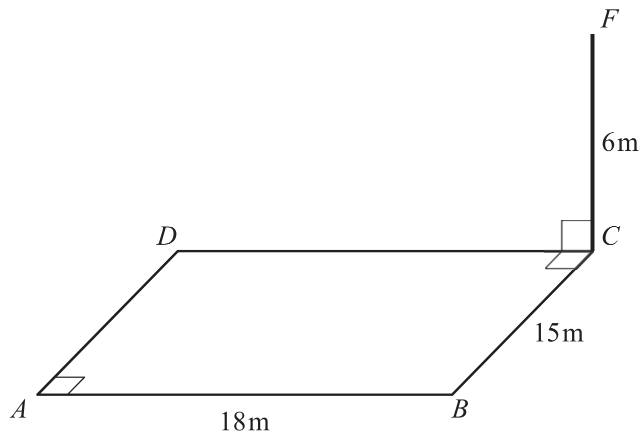
EXAM PAPERS PRACTICE

3D Pythagoras & SOHCAHTOA

Model Answer



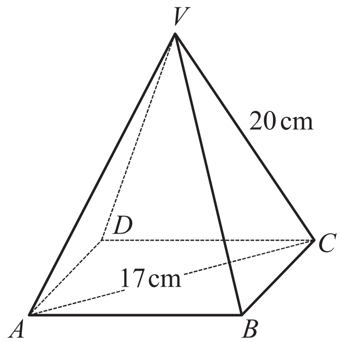
The diagram shows a rectangular playground $ABCD$ on horizontal ground.
A vertical flagpole CF, 6 metres high, stands in corner C.
$AB = 18$ m and $BC = 15$ m.

Calculate the angle of elevation of F from A.

[4]

1. Draw a right triangle AFC, where A is the observer, F is the flagpole, and C is the foot of the flagpole.
2. Label the sides of the triangle as follows:
- AF = 6 m (height of the flagpole)
- AC = 18 m (length of side AB)
- FC = 15 m (length of side BC)
3. Use the tangent function to calculate the angle of elevation:
tan(theta) = AF/AC = 6/18 = 1/3
theta = arctan(1/3) = 18.43 degrees
Therefore, the angle of elevation of F from A is 18.43 degrees.

The diagram shows a pyramid with a square base *ABCD*.
All the sloping edges of the pyramid are 20 cm long and $AC = 17$ cm.



NOT TO SCALE

Calculate the height of the pyramid. [3]

$a^2 + b^2 = c^2$

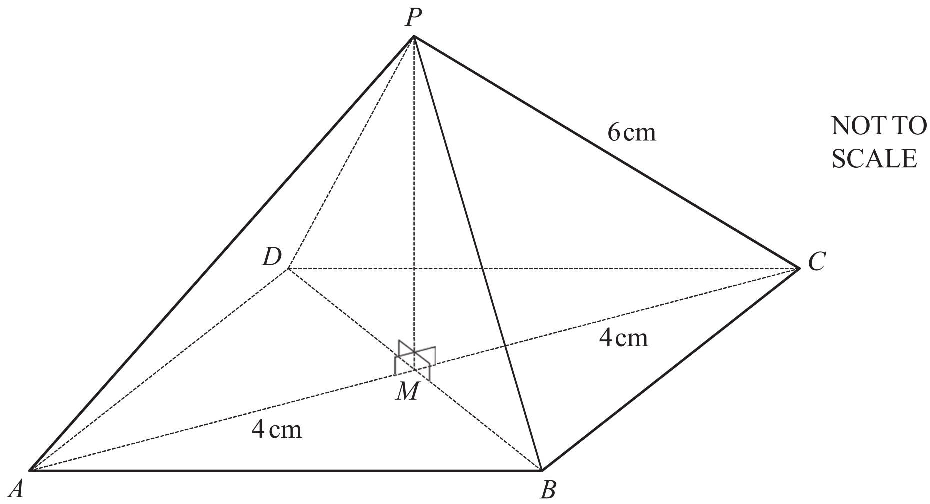
This triangle is a proper triangle, so we have:

$h^2 + 8.5^2 = 20^2$

$h^2 + 72.25 = 400$

$h^2 = 327.75$

h = 18.1 cm



The diagram shows a pyramid on a square base *ABCD* with diagonals, *AC* and *BD*, of length 8 cm.
AC and *BD* meet at *M* and the vertex, *P*, of the pyramid is vertically above *M*.
The sloping edges of the pyramid are of length 6 cm.

