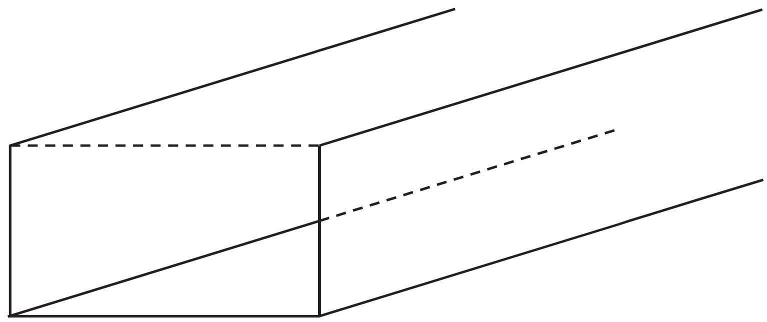
EXAM PAPERS PRACTICE

3D Areas & Volume

Model Answer

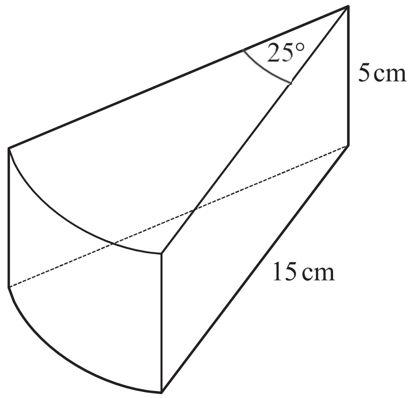


The diagram shows a channel for water.
The channel lies on horizontal ground.
This channel has a constant rectangular cross section with area 0.95 m^2.
The channel is full and the water flows through the channel at a rate of 4 metres/**minute**.

Calculate the number of cubic metres of water that flow along the channel in 3 **hours**.

$$V = S \cdot l$$
$$= 0.95 \times (4 \times 3 \times 60)$$
$$= 684 \text{ m}^3$$

[3]



NOT TO SCALE

The diagram shows a wooden prism of height 5 cm.
The cross section of the prism is a sector of a circle with sector angle 25°.
The radius of the sector is 15 cm.

Calculate the **total** surface area of the prism.

Find the sector angle in radians

$a = 25 \times (\pi/180)$

$a = (25\pi)/(180)\text{rad}$

Find the area of the prism:

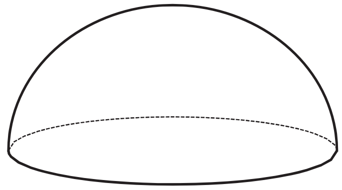
$A = 2 \times 15 \times 5 + 15 \times 15 \times \alpha + 5 \times 15 \times \alpha$

$A = 150 + 15 \times 15 \times (25\pi)/(180) + 5 \times 15 \times (25\pi)/(180)$

$A = 280.890\ \text{cm}^2$

[5]

The diagram shows a solid hemisphere.



The **total** surface area of this hemisphere is 243π.
The volume of the hemisphere is $k\pi$.

Find the value of k.

[The surface area, A, of a sphere with radius r is $A = 4\pi r^2$.]
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [4]

according to the known:

$243\pi = 4\pi r^2$

$243 = 4r^2$

$r^2 = \frac{243}{4}$

$r = \pm\frac{9\sqrt{3}}{2}$

$\because r > 0$

$\therefore r = \frac{9\sqrt{3}}{2}$

$v = \frac{4}{3}\pi r^3 = k\pi$

$\frac{4}{3}\pi r^3 = k\pi$

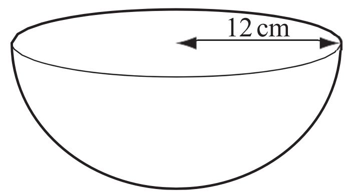
$\frac{4}{3}r^3 = k$

$\frac{4}{3} \times \left(\frac{9\sqrt{3}}{2}\right)^3 = k$

$\frac{4}{3} \times \frac{2187\sqrt{3}}{8} = k$

$k = \frac{729\sqrt{3}}{2}$

A **hemisphere** has a radius of 12cm.



