Exam Papers Practice

3.5 Trigonometric Functions & Graphs

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Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.5 Trigonometric Functions & Graphs
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1

$\sin^{-1}(1) = 90^\circ$ so $\sin(90^\circ) = 1$

IF $x = 45^\circ$, $\sin(2x) = \sin(90^\circ) = 1$ MAXIMUM

a) **Point B has coordinates $(45^\circ, 1)$**

$\sin 2x$ is a horizontal stretch of $\sin x$, with scale factor $\frac{1}{2}$ (i.e., a 'squash'), around the y-axis. $\sin x$ repeats every 360°, so $\sin 2x$ repeats every 180°.

b) $45 - (-15) = 60$ } horizontal distance from point P to point B

$45 + 60 = 105$

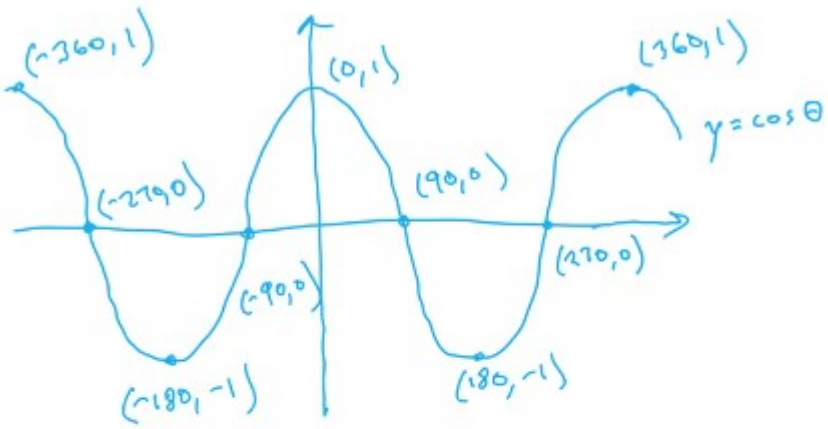
Point Q has coordinates $(105^\circ, -\frac{1}{2})$

$-15 + 180 = 165$

Point R has coordinates $(165^\circ, -\frac{1}{2})$

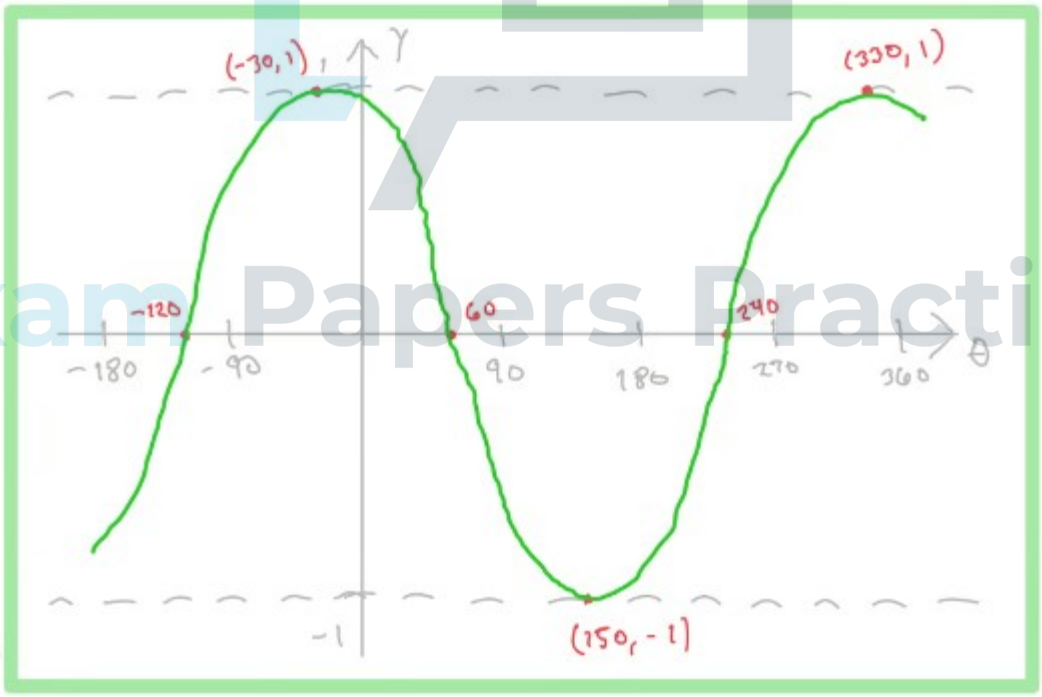
Note: Remember that you can use your GDC to check your answers on a question like this!

