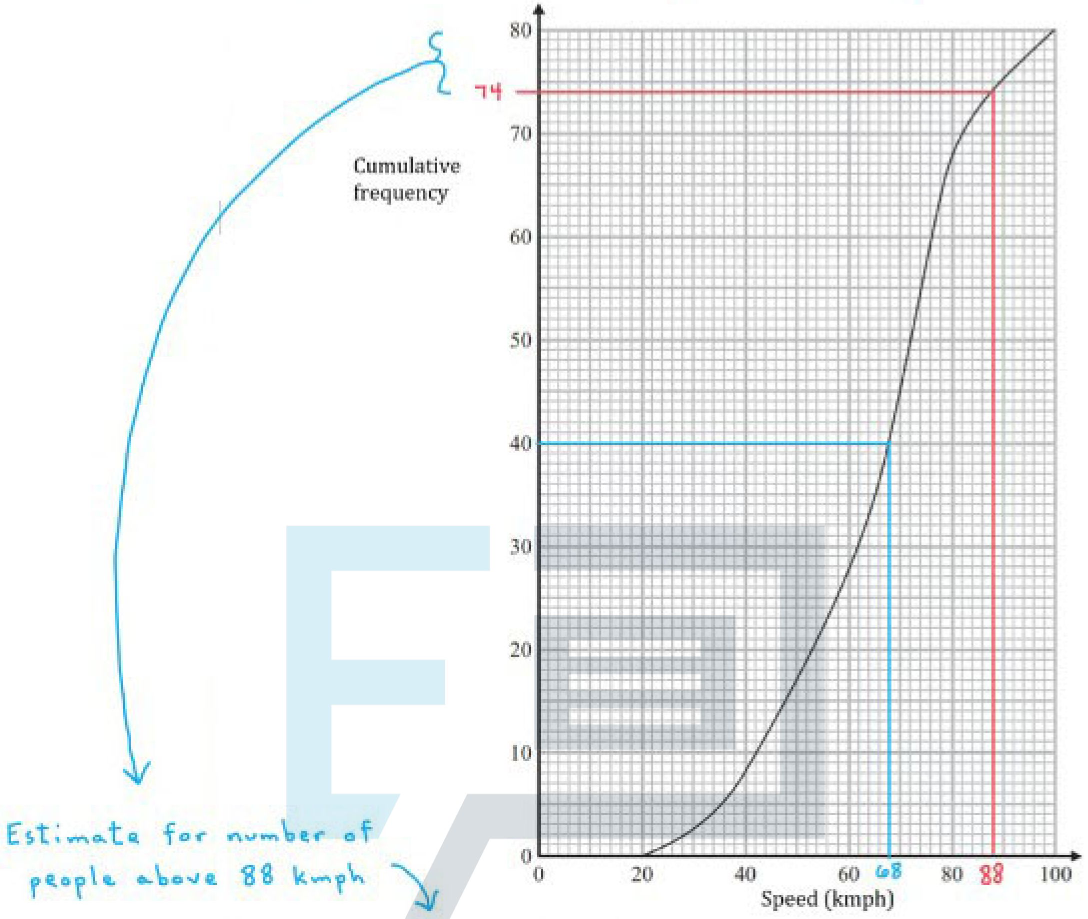
Question 8

a)



$\boxed{\text{median} = 68 \text{ kmph}}$

b) 10% over 80 kmph = 88 kmph



Estimate for number of people above 88 kmph

80 - 74 = 6

$\frac{6}{80} \times 100 = \boxed{7.5\%}$

Question 9

a) $\boxed{\text{Median} = 3.7 \text{ hours}}$

b) 61 - 17 = 44

$\boxed{44 \text{ students}}$

c) LQ = 2.9 UQ = 4.6

4.6 - 2.9 = 1.7 IQR = UQ - LQ

$\boxed{\text{IQR} = 1.7 \text{ hours}}$

d) p = 48 - 9 = 39

q = 74 - 48 = 26

$\boxed{p = 39 \quad q = 26}$

9 + 39 + 26 + 6 = 80 ✓

e) 10% of 80 = 8

80 - 8 = 72

$\boxed{d = 6.4}$

f) $4000 \times \frac{21}{80} = 1050$

$\boxed{\text{Approximately 1050 students took less than 3 hours.}}$