Calculate its volume.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [2]

$$V = \frac{4}{3}\pi r^3 \times \frac{1}{2}$$
$$= \frac{2}{3}\pi(12)^3$$
$$= 1152\pi \text{cm}^3.$$

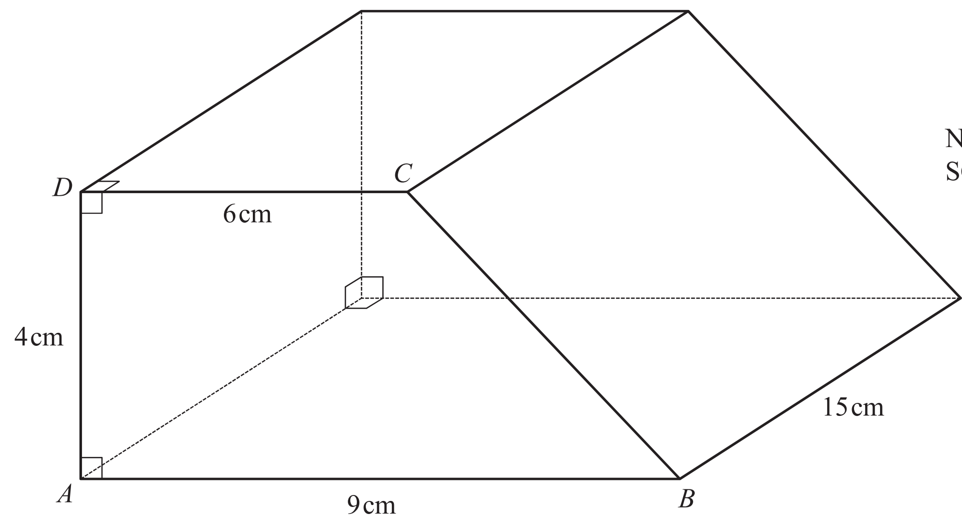
A water pipe has a circular cross section of radius 0.75 cm.
Water flows through the pipe at a rate of 16 cm/s.

Calculate the time taken for 1 litre of water to flow through the pipe. [3]

$0.001\ \mathrm{m}^3 = \pi(0.75\ \mathrm{m})^2(0.16\ \mathrm{m/s})t$

Solving for t, we get:

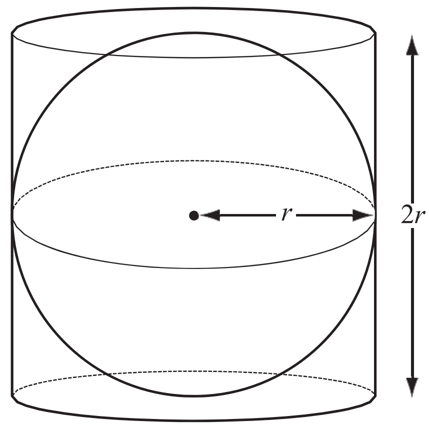
$t = \frac{0.001\ \mathrm{m}^3}{\pi(0.75\ \mathrm{m})^2(0.16\ \mathrm{m/s})} \approx 35.3\ \mathrm{s}$



The diagram shows a solid prism of length 15 cm.
The cross section of the prism is the trapezium *ABCD*.
Angle *DAB* = angle *CDA* = 90°.
AB = 9 cm, *DC* = 6 cm and *AD* = 4 cm.

Calculate the **total** surface area of the prism. [5]

The length of the prism $L = 15$ cm
Now, cross section of the prism is the trapezium ABCD
And , $\angle DAB = \angle CDA = 90°$
The measure of sides of the prism are
AB = 9 cm
DC = 6 cm
And, AD = 4 cm
Now, total surface area of prism is $A = 2B + ph$
On simplifying, we get
$S_1 = (1/2)(6+9)(4 \times 2) + (6 \times 15) + (4 \times 15) + (9 \times 15)$
$S_1 = 345 \text{ cm}^2$
Now, the perpendicular CE to AB is drawn and
DC = AE = 6 cm
And , BE = AB − AE = 3 cm
$BC^2 = CE^2 + BE^2$
On simplifying, we get
BC = 5 cm
$S_2 = 5 \times 15 = 75 \text{ cm}^2$
So, the total surface area $S = S_1 + S_2 = 420 \text{ cm}^2$
Hence, the surface area of prism is $S = 420 \text{ cm}^2$



NOT TO SCALE

The sphere of radius r fits exactly inside the cylinder of radius r and height $2r$.
Calculate the percentage of the cylinder occupied by the sphere.