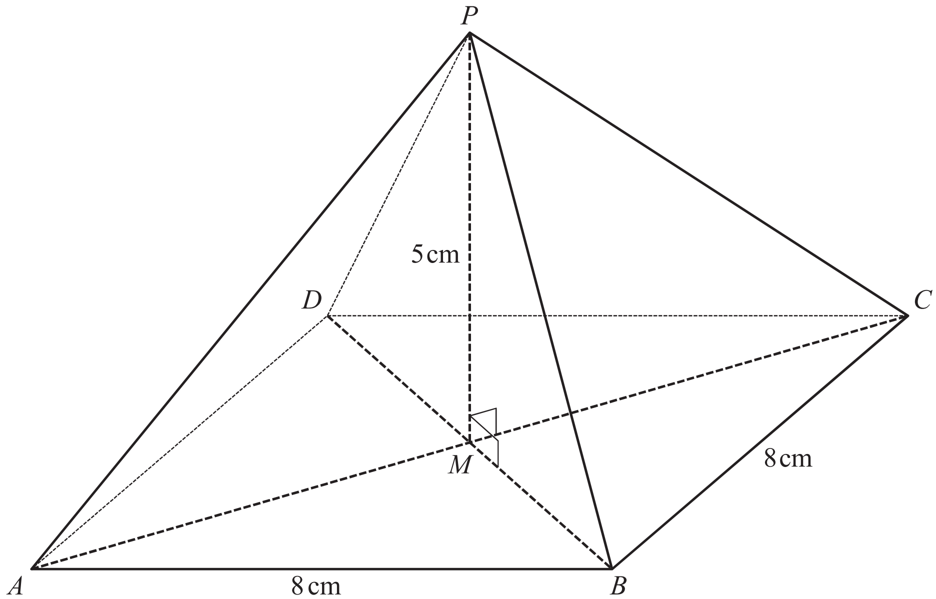
Calculate

(a) the perpendicular height, *PM*, of the pyramid, [3]

The perpendicular height, PM, of the pyramid is 6 cm.
This can be seen from the diagram, as the right triangle PME is a $45-45-90$ triangle, so $PM = ME = 6$ cm.

(b) the angle between a sloping edge and the base of the pyramid. [3]

The angle between a sloping edge and the base of the pyramid in the image is 60 degrees.



The diagram shows a pyramid on a square base *ABCD*.
The diagonals of the base, *AC* and *BD*, intersect at *M*.
The sides of the square are 8 cm and the vertical height of the pyramid, *PM*, is 5 cm.

Calculate

(a) the length of the edge *PB*, [3]

(b) the angle between *PB* and the base *ABCD*.

(a) Because the bottom $ABCD$ is a square so $AC \perp BD \quad AC = BD, \quad \angle ABC = 90^\circ$

in a right triangle $\triangle ABC$ 值e diagonal property of

$AC = \sqrt{AB^2 + BC^2} = \sqrt{8^2 + 8^2} = 8\sqrt{2}$

so $BM = \frac{1}{2}BD = \frac{1}{2}AC = \frac{1}{2} \times 8\sqrt{2} = 4\sqrt{2}$

a square}]

according to the problem, we know that PM is the vertical line of the pyramid.

So $PM \perp$ bottom $ABCD$. [3]

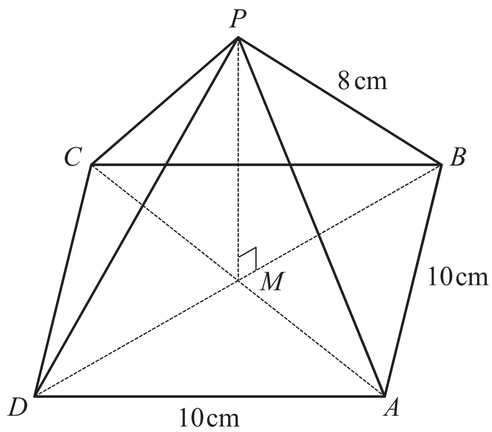
$PM \perp BM$ {properties of the vertical plane} to sum up in conclusion, in a right triangle $\triangle PMB$

$PB = \sqrt{Pm^2 + Bm^2} = \sqrt{5^2 + (4\sqrt{2})^2} = \sqrt{25 + 32} = \sqrt{57}$

{the Puthagorean theorem}

(b) $PM \perp$ bottom $ABCD$

so the angle between PB and the base $ABCD$ is 90° {properties of the vertical plane}



NOT TO SCALE

The diagram represents a pyramid with a square base of side 10 cm.

