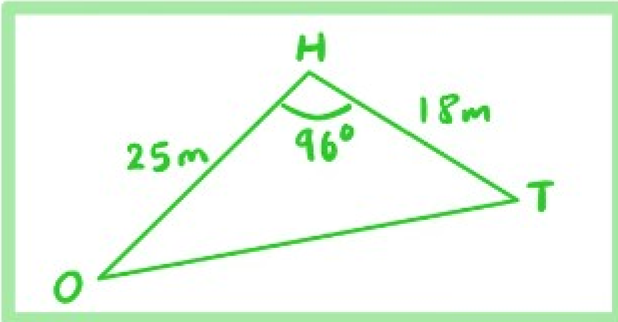
3.3 Trigonometry

Mark Schemes

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.3 Trigonometry
Difficulty	Medium

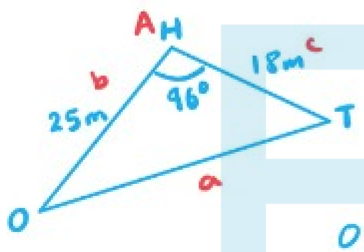
To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1 (a) (i) THREE POINTS CREATE TRIANGLE OHT



ORIENTATION OF TRIANGLE MAY DIFFER

(ii) OT = SIDE OPPOSITE GIVEN ANGLE



USING COSINE RULE

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$OT^2 = OH^2 + HT^2 - 2(OH)(HT)\cos(O\hat{H}T)$$

SUB IN VALUES

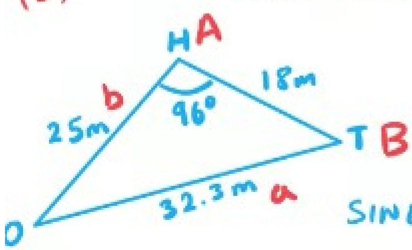
$$OT^2 = 25^2 + 18^2 - 2(25)(18)\cos(96)$$

$$OT = \sqrt{25^2 + 18^2 - 2(25)(18)\cos(96)}$$

$$OT = 32.29668121$$

$$\boxed{OT = 32.3\text{ m (3sf)}}$$

(b) $O\hat{T}H$ = ANGLE OPPOSITE SIDE OH



TWO PAIRS OF OPPOSITE SIDES AND ANGLES = SINE RULE

SINE RULE $\frac{\sin B}{b} = \frac{\sin A}{a}$

$$\frac{\sin(O\hat{T}H)}{OH} = \frac{\sin(O\hat{H}T)}{OT}$$

SUB IN VALUES

$$\frac{\sin(O\hat{T}H)}{25} = \frac{\sin(96)}{32.2966...}$$ (USE ANSWER FROM a)

$$\sin(O\hat{T}H) = \frac{\sin(96)}{32.2966...} \times 25$$

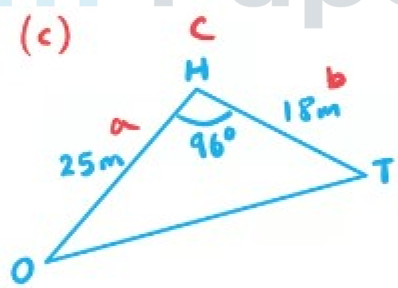
$$O\hat{T}H = \sin^{-1}\left(\frac{\sin(96)}{32.2966...} \times 25\right)$$

$$O\hat{T}H = 50.33888476$$

$$\boxed{O\hat{T}H = 50.3° \text{ (3sf)}}$$

SAME ANSWER FROM OT = 32.3

(c)