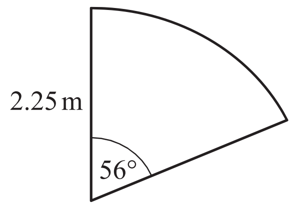
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [3]

According to the topic

$$V_{cylinder} = sh = \pi r^2 \cdot 2r = 2\pi r^3$$

$$V_{\text{sphere}} = \frac{4}{3}\pi r^3$$

$$\frac{V_{\text{sphere}}}{V_{\text{cylinder}}} = \frac{\frac{4}{3}\pi r^3}{2\pi r^3} = \frac{2}{3}$$



The diagram shows a sand pit in a child's play area.
The shape of the sand pit is a sector of a circle of radius 2.25m and sector angle 56°.

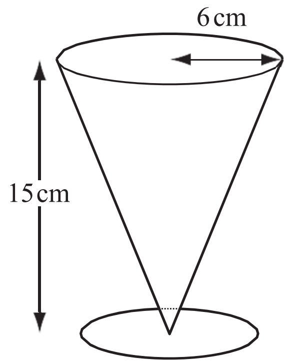
(a) Calculate the area of the sand pit. [2]

$$\frac{56}{2 \times 80} \cdot \pi \cdot (2.25)^2 = \frac{63}{80}\pi \mathrm{m}^2$$

(b) The sand pit is filled with sand to a depth of 0.3 m.

Calculate the volume of sand in the sand pit. [1]

$$v = A \cdot h = \frac{63}{80}\pi \times 0.3 = \frac{189}{800}\pi \mathrm{m}^3$$



NOT TO SCALE

The diagram shows a glass, in the shape of a cone, for drinking milk.
The cone has a radius of 6 cm and height 15 cm.
A bottle of milk holds 2 litres.

(a) How many times can the glass be completely filled from the bottle?
[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [4]

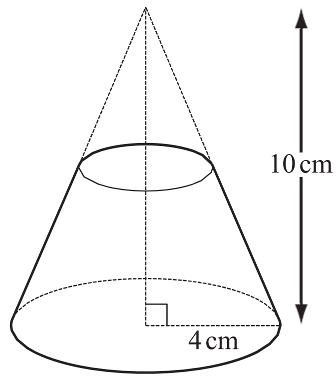
$$V = \frac{1}{3}\pi r^2 h$$
$$= \frac{1}{3}\pi 6^2 \cdot 15$$
$$= \frac{1}{3}\pi 36 \cdot 15$$
$$= 565.2 \text{ cm}^3$$
$$2L = 2000 \text{ mL}$$
$$2000 \div 565.2 \approx 3.5$$

The glass be completely filled from the bottle 3 times

(b) Calculate the volume of milk left in the bottle.
Give your answer in cm^3. [3]

The volume of millk
left in the bottle.

$$2000 - 180\pi \times 3 = 303.54 \left(\text{cm}^3\right)$$



A **solid** cone has base radius 4 cm and height 10 cm.
A mathematically similar cone is removed from the top as shown in the diagram.
The volume of the cone that is removed is $\frac{1}{8}$ of the volume of the original cone.

(a) Explain why the cone that is removed has radius 2 cm and height 5 cm. [2]

Volume of a cone: $\frac{1}{3}\pi R^2 h$

$= \frac{1}{3} \times 16 \times 10 \times 3.14 = 167.4$

Small cone: $167.4 \times \frac{1}{8} = 20.925$

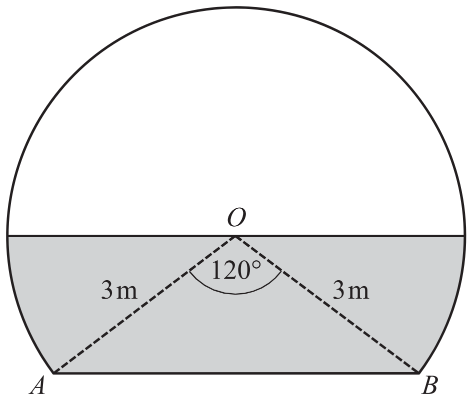
$\frac{1}{3} \times 4 \times 5 \times 3.14 = 20.1$

(b) Calculate the volume of the remaining solid.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.] [4]

$167.4 - 20.925 = 146.475$

The diagram shows the entrance to a tunnel.
The circular arc has a radius of 3m and centre O.
AB is horizontal and angle $AOB = 120°$.