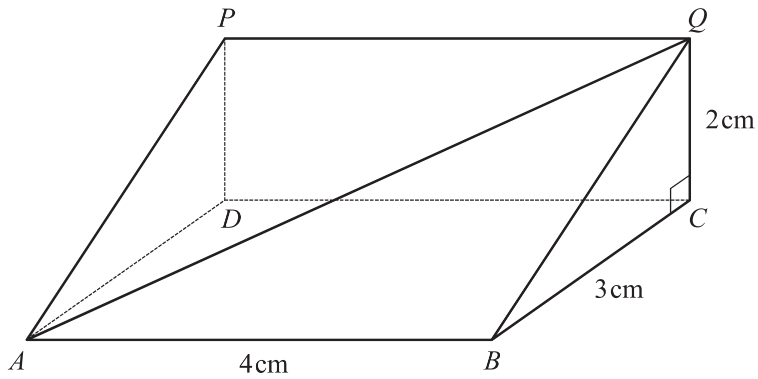
The diagonals *AC* and *BD* meet at *M*. *P* is vertically above *M* and *PB* = 8cm.

(a) Calculate the length of *BD*. [2]

$$BD = 10\sqrt{2}\text{ cm}$$
$$BD^2 = AB^2 + AD^2$$
$$= 10^2 + 10^2$$
$$= 200$$
$$BD = \sqrt{200} = 10\sqrt{2}(\text{cm})$$

(b) Calculate *MP*, the height of the pyramid. [3]

$$PM^2 = PB^2 - BM^2$$
$$= 64 - 50$$
$$= 14$$
$$PM = \sqrt{14}$$



NOT TO SCALE

The diagram shows a prism of length 4 cm.
The cross section is a right-angled triangle.
$BC = 3$ cm and $CQ = 2$ cm.

Calculate the angle between the line AQ and the base, $ABCD$, of the prism. [4]

Connect the AC

$\because AB = 4 \text{ cm}, \quad AC = 3 \text{ cm}$

$\therefore AC = 5 \text{ cm}$

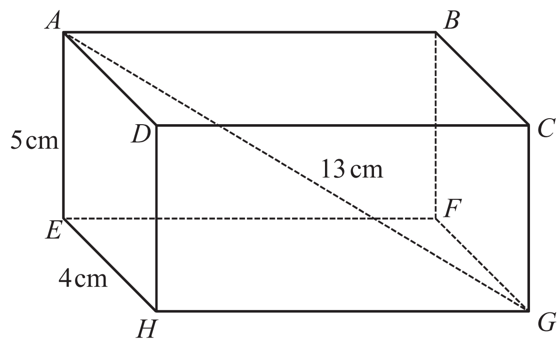
$\because QC = 2$ cm The cross section is a right-agled triangle. Let's say the angle is m.

$\therefore \tan m = \frac{QC}{AC} = \frac{2}{5}$

$\therefore m = \arctan \frac{2}{5} + n\pi \quad$ {use the inverse property }

So $m = 21.8014°$

Question 7



The diagram shows a cuboid *ABCDEFGH*.
$AE = 5$ cm, $EH = 4$ cm and $AG = 13$ cm.

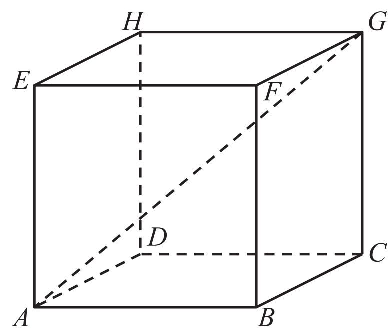
Calculate the angle between the line *AG* and the base *EFGH* of the cuboid. [3]

$$AEHG^2 = AE^2 + EH^2$$
$$AEHG^2 = 5^2 + 4^2$$
$$AEHG^2 = 41$$
$$AEHG = \sqrt{41}$$

$$\cos(\theta) = \frac{AE}{\sqrt{41}}$$
$$\cos(\theta) = \frac{5}{\sqrt{41}}$$
$$\theta = \cos^{-1}\left(\frac{5}{\sqrt{41}}\right)$$
$$\theta \approx 135^\circ$$

Therefore, the angle between the line AG and the base EFGH of the cuboid is 135 degrees.