Question 2



$y = \cos(\theta + 30°)$ is $y = \cos\theta$ translated $30°$ to the left

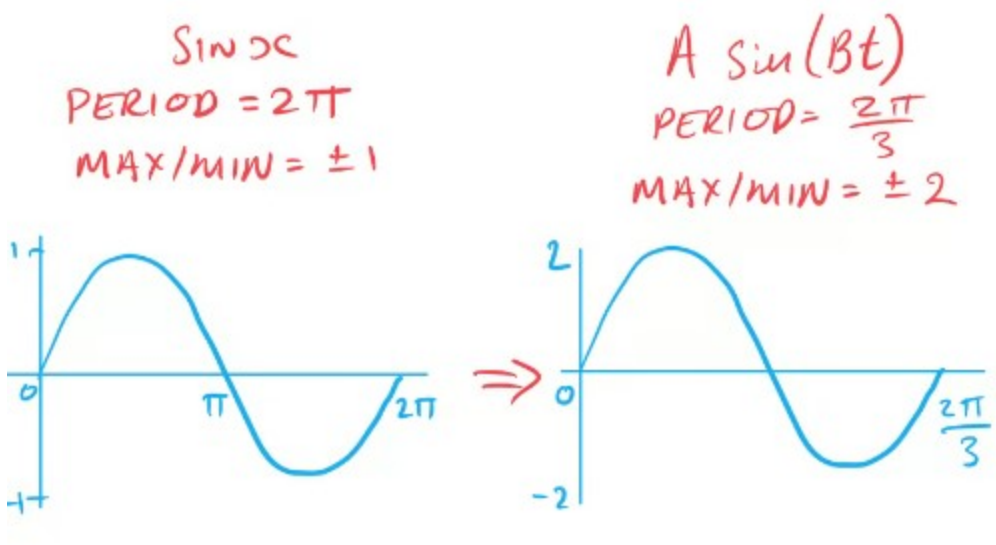
(i)



(ii) From the graph

$\theta = -120°, 60°, 240°$

Question 3



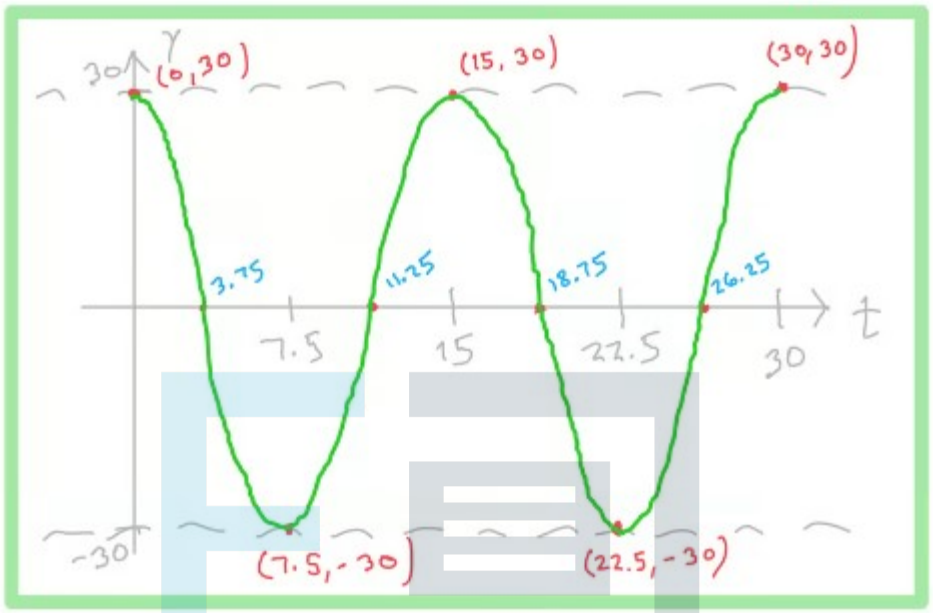
A = VERTICAL STRETCH SF 2 = 2

B = HORIZONTAL STRETCH SF $\frac{1}{3}$ = 3

$h = 2\sin(3t)$

Question 4

(i) $0 \leq t \leq 30$ means $0 \leq 24t \leq 720$ (2 complete cycles of cos)