AREA = $\frac{1}{2}ab\sin C$

$$A = \frac{1}{2}(OH)(HT)\sin(O\hat{H}T)$$

SUB IN VALUES

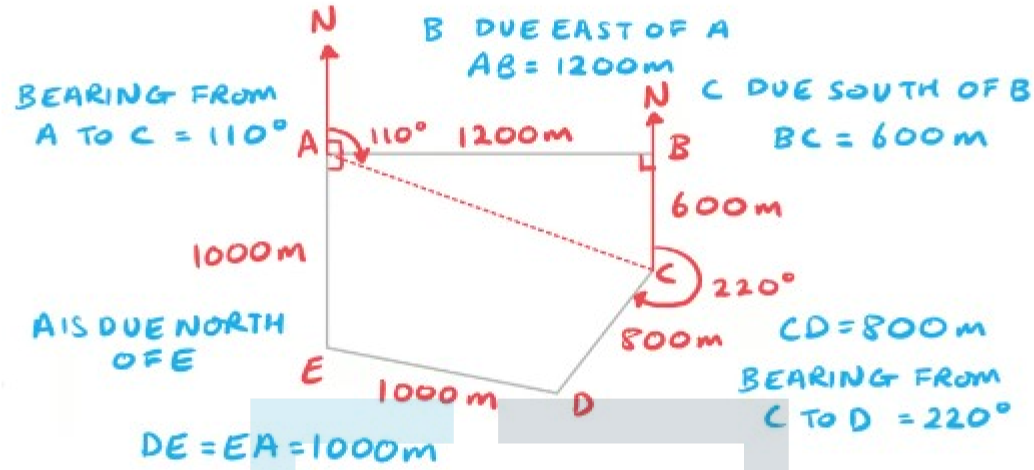
$$A = \frac{1}{2}(25)(18)\sin(96)$$

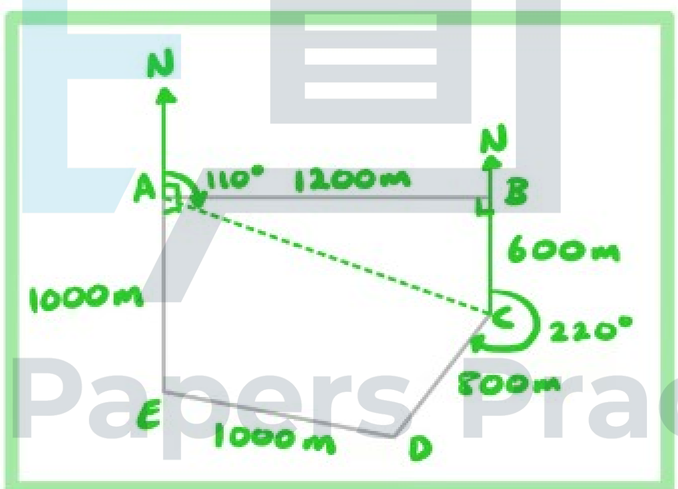
$$A = 223.7674265$$

$$\boxed{\text{AREA} = 224 \text{ m}^2 \text{ (3sf)}}$$

Question 2

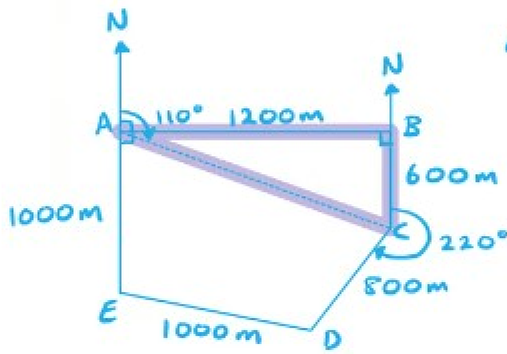
(a) LOOK OUT FOR RIGHT ANGLES WHEN BEARINGS ARE USED, WORK THROUGH STATEMENTS SYSTEMATICALLY BEARINGS ARE MEASURED CLOCKWISE FROM NORTH

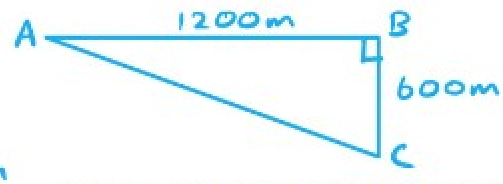




(b) USE DIAGRAM CONSTRUCTED IN PART (a)

AC = HYPOTENUSE OF RIGHT ANGLED TRIANGLE ABC





USING PYTHAGORAS' THEOREM

$$a^2 + b^2 = c^2$$

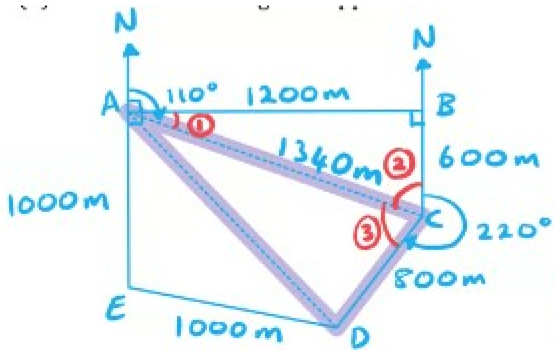
$$AB^2 + BC^2 = AC^2$$

$$AC^2 = 1200^2 + 600^2$$

$$AC = \sqrt{1200^2 + 600^2}$$

$$AC = 1341.640786$$

$$\boxed{AC = 1340\text{ m (3sf)}}$$



[4]