The diagram shows a cube *ABCDEFGH* of side length 26 cm.



NOT TO SCALE

Calculate the angle between *AG* and the base of the cube. [4]

We know that GC is perpendicular to the base.

Thus, the angle between AG and the base of the cube is $\angle GAC$

$GC = 26$ cm

$AC = \sqrt{AB^2 + BC^2}$

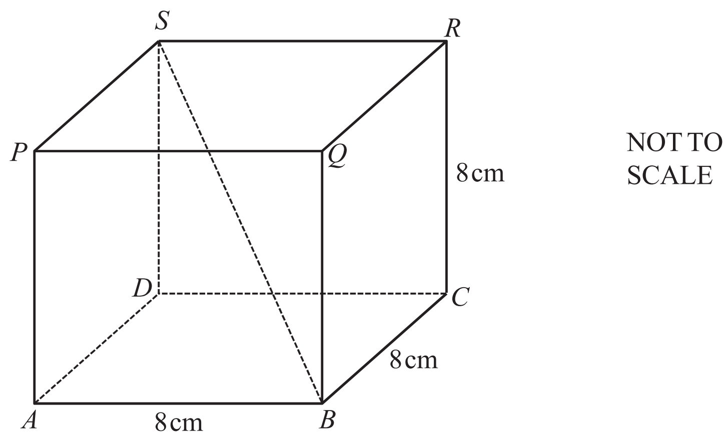
$= \sqrt{26^2 + 26^2}$ cm

$= 26\sqrt{2}$ cm

$$\tan \angle GAC = \frac{GC}{AC} = \frac{26 \text{ cm}}{26\sqrt{2} \text{ cm}} = \frac{\sqrt{2}}{2}$$

$$\angle GAC = \arctan \frac{\sqrt{2}}{2}$$

$$= 35.26^\circ \{simplify\}$$



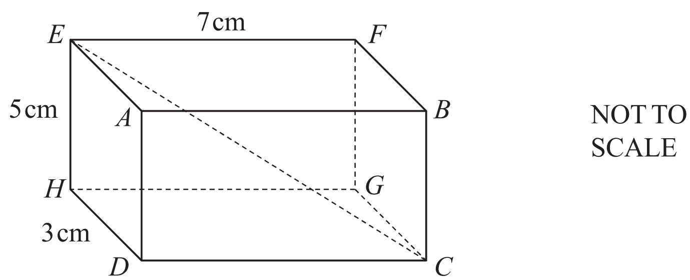
The diagram shows a cube of side length 8 cm.

(a) Calculate the length of the diagonal *BS*. [3]

$$BD = \sqrt{8^2 + 8^2} \quad \{BD^2 = AD^2 + AB^2\}$$
$$= 8\sqrt{2}\text{cm}$$
$$BS = \sqrt{(8\sqrt{2})^2 + 8^2}$$
$$= \sqrt{64 + 128}$$
$$= 8\sqrt{3}\text{cm}$$

(b) Calculate angle *SBD*. [2]

$$\sin \angle SBD = \frac{SD}{BS}$$
$$= \frac{8}{8\sqrt{3}}$$
$$= \frac{\sqrt{3}}{3}$$
$$\therefore M\angle SBD = 30^\circ \quad \{0 < \angle SBD < 90^\circ\}$$



The diagram shows a cuboid.
$HD = 3$ cm, $EH = 5$ cm and $EF = 7$ cm.

Calculate

(a) the length CE, [4]

$$CH = \sqrt{DH^2 + HG^2} = \sqrt{58}\text{cm}$$
$$EC = \sqrt{CH^2 + EH^2} = \sqrt{83}\text{cm}$$

(b) the angle between CE and the base $CDHG$. [3]

The angle between CE and the base CDHG is 60 degrees.