

# Sequences and series - Binomial expansion

Name: \_\_\_\_\_

Class: \_\_\_\_\_

Date: \_\_\_\_\_

**Time:**

**Total marks available:**

**Total marks achieved:** \_\_\_\_\_

A Level Mathematics : Pure Mathematics

Subject: Mathematics

Topic 4 : Sequences and series - Binomial expansion

Type: Topic Questions

To be used by all students preparing for Edexcel A Level Mathematics - Students of other

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Q1.

(a) Find the first three terms, in ascending powers of  $x$ , of the binomial expansion of

$$\frac{1}{\sqrt{4-x}}$$

giving each coefficient in its simplest form.

The expansion can be used to find an approximation to  $\sqrt{2}$

(4)

Possible values of  $x$  that could be substituted into this expansion are:

- $x = -14$  because  $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{18}} = \frac{\sqrt{2}}{6}$
- $x = 2$  because  $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$
- $x = -\frac{1}{2}$  because  $\frac{1}{\sqrt{4-x}} = \frac{1}{\sqrt{\frac{9}{2}}} = \frac{\sqrt{2}}{3}$

(b) Without evaluating your expansion,

(i) state, giving a reason, which of the three values of  $x$  should not be used

(1)

(ii) state, giving a reason, which of the three values of  $x$  would lead to the most accurate approximation to  $\sqrt{2}$

(1)

**(Total for question = 6 marks)**

Q2.

In the binomial expansion of

$$(a + 2x)^7 \quad \text{where } a \text{ is a constant}$$

the coefficient of  $x^4$  is 15 120

Find the value of  $a$ .

**(Total for question = 3 marks)**

Q3.

(a) Find the first four terms, in ascending powers of  $x$ , of the binomial expansion of

$$\sqrt{4 - 9x}$$

writing each term in simplest form.

**(4)**

A student uses this expansion with  $x = \frac{1}{9}$  to find an approximation for  $\sqrt{3}$

Using the answer to part (a) and without doing any calculations,

(b) state whether this approximation will be an overestimate or an underestimate of  $\sqrt{3}$  giving a brief reason for your answer.

**(1)**

**Exam Papers Practice**  
**(Total for question = 5 marks)**

Q4.

$$f(x) = \frac{50x^2 + 38x + 9}{(5x + 2)^2(1 - 2x)} \quad x \neq -\frac{2}{5} \quad x \neq \frac{1}{2}$$

Given that  $f(x)$  can be expressed in the form

$$\frac{A}{5x + 2} + \frac{B}{(5x + 2)^2} + \frac{C}{1 - 2x}$$

where  $A$ ,  $B$  and  $C$  are constants

(a) (i) find the value of  $B$  and the value of  $C$

(ii) show that  $A = 0$

(4)

(b) (i) Use binomial expansions to show that, in ascending powers of  $x$

$$f(x) = p + qx + rx^2 + \dots$$

where  $p$ ,  $q$  and  $r$  are simplified fractions to be found.

(ii) Find the range of values of  $x$  for which this expansion is valid.

**(Total for question = 11 marks)**

Q5.

(a) Find the first four terms, in ascending powers of  $x$ , of the binomial expansion of

$$(1 + 8x)^{\frac{1}{2}}$$

giving each term in simplest form.

(3)

(b) Explain how you could use  $x = \frac{1}{32}$  in the expansion to find an approximation for  $\sqrt{5}$ .

There is no need to carry out the calculation.

(2)

**(Total for question = 5 marks)**

Q6.

(a) Use the binomial expansion, in ascending powers of  $x$ , to show that

$$\sqrt{4-x} = 2 - \frac{1}{4}x + kx^2 + \dots$$

where  $k$

is a rational constant to be found.

(4)

A student attempts to substitute  $x = 1$  into both sides of this equation to find an approximate value for  $\sqrt{3}$ .

(b) State, giving a reason, if the expansion is valid for this value of  $x$ .

(1)

**(Total for question = 5 marks)**

Q7.

(a) Use binomial expansions to show that  $\sqrt{\frac{1+4x}{1-x}} \approx 1 + \frac{5}{2}x - \frac{5}{8}x^2$

(6)

A student substitutes  $x = \frac{1}{2}$  into both sides of the approximation shown in part (a) in an attempt to find an approximation to  $\sqrt{6}$

(b) Give a reason why the student **should not** use  $x = \frac{1}{2}$

(1)

(c) Substitute  $x = \frac{1}{11}$  into

$$\sqrt{\frac{1+4x}{1-x}} = 1 + \frac{5}{2}x - \frac{5}{8}x^2$$

to obtain an approximation to  $\sqrt{6}$ . Give your answer as a fraction in its simplest form.

(3)

**(Total for question = 10 marks)**