NEED TO FIND MORE INFO FOR TRIANGLE ACD

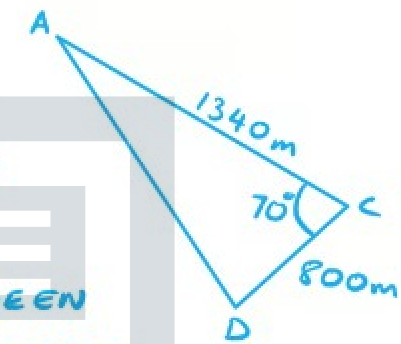
USE BEARINGS TO LABEL MORE ANGLES ON DIAGRAM

(c) USE DIAGRAM CONSTRUCTED IN PART (a)

① $110 - 90 = 20°$

② $180 - (90 + 20) = 70°$

③ $360 - (220 + 70) = 70°$



WE NOW HAVE ANGLE BETWEEN TWO SIDES SO CAN USE COSINE RULE

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$AD^2 = AC^2 + CD^2 - 2(AC)(CD)\cos C$$

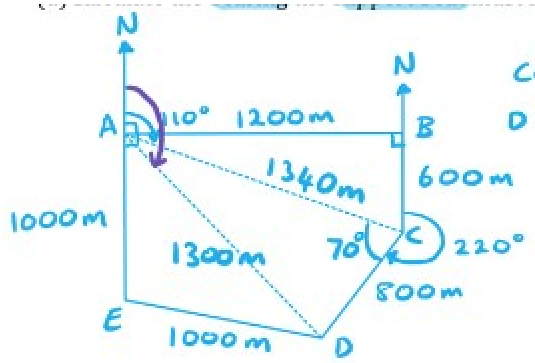
SUB IN VALUES

$$AD^2 = 1340^2 + 800^2 - 2(1340)(800)\cos(70)$$

$$AD = \sqrt{1340^2 + 800^2 - 2(1340)(800)\cos(70)}$$

$$AD = 1304.72557$$

$$\boxed{AD = 1300\text{ m } (3sf)}$$



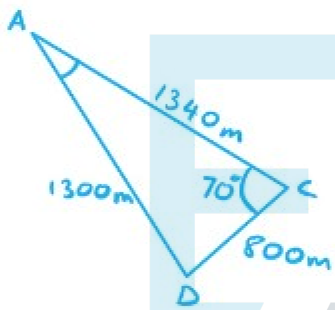
[4]

CONTINUE LABELLING DIAGRAM WITH NEW VALUES

BEARING FROM A TO D

$$= \text{BEARING FROM A TO C} + D\hat{A}C$$

(d) $D\hat{A}C$ CAN BE FOUND USING SINE RULE



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin D\hat{A}C}{DC} = \frac{\sin A\hat{C}D}{AD}$$