NOT TO SCALE

During a storm the tunnel filled with water, to the level shown by the shaded area in the diagram.

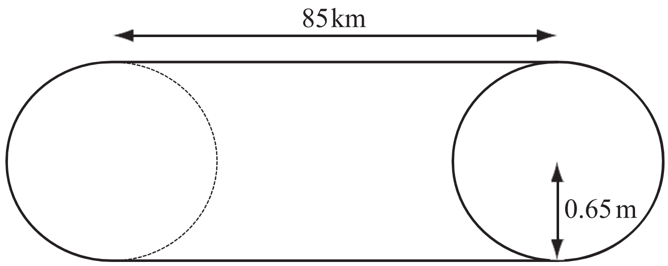
(a) Calculate the shaded area. [4]

The shaded area is 3 m^2.

(b) The tunnel is 50 m long.

Calculate the volume of water in the tunnel. [1]

The volume of water in the tunnel is 50 cubic meters.



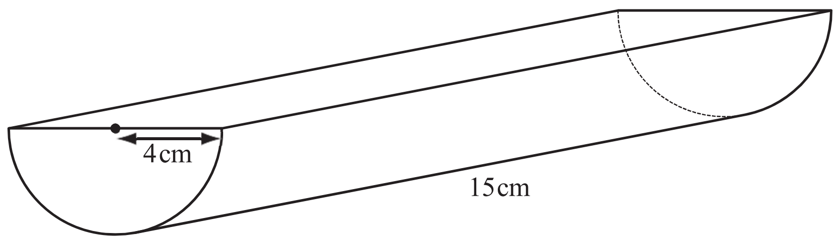
NOT TO SCALE

A water pipeline in Australia is a cylinder with **radius** 0.65 **metres** and length 85 **kilometres**.

Calculate the volume of water the pipeline contains when it is full.
Give your answer in cubic metres.

The volume of the water pipeline is 35912.5π cubic meters.

[3]



NOT TO SCALE

The diagram shows a solid prism of length 15 cm.
The cross-section of the prism is a semi-circle of radius 4 cm.

Calculate the total surface area of the prism.

The total surface area of the prism is 76π square cm.

[4]

A cylinder has a height of 12 cm and a volume of 920 cm^3.

Calculate the radius of the base of the cylinder. [3]

$920\ \text{cm}^3 = \pi r^2 \cdot 12\ \text{cm}$

Dividing both sides by 12 cm and simplifying, we get:

$77\ \text{cm}^2 = \pi r^2$

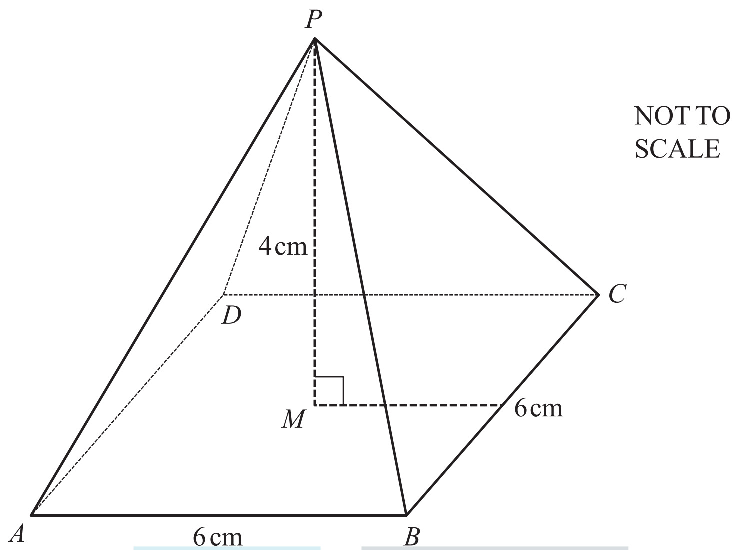
Dividing both sides by π, we get:

$24.67\ \text{cm} = r^2$

Taking the square root of both sides, we get:

$r = 4.94\ \text{cm}$

Therefore, the radius of the base of the cylinder is 4.94 cm.

