Exam Papers Practice

3.4 Voronoi Diagrams

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Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.4 Voronoi Diagrams
Difficulty	Medium

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful

Question 1

a) i) Amy's house lies on the perpendicular bisector of sites B and C.

∴ Amy's house is closest to sites B and C.

ii) The distance is the same to either site.

Distance between two points formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ (in formula booklet)

H(-1, 1) B(1, 4)

Sub H and B into formula.

$d = \sqrt{(-1 - 1)^2 + (1 - 4)^2}$

$d = \sqrt{13} = 3.6055...$

d = 3.61 km (3sf)

b) Kayla's apartment is at the midpoint of sites A and C.

Midpoint formula

Midpoint = $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ (in formula booklet)

A(-2, -7) C(-4, -1)

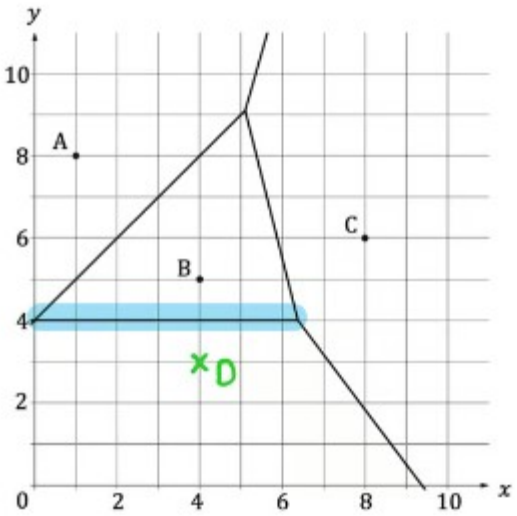
Sub A and C into formula.

Midpoint = $\left(\frac{-2 + (-4)}{2}, \frac{-7 + (-1)}{2}\right)$

Midpoint = (-3, -4)

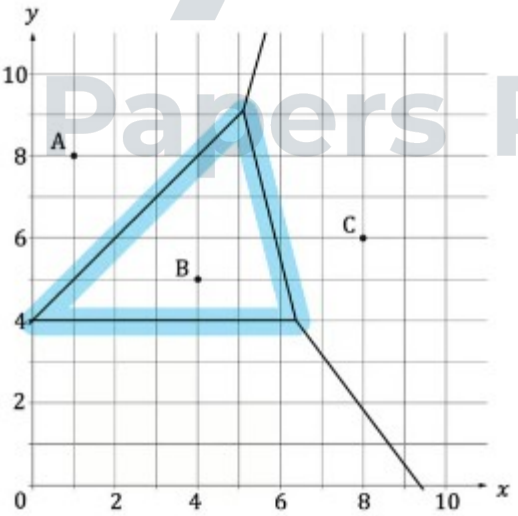
∴ Kayla's apartment is at (-3, -4).

Question 2



a) Site D will be an equal distance from the edge below site B.

D (4, 3)



b) Area of a triangle formula

$A = \frac{1}{2}bh$ (in formula booklet)

b is the base, h is the perpendicular height

$b = 6.375$ $h = 9.1 - 4 = 5.1$

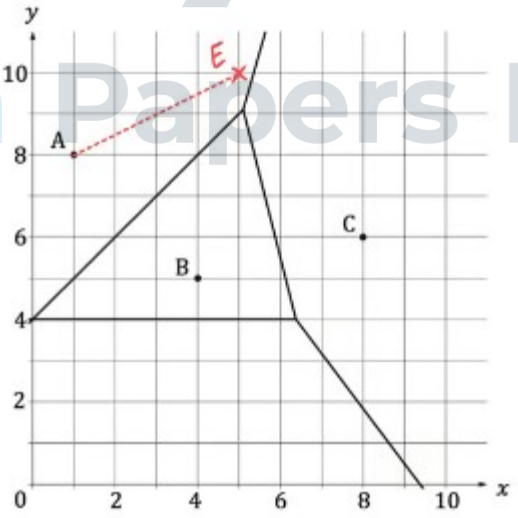
Sub b and h into formula.

$A = \frac{1}{2}(6.375)(5.1)$

$A = 16.25625$

$A = 16.3$ units^2 (3sf)

$A = 1630$ km^2 (3sf)



[1]

c) Closest site to E(5, 10) is site A(1, 8).

Distance between two points formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ (in formula booklet)

E(5, 10) A(1, 8).

Sub E and A into formula.

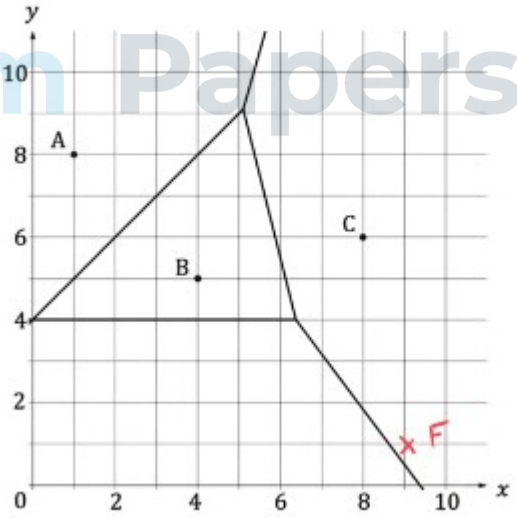
$d = \sqrt{(5 - 1)^2 + (10 - 8)^2}$

$d = \sqrt{20}$

$d = 4.4721...$

$d = 4.47$ (3sf)

d = 44.7 km (3sf)



d) Nearest neighbour interpolation

Nearest site to F(9,1) is site C(8,6)

∴ 312 watts produced per day

Question 3

a) Gradient of a line formula.