SUB IN VALUES

$$\frac{\sin D\hat{A}C}{800} = \frac{\sin 70}{1300}$$

$$\sin D\hat{A}C = \frac{\sin 70}{1300} \times 800$$

$$D\hat{A}C = \sin^{-1}\left(\frac{\sin 70}{1300} \times 800\right)$$

$$D\hat{A}C = 35.32912271$$

BEARING FROM A TO D = 110 + 35.3 … = 145.3 …

BEARINGS ARE ALWAYS GIVEN AS 3 FIGURES

$$\boxed{\text{BEARING A TO D} = 145°}$$

Question 3

(a) AREA $= \frac{1}{2} ab \sin C$

$$A = \frac{1}{2}(AC)(CB)\sin(A\hat{C}B)$$

SUB IN VALUES

$$A = \frac{1}{2}(21)(15)\sin(75)$$

$$A = 152.133\ldots$$

$$\boxed{\text{AREA} = 152 \text{ km}^2 \text{ (3sf)}}$$

(b) AB = SIDE OPPOSITE GIVEN ANGLE

USING COSINE RULE $a^2 = b^2 + c^2 - 2bc\cos A$

$$AB^2 = AC^2 + CB^2 - 2(AC)(CB)\cos(A\hat{C}B)$$

SUB IN VALUES

$$AB^2 = 21^2 + 15^2 - 2(21)(15)\cos(75)$$

$$AB = \sqrt{21^2 + 15^2 - 2(21)(15)\cos(75)}$$

$$AB = 22.4264\ldots$$

$$\boxed{AB = 22.4 \text{ km (3sf)}}$$

(c) $C\hat{A}B$ = ANGLE OPPOSITE SIDE CB

TWO PAIRS OF OPPOSITE SIDES AND ANGLES

= SINE RULE $\frac{\sin B}{b} = \frac{\sin A}{a}$

$$\frac{\sin(C\hat{A}B)}{CB} = \frac{\sin(A\hat{C}B)}{AB}$$

SUB IN VALUES

$$\frac{\sin(C\hat{A}B)}{15} = \frac{\sin(75)}{22.4264...}$$ (USE ANSWER FROM b)

$$\sin(C\hat{A}B) = \frac{\sin(75)}{22.4264...} \times 15$$

$$C\hat{A}B = \sin^{-1}\left(\frac{\sin(75)}{22.4264...} \times 15\right)$$

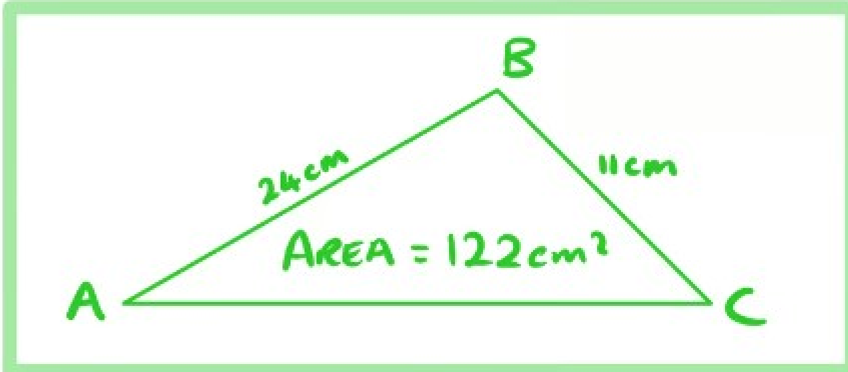
$$C\hat{A}B = 40.2454...$$

$$\boxed{C\hat{A}B = 40.2^\circ \ (3SF)}$$

$C\hat{A}B = 40.3$
USING AB = 22.4

Question 4

(a) TWO POSSIBLE VALID DIAGRAMS



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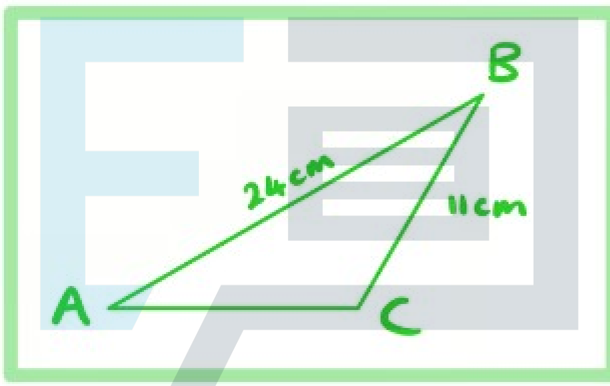
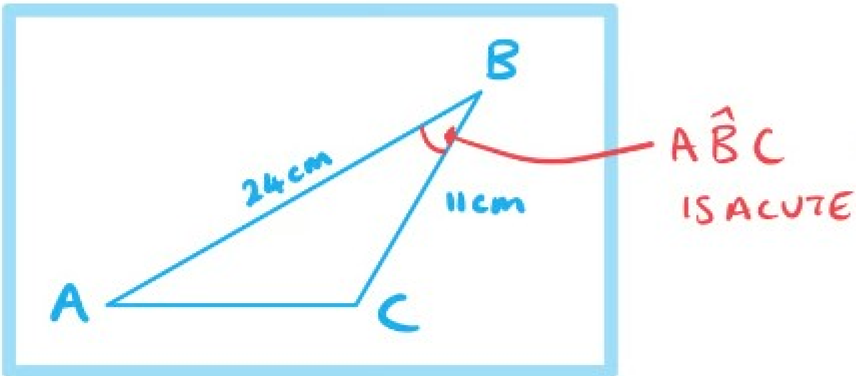