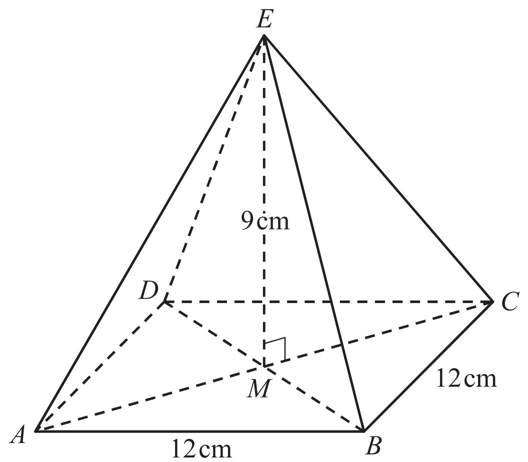


The diagram shows a pyramid with a square base *ABCD* of side 6 cm.

The height of the pyramid, *PM*, is 4 cm, where *M* is the centre of the base.

Calculate the total surface area of the pyramid. [5]

The total surface area of the pyramid is the sum of the areas of the lateral faces and the base, *which is equal to* $48\ \text{cm}^2 + 36\ \text{cm}^2 = 84\ \text{cm}^2$.



NOT TO SCALE

The diagram shows a square-based pyramid *ABCDE*.
The diagonals of the square meet at *M*.
E is vertically above *M*.
AB = *BC* = 12 cm and *EM* = 9 cm.

Calculate the angle between the edge *EC* and the base, *ABCD*, of the pyramid. [4]

To find the angle between EC and the base $ABCD$, we can use right triangle ECD.
$CE = 15$ cm and $ED = 12$ cm. Using the tangent function, we can find that $\theta = \tan^{-1}\left(\frac{4}{5}\right) \approx 38.9°$.

Question 17

Calculate the volume of a **hemisphere** with radius 3.2 cm.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [2]

$V = 68.63\ \text{cm}^9$

Step-by-step explanation:

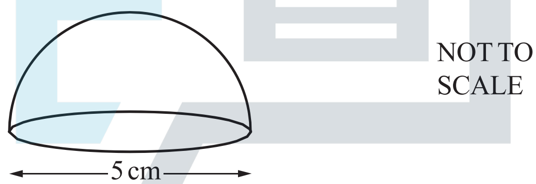
the volume (V) of a sphere is calculated as

$V = \frac{4}{3}\pi r^3$

the volume of a hemisphere is half the volume of a sphere, so $V = \frac{1}{2} \times \frac{4}{3}\pi r^3 = \frac{2}{3}\pi r^3$, then

$$V = \frac{2}{3}\Pi \times 3.2^9$$
$$= \frac{2}{3}\Pi \times 32.768$$
$$= 68.63\ \text{cm}^9 \text{ (to 2 dec. places)}$$

Question 18



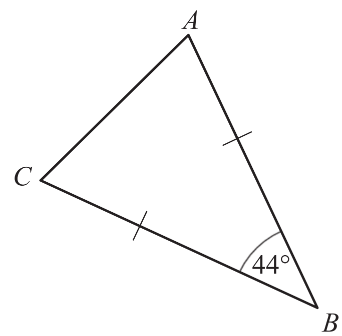
The diagram shows a hemisphere with diameter 5 cm.

Calculate the volume of this hemisphere.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [2]

$$\text{diameter } = 5 \text{ cm}$$
$$\Rightarrow \text{ radius } r = \frac{5}{2} \text{ cm}$$
$$\Rightarrow \text{ volume } = \frac{2}{3}\pi r^3$$
$$= \frac{2}{3} \times 3.14 \times \left(\frac{5}{2}\right)^3$$
$$= \frac{z}{3} \times 3.14 \times \frac{125}{4}$$
$$= \frac{3.14 \times 125}{12}$$
$$= 32.71\ \text{cm}^3$$

(a)



NOT TO SCALE

Triangle *ABC* is an isosceles triangle with *AB* = *CB*.
Angle *ABC* = 44°.