(ii) Twice from the graph

(iii) $24t = 180$

$t = 180 \div 24 = 7.5$

7.5 seconds

Question 5

a)

NATURAL HEIGHT = A m

MAX = (A + B) m

MIN = (A - B) m

b) MAX = A + B MIN = A - B

NATURAL = A

MAX = A + 3 B = 3

MAX = 3(MIN)

A + 3 = 3(A - 3)

A + 3 = 3A - 9

2A = 12

A = 6

NATURAL LEVEL = 6 m

MIN = 3m MAX = 9m

c) $h(t) = 6 + 3\sin\left(\frac{\pi}{6}t\right)$

i) $2\pi \times \frac{6}{\pi} = 12$ HOURS

MAX 2 TIMES PER DAY

ii) $6 + 3\sin\left(\frac{\pi}{6}t\right) = 3$ MIN = 3

CONSIDER RANGE $0 \leq t \leq 24$

$\sin\left(\frac{\pi}{6}t\right) = -1$ $0 \leq \frac{\pi}{6}t \leq 4\pi$

$\sin^{-1}(-1) = -\frac{\pi}{2}$

$+2\pi$ $+4\pi$

$\frac{\pi}{6}t = \frac{3}{2}\pi, \frac{7}{2}\pi$

$\div \frac{\pi}{6}$

9, 21

9AM AND 9PM

Question 6

a) $h = 3e^{-0.7 \times 0.742} \cos(4 \times 0.742)$

$= -1.75781\ldots$ BELOW SEA LEVEL

TOTAL DISTANCE

$3 + 1.7578\ldots = 4.7578\ldots$

4.76 m (3sf)

b) $h = y = 0$ CAN USE GDC TO SOLVE

AS $3e^{-0.7t} \neq 0$

$\cos(4t) = 0$

$\cos^{-1}(0) = \frac{\pi}{2}$

$+\pi \quad +2\pi$

$\div 4$ $4t = \frac{\pi}{2}, \frac{3}{2}\pi, \frac{5}{2}\pi$

$t = \frac{\pi}{8}, \frac{3}{8}\pi, \frac{5}{8}\pi$ SECONDS

c) i) $t = 6.2$

$3e^{-0.7 \times 6.2} = 0.0391095.$

0.0391 (3sf)