(b)(i) AREA = $\frac{1}{2}ab\sin C$ $A = \frac{1}{2}(AB)(BC)\sin(A\hat{B}C)$

SUB IN VALUES AND REARRANGE

$$122 = \frac{1}{2}(24)(11)\sin(A\hat{B}C)$$

$$\sin(A\hat{B}C) = \frac{122}{132}$$

$$A\hat{B}C = \sin^{-1}\left(\frac{122}{132}\right) = 67.55439...$$

$$\boxed{A\hat{B}C = 67.6° \ (3sf)}$$

(ii) TWO SIDES GIVEN, USE COSINE RULE FOR THIRD SIDE $a^2 = b^2 + c^2 - 2bc\cos A$

$$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos A\hat{B}C$$

From (i)

$$AC^2 = 24^2 + 11^2 - 2(24)(11)\cos 67.55439...$$

$$AC = \sqrt{24^2 + 11^2 - 2(24)(11)\cos 67.55439...}$$

$$AC = 22.25772561$$

$$\boxed{AC = 22.3 \text{ cm } (3sf)}$$

Question 5

(a) AB = SIDE OPPOSITE GIVEN ANGLE

USING COSINE RULE $a^2 = b^2 + c^2 - 2bc \cos A$

$BD^2 = AB^2 + AD^2 - 2(AB)(AD)\cos(D\hat{A}B)$

SUB IN VALUES

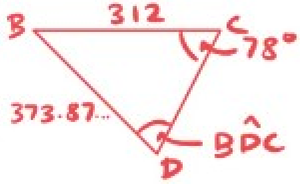
$BD^2 = 246^2 + 257^2 - 2(246)(257)\cos(96)$

$BD = \sqrt{246^2 + 257^2 - 2(246)(257)\cos(96)}$

$BD = 373.8743064$

$\boxed{BD = 374\text{ m (3sf)}}$

(b) AREA = $\frac{1}{2}ab\sin C$ SO FIRST NEED TO CALCULATE $D\hat{B}C$



SINE RULE TO FIND $B\hat{D}C$

$$\frac{\sin B\hat{D}C}{312} = \frac{\sin 78}{373.87...}$$ ANSWER FROM (a)

$$B\hat{D}C = \sin^{-1}\left(312 \times \frac{\sin 78}{373.87...}\right)$$

$$B\hat{D}C = 54.71304365$$ 54.685... IF USING 374m

$$D\hat{B}C = 180 - 54.173... - 78 = 47.286...$$

$$AREA_{ABCD} = AREA_{ABD} + AREA_{DBC}$$

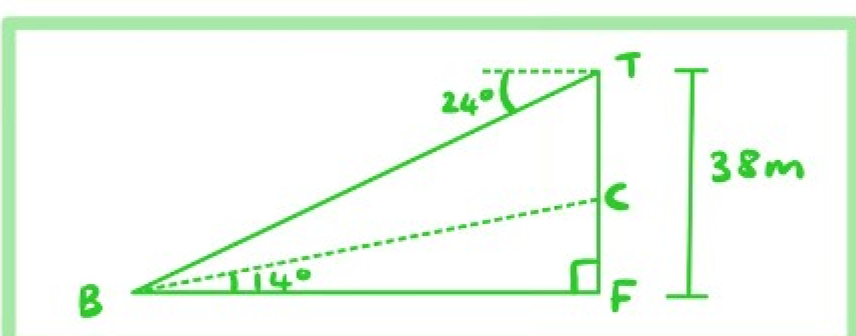
$AREA_{ABCD} =$

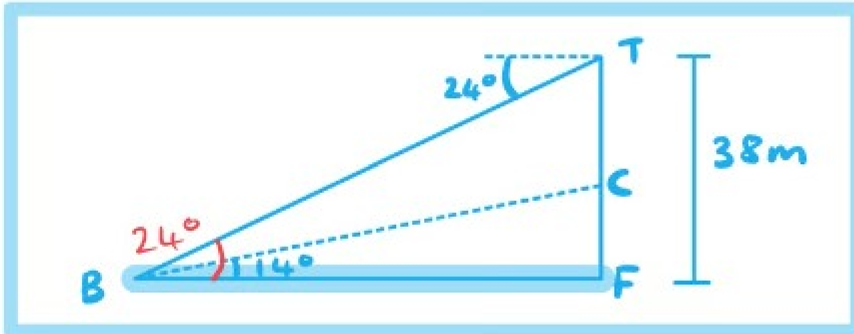
$$\frac{1}{2}(246)(257)\sin 96 + \frac{1}{2}(312)(373.87...)\sin 47.286...$$