Find angle *ACB*. [1]

$\angle ACB = \angle CAB$

Sum of angle in a triagle $= 180°$

$m\angle ACB + m\angle ACB + m\angle ABC = 180°$

$2m\angle ACB + 44° = 180°$

$2m\angle ACB = 180° - 44° = 136°$

$m\angle ACB = \frac{136°}{2} = 68°$

(b) A regular polygon has an exterior angle of 40°.

Work out the number of sides of this polygon. [2]

$140n = (2n - 4)90° = 180\text{n} - 360°$

$360° = 180n - 140n = 40n$

$n = \frac{360}{40}$

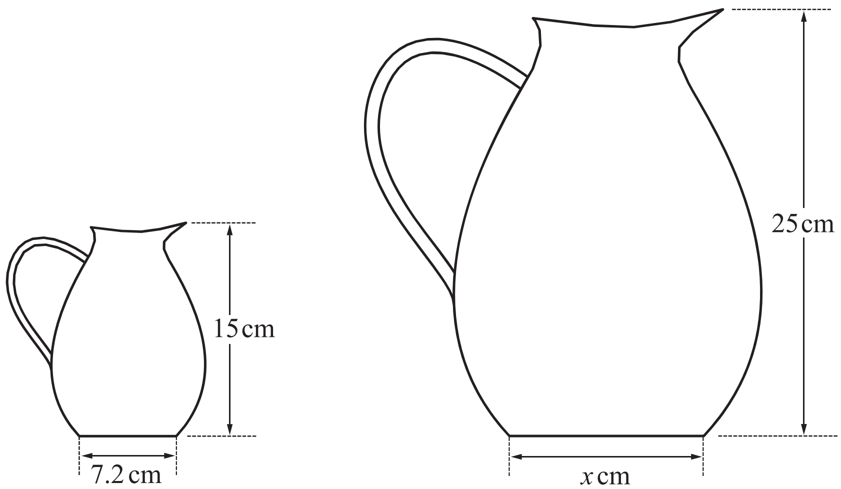
$\because n = 9$

Calculate the volume of a hemisphere with radius 5 cm.

[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [2]

$$\begin{aligned} V &= \frac{4}{3}\pi \times 5^3 \\ &= \frac{4}{3}\pi \times 125 \\ &= \frac{500}{3}\pi = 523.6 \text{ cm}^3 \end{aligned}$$

(a)



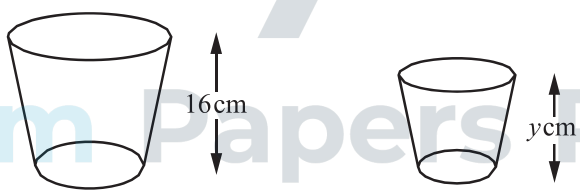
The diagram shows two jugs that are mathematically similar.

Find the value of x. [2]

$25 : 15 = \frac{25}{15} = \frac{5}{3} = 5 : 3$

so, $\frac{5}{3} = \frac{x}{7.2}$ or, $x = \frac{5 \times 7.2}{3} = 12$ cm

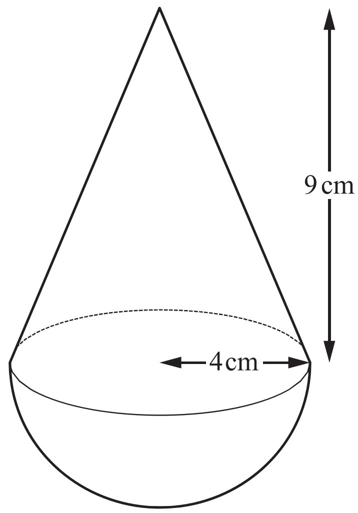
(b)



The diagram shows two glasses that are mathematically similar.
The height of the larger glass is 16 cm and its volume is 375 cm^3.
The height of the smaller glass is y cm and its volume is 192 cm^3.