ii) As t INCREASES $3e^{-0.7t}$ WILL DECREASE

SO FOR $t \geq 6.2$

$3e^{-0.7t} < 0.04$

GIVEN THAT $-1 \leq \cos(4t) \leq 1$

AT $t \geq 6.2$ THEN

$3e^{-0.7t} \cos(4t) \leq \pm 0.04$

SO LIFE JACKET WILL ALWAYS BE WITHIN 4CM OF SEA LEVEL AFTER 6.2S

Question 7

a) $d = 100$

i) $h = 12 + \frac{9}{2}\sin\left(\frac{2\pi}{365} \cdot 100\right)$

$= 16.449\ldots$

16.4 HOURS

ii) SET FUNCTION TO EQUAL 9 AND SOLVE

$12 + \frac{9}{2}\left(\sin\frac{2\pi}{365}d\right) = 9$ CAN USE GDC TO SOLVE

$\sin\left(\frac{2\pi}{365}d\right) = -3 \times \frac{2}{9} = -\frac{2}{3}$

$\sin^{-1}\left(-\frac{2}{3}\right) = -0.7297\ldots$

$\pi + 0.729\ldots \quad 2\pi - 0.729\ldots$

$3.871\ldots, \quad 5.553\ldots$

$\div \frac{2\pi}{365}$

$224.891\ldots, \quad 322.609\ldots$

CANNOT RELY JUST ON ROUNDING VALUES AS ONLY INTEGER VALUES OF d CAN BE USED

FOUR POSSIBLE DAYS = 224, 225, 322, 323

CHECK EACH VALUE IN FUNCTION TO FIND EXACT NUMBER OF HOURS

$h(224) = 9.052$ $h(225) = \underline{8.994}$

$h(322) = 8.965$ $h(323) = \underline{9.023}$

TWO DAYS CLOSEST TO 9 HOURS

225 AND 323 DAYS

b) $12 + \frac{4}{2}\sin\left(\frac{2\pi}{365}d\right) = 15$

$\sin\left(\frac{2\pi}{365}d\right) = \frac{2}{3}$ CAN USE GDC TO SOLVE

$\sin^{-1}\left(\frac{2}{3}\right) = 0.7297...$

$\pi - 0.7297...$

$\frac{2\pi}{365}d = 0.7297...,\ 2.4118...$

$\div \frac{2\pi}{365}$ → $42.389...$ $140.108...$

CANNOT RELY JUST ON ROUNDING VALUES AS ONLY INTEGER VALUES OF d CAN BE USED

SO DAY LIGHT WILL BE $<$ IS FOR 42
$>$ IS FOR 43

$<$ IS FOR 141
$>$ IS FOR 140

DAY LIGHT GREATER THAN IS BETWEEN

43RD DAY AND 140TH DAY INCLUSIVE

98 DAYS

BE CAREFUL NOT TO USE 140 - 43 = 97
AS THIS IS NOT INCLUSIVE

Question 8

a)

i) AT START OF ROUTINE $t = 0$

$d = 12\cos(0) + 15$

EXACT VALUE
$\cos 0 = 1$

$d = 12 + 15 = 27$

$d = 27$ m

ii) $t = 15$

$d = 12\cos\left(\frac{15\pi}{30}\right) + 15$

$\frac{15\pi}{30} = \frac{\pi}{2}$

EXACT VALUE
$\cos\left(\frac{\pi}{2}\right) = 0$

$d = 15$ m

b) LAP CIRCUMFERENCE $C = 2\pi r = \pi d$

CIRCLE DIAMETER $= d_{max} - d_{min}$

$-1 \leq \cos\frac{\pi}{30}t \leq 1$

$d_{max} = 12(1) + 15 = 27$ m

$d_{min} = 12(-1) + 15 = 3$ m

DIAMETER $= 27 - 3 = 24$

$C = 24\pi$

CIRCUMFERENCE $= 24\pi$ m

c) $\text{SPEED} = \frac{\text{DISTANCE}}{\text{TIME}}$

DISTANCE FROM (b) $= 24\pi$

TIME TAKEN FOR ONE LAP IS PERIOD OF FUNCTION

$2\pi = \frac{\pi}{30}t$ $\qquad$ TIME = PERIOD $\Rightarrow \frac{2\pi}{|B|}$

$t = 2\pi \times \frac{30}{\pi} = 60$

$\text{SPEED} = \frac{24\pi}{60} = \frac{2\pi}{5}$

SPEED $= \frac{2\pi}{5}$ M/S OR ms^{-1}

d) EACH LAP TAKES 60 SECONDS (PERIOD FROM c)

3 LAPS = 180 SECONDS

SOLVE FOR $d = 21$ $\quad 0 \leq t \leq 180$

$12\cos\frac{\pi}{30}t + 15 = 21$

$-15 \qquad -15$

$12\cos\frac{\pi}{30}t = 6$

$\div 12 \qquad \div 12$

$\cos\frac{\pi}{30}t = \frac{1}{2}$

EXACT VALUES $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$ OR $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$

$\frac{\pi}{30}t = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \frac{11\pi}{3}, \frac{13\pi}{3}, \frac{17\pi}{3}$

$t = \frac{\pi}{3} \times \frac{30}{\pi} = \frac{30}{3} = 10$ SECONDS ...

$t = 10s, 50s, 70s, 110s, 130s, 170s$