$$= 74292.27283$$ USING VALUES TO 3SF AREA = 74315

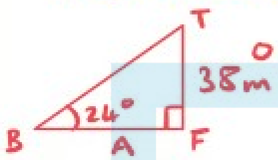
$$\boxed{AREA = 74300\ m^2\ (3sf)}$$

Question 6 (a) DEPRESSION = DOWN FROM HORIZONTAL
ELEVATION = UP FROM HORIZONTAL





(b) RIGHT ANGLED TRIG USING PARALLEL SEA AND DEPRESSION GIVES $T\hat{B}F = 24°$

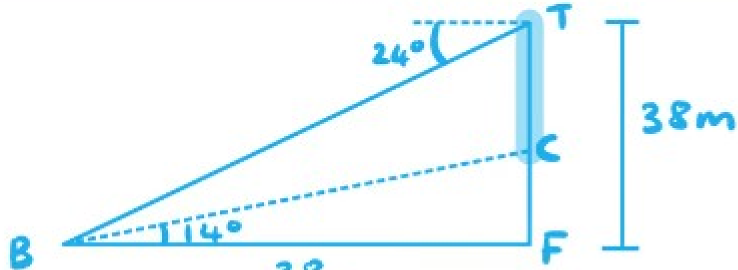


$\tan\theta = \frac{O}{A}$ $\quad \tan T\hat{B}F = \frac{TF}{BF}$

$$BF = \frac{TF}{\tan T\hat{B}F}$$

$$BF = \frac{38}{\tan 24} = 85.34939741$$

$$\boxed{BF = 85.3 \text{ m}}$$

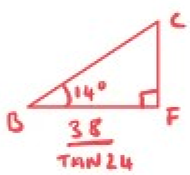


$$BF = \frac{38}{\tan 24°} = 85.349\ldots$$

(c) TO FIND CLIMB DISTANCE CT

$$CT = 38 - FC$$

USE RIGHT ANGLED TRIG ON BFC



$$FC = \tan 14° \times \frac{38}{\tan 24°}$$

BF FROM (b) TO RETAIN ACCURACY

$$FC = 21.2799948$$

$$CT = 38 - 21.2799948$$

$$CT = 16.7200052$$

$$\boxed{CT = 16.7 \text{ m } (3sf)}$$

Question 7

(a) THREE SIDES FINDING ANGLE = COSINE RULE

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

$$\cos Y\hat{Z}W = \frac{WZ^2 + YZ^2 - WY^2}{2(WZ)(YZ)}$$

$$Y\hat{Z}W = \cos^{-1}\left(\frac{4.2^2 + 5.4^2 - 5.8^2}{2(4.2)(5.4)}\right)$$

$$Y\hat{Z}W = 73.13465266$$

$$\boxed{Y\hat{Z}W = 73.1^\circ \ (3sf)}$$

(b)

$$AREA = \frac{1}{2}ab\sin C$$

$$= \frac{1}{2}(XW + WZ)(YZ)\sin(Y\hat{Z}W)$$

$$AREA_{XYZ} = \frac{1}{2}(5.6 + 4.2)(5.4)\sin(73.13465266)$$

$$AREA_{XYZ} = 25.32193494$$

$$\boxed{AREA_{XYZ} = 25.3\ cm^2 \ (3sf)}$$

(c)