Find the value of y. [3]

The height of the smaller glass is 8.19 cm.

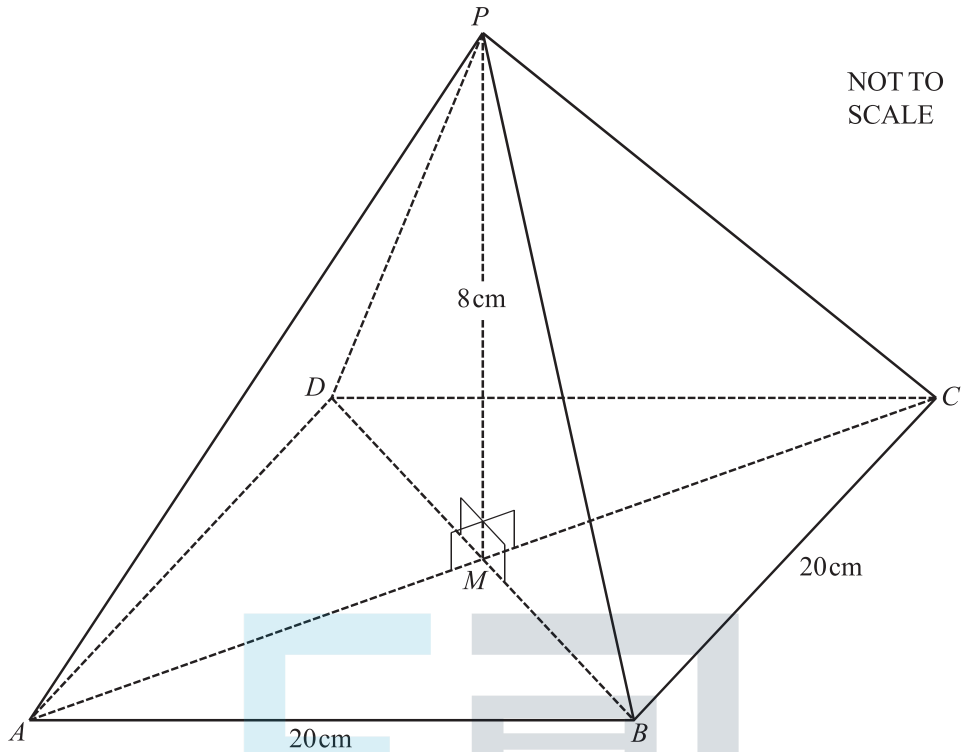


The diagram shows a toy.
The shape of the toy is a cone, with radius 4 cm and height 9 cm, on top of a hemisphere with radius 4 cm.

Calculate the volume of the toy.
Give your answer correct to the nearest cubic centimetre.

[The volume, V, of a cone with radius r and height h is $V = \frac{1}{3}\pi r^2 h$.]
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.] [4]

The volume of the toy is 285 cm^3.



The diagram shows a solid pyramid on a square horizontal base *ABCD*.
The diagonals *AC* and *BD* intersect at *M*.
P is vertically above *M*.
AB = 20 cm and *PM* = 8 cm.

Calculate the total surface area of the pyramid. [5]

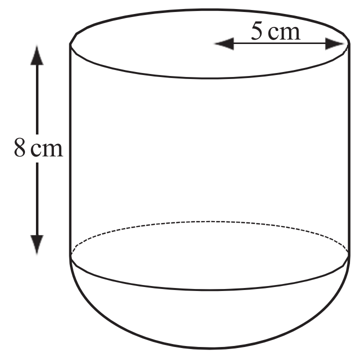
The total surface area of the pyramid is 720 square cm.

The base of a rectangular tank is 1.2 metres by 0.9 metres.
The water in the tank is 53 **centimetres** deep.

Calculate the number of litres of water in the tank. [2]

The tank can hold 572.4 liters of water.

The diagram shows a child's toy.



NOT TO SCALE

The shape of the toy is a cylinder of radius 5 cm and height 8 cm on top of a hemisphere of radius 5 cm.

Calculate the volume of the toy.
[The volume, V, of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.]

[5]

The volume of the toy is $850\pi/3$ cubic cm.