$m = \frac{y_2 - y_1}{x_2 - x_1}$ (in formula booklet)

C(2.5, 3.5) D(4.5, 4.5)

Sub C and D into formula.

$m_{CD} = \frac{4.5 - 3.5}{4.5 - 2.5}$

$m_{CD} = \frac{1}{2}$

b) The perpendicular bisector of CD is needed.

Perpendicular gradient formula

$m_2 = -\frac{1}{m_1}$ (in formula booklet)

$m_{CD} = m_1 = \frac{1}{2}$ $\therefore m_{\perp CD} = m_2 = -2$

Midpoint formula

$MP = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ (in formula booklet)

C(2.5, 3.5) D(4.5, 4.5)

Sub C and D into formula.

$MP = \left(\frac{2.5 + 4.5}{2}, \frac{3.5 + 4.5}{2}\right)$

$MP = (3.5, 4)$

Sub MP and m into $y - y_1 = m(x - x_1)$.

$y - 4 = -2(x - 3.5)$ } expand and rearrange into form

$2x + y - 11 = 0$

c) That Voronoi cell contains all the points that are closer to C than they are to A, B, D and E.
Therefore the closest mechanic for the houses that lie in cell C is site C.

Question 4

a) Gradient of a line formula.

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$ (in formula booklet)

B(3, 9) C(5, 5)

Sub B and C into formula.

$$m = \frac{5 - 9}{5 - 3}$$

$$\boxed{m = -2}$$

b) The perpendicular bisector of BC is needed.

Perpendicular gradient formula

$$m_2 = -\frac{1}{m_1}$$ (in formula booklet)

$m_{BC} = m_1 = -2$ $\therefore m_{\perp BC} = m_2 = \frac{1}{2}$

Midpoint formula

$$MP = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$ (in formula booklet)

B(3, 9) C(5, 5)

Sub B and C into formula.

$$MP = \left(\frac{3+5}{2}, \frac{9+5}{2}\right)$$

$MP = (4, 7)$

Sub MP and m into $y - y_1 = m(x - x_1)$.

$y - 7 = \frac{1}{2}(x - 4)$ } expand and rearrange into form

$$\boxed{x - 2y + 10 = 0}$$

c) The perpendicular bisector of DE is needed.

D(8, 2) E(9, 10)

$m_{DE} = \frac{10-2}{9-8} = 8$

$\therefore m_{\perp DE} = -\frac{1}{8}$

$MP = \left(\frac{8+9}{2}, \frac{2+10}{2}\right)$

$MP = (8.5, 6)$

Sub MP and $m_{\perp DE}$ into $y - y_1 = m(x - x_1)$.

$y - 6 = -\frac{1}{8}(x - 8.5)$

expand and rearrange into form

$2x + 16y - 113 = 0$

d) The rabbits are in wolfpack O's territory.

Question 5

a) Perpendicular bisector of AB.

A(3, 15) B(6, 27)

Find midpoint, MP.

$MP = \left(\frac{3+6}{2}, \frac{15+27}{2}\right)$

$MP = (4.5, 21)$

Find perpendicular gradient, $m_{\perp AB}$.

$m_{AB} = \frac{27-15}{6-3} = 4 \quad \therefore m_{\perp AB} = -\frac{1}{4}$

Sub MP and $m_{\perp AB}$ into $y - y_1 = m(x - x_1)$.

$y - 21 = -\frac{1}{4}(x - 4.5)$

expand and rearrange into form

$2x + 8y - 177 = 0$

b) Simultaneous equations

$2x + 8y - 177 = 0, \quad y = 2x + 2$

Sub ① and ② into your GDC.

$x = \frac{161}{18}$ $\quad y = \frac{179}{9}$

$x = 8.9444...$ $\quad y = 19.8888...$

$x = 8.94$ (3sf) $\quad y = 19.9$ (3sf)

The bus stop should be located at $\left(\frac{161}{18}, \frac{179}{9}\right)$ or (8.94, 19.9).

Question 6

a) The population centre (P) is closest to volcano B.

Distance between two points formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ (in formula booklet)

P(7, 5) B(8, 1)

Sub P and B into formula.

$d = \sqrt{(7-8)^2 + (5-1)^2}$

$d = \sqrt{17} = 4.1231...$

$d = 41.2$ km

b) The optimal position will be the coordinates of the largest empty circle.

This will be either E(5, 4) or F(6, 5).

E is equidistant from A, B and C.

A(2, 7) E(5, 4)

$d_{AE} = \sqrt{(2-5)^2+(7-4)^2}$

$d_{AE} = \sqrt{18}$ units (42.4 km)

F is equidistant from A, B and D.

A(2, 7) F(6, 5)

$d_{AF} = \sqrt{(2-6)^2+(7-5)^2}$

$d_{AF} = \sqrt{20}$ units (44.7 km)

∴ F(6, 5) is the optimal position for the shopping centre.

c) The distance is 44.7 km, therefore the position is not safe.