$$\text{AREA} = \frac{1}{2}ab\sin C$$

$$\text{AREA}_{XYW} = \text{AREA}_{XYZ} - \text{AREA}_{WYZ}$$

$$\text{AREA}_{XYW} = \text{AREA}_{XYZ} - \frac{1}{2}(4.2)(5.4)\sin(Y\hat{Z}W)$$

USING VALUES FOR AREA_{XYZ} AND $Y\hat{Z}W$ FROM (a) AND (b) TO KEEP ACCURACY

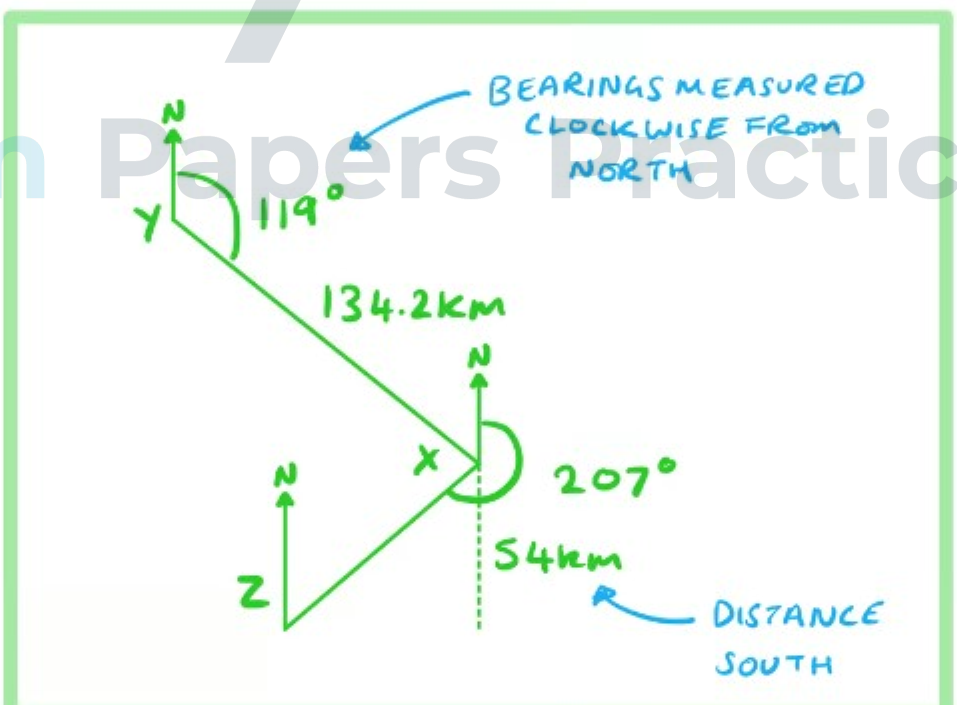
$$\text{AREA}_{XYW} = 25.321\ldots - \frac{1}{2}(4.2)(5.4)\sin(73.134\ldots)$$

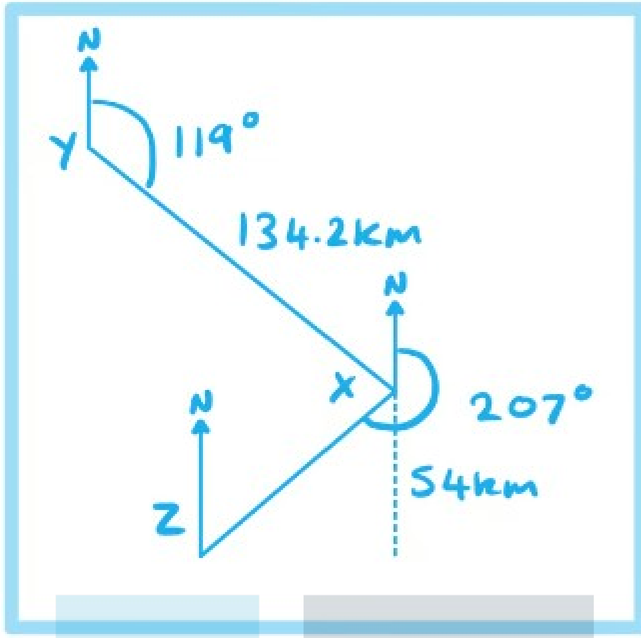
$$\text{AREA}_{XYW} = 14.46967711$$

$$\boxed{\text{AREA}_{XYW} = 14.5\ \text{cm}^2\ (3sf)}$$

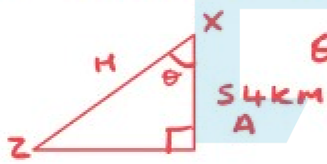
Question 8

(a)





(b) DISTANCE XZ USES RIGHT ANGLED TRIG



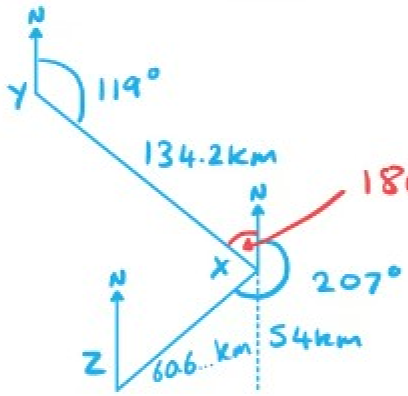
$\theta = 207 - 180 = 27°$

$\cos\theta = \dfrac{A}{H}$ ← XZ

$XZ = \dfrac{54}{\cos 27°} = 60.60561683$

$\boxed{XZ = 60.6 \text{ km} \quad (3sf)}$

(c) USE BEARINGS TO FIND $Y\hat{X}Z$



$180 - 119 = 61°$

$Y\hat{X}Z = 360 - 207 - 61$
$= 92°$

USE COSINE RULE TO FIND YZ

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$YZ^2 = YX^2 + XZ^2 - 2(YX)(XZ)\cos(Y\hat{X}Z)$$

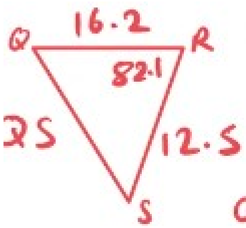
$$YZ^2 = (134.2)^2 + \left(\frac{54}{\cos 27}\right)^2 - 2(134.2)\left(\frac{54}{\cos 27}\right)\cos(92)$$

$$YZ = \sqrt{(134.2)^2 + \left(\frac{54}{\cos 27}\right)^2 - 2(134.2)\left(\frac{54}{\cos 27}\right)\cos(92)}$$

$$YZ = 149.1655963$$

$$\boxed{YZ = 149\text{km } (3sf)}$$

Question 9

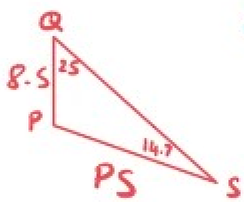


COSINE RULE $a^2 = b^2 + c^2 - 2bc \cos A$

$QS^2 = (16.2)^2 + (12.5)^2 - 2(16.2)(12.5)\cos(82.1)$

$QS = \sqrt{(16.2)^2 + (12.5)^2 - 2(16.2)(12.5)\cos(82.1)}$

$QS = 19.05321387$ km



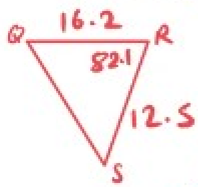
SINE RULE $\dfrac{a}{\sin A} = \dfrac{b}{\sin B}$

$$\frac{PS}{\sin 25} = \frac{8.5}{\sin 14.7}$$

$$PS = \frac{8.5}{\sin 14.7} \times \sin 25$$

$PS = 14.15622762$ km

TO FIND PR FIRST FIND $S\hat{Q}R$ USING SINE RULE

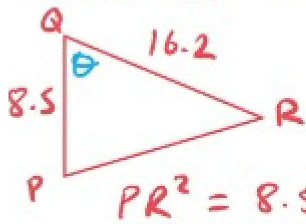


$$\frac{\sin S\hat{Q}R}{12.5} = \frac{\sin 82.1}{QS}$$

$$S\hat{Q}R = \sin^{-1}\left(\frac{\sin 82.1}{19.05321387} \times 12.5\right)$$

$S\hat{Q}R = 40.52885831$

THEN USE $\theta = P\hat{Q}S + S\hat{Q}R$ TO FIND PR



$\theta = 25 + 40.52885831$
$= 65.52885831$

$PR^2 = 8.5^2 + 16.2^2 - 2(8.5)(16.2)\cos\theta$

$PR = \sqrt{8.5^2 + 16.2^2 - 2(8.5)(16.2)\cos(65.528\ldots)}$

$PR = 14.85293632$ km

TOTAL DISTANCE $= QS + SP + PR$

$19.05321387 + 14.15622762 + 14.85293632$

TOTAL $= 48.0623778$ km

$\boxed{\text{TOTAL} = 48.1 \text{ km (3sf)}}$

Question 10

(a) DRAW DIAGRAM

$\sin\theta = \frac{O}{H}$

$MN = \frac{10}{\sin 21.6}$

$MN = 27.16471892$

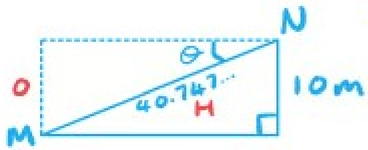
$\boxed{MN = 27.2 \text{ m (3sf)}}$

(b) LN = 1.5 × MN

LN = 1.5 × 27.16471892

LN = 40.74707837

DRAW DIAGRAM DEPRESSION IS DOWN FROM HORIZONTAL



$\sin\theta = \frac{O}{H}$ ← LN

$\theta = \sin^{-1}\left(\frac{10}{LN}\right) = \sin^{-1}\left(\frac{10}{40.74707837}\right)$

$\theta = 14.20644114$

$\boxed{\theta = 14.2^\circ \text{ (3sf)}}$