Exam Papers Practice

3.3 Trigonometry

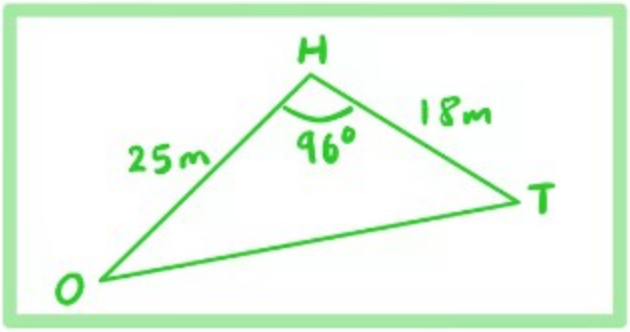
Mark Schemes

Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.3 Trigonometry
Difficulty	Medium

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful

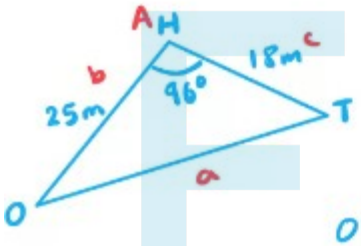
Question 1

(a) (i) THREE POINTS CREATE TRIANGLE OHT



ORIENTATION OF TRIANGLE MAY DIFFER

(ii) OT = SIDE OPPOSITE GIVEN ANGLE



USING COSINE RULE

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$OT^2 = OH^2 + HT^2 - 2(OH)(HT)\cos(O\hat{H}T)$$

SUB IN VALUES

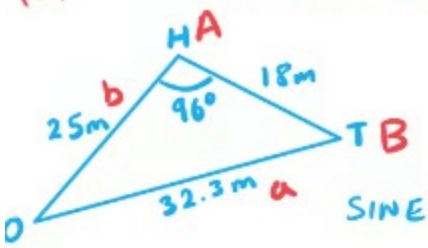
$$OT^2 = 25^2 + 18^2 - 2(25)(18)\cos(96)$$

$$OT = \sqrt{25^2 + 18^2 - 2(25)(18)\cos(96)}$$

$$OT = 32.29668121$$

$$OT = 32.3\text{ m (3SF)}$$

(b) $O\hat{T}H$ = ANGLE OPPOSITE SIDE OH



TWO PAIRS OF OPPOSITE SIDES AND ANGLES = SINE RULE

SINE RULE $\frac{\sin B}{b} = \frac{\sin A}{a}$

$$\frac{\sin(O\hat{T}H)}{OH} = \frac{\sin(O\hat{H}T)}{OT}$$

SUB IN VALUES

$$\frac{\sin(O\hat{T}H)}{25} = \frac{\sin(96)}{32.2966...}$$ (USE ANSWER FROM a)

$$\sin(O\hat{T}H) = \frac{\sin(96)}{32.2966...} \times 25$$

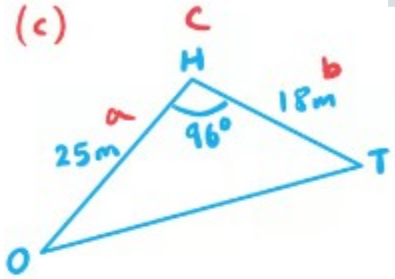
$$O\hat{T}H = \sin^{-1}\left(\frac{\sin(96)}{32.2966...} \times 25\right)$$

$$O\hat{T}H = 50.338884 76$$

$O\hat{T}H = 50.3°$ (3SF)

SAME ANSWER FROM OT=32.3

(c)



AREA = $\frac{1}{2}ab\sin C$

$A = \frac{1}{2}(OH)(HT)\sin(O\hat{H}T)$

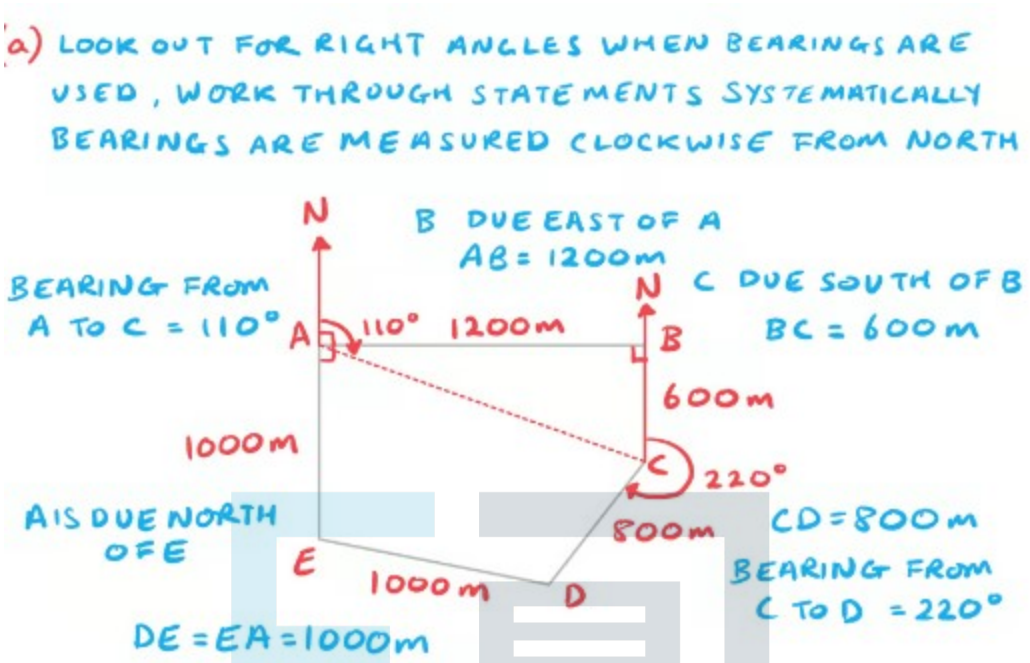
SUB IN VALUES

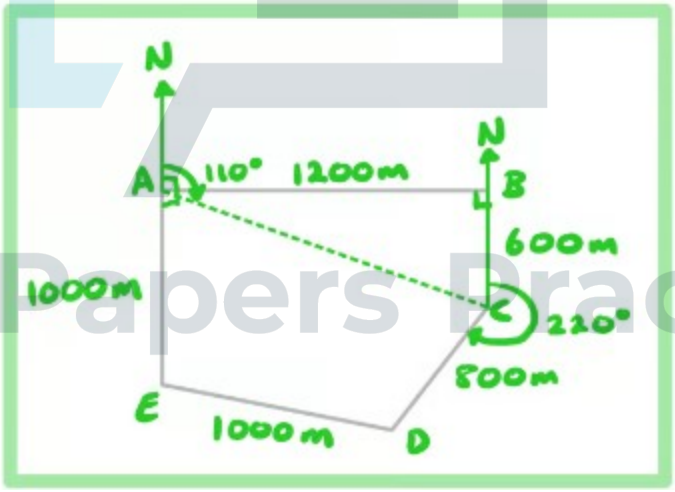
$A = \frac{1}{2}(25)(18)\sin(96)$

$A = 223.7674265$

AREA = 224 m^2 (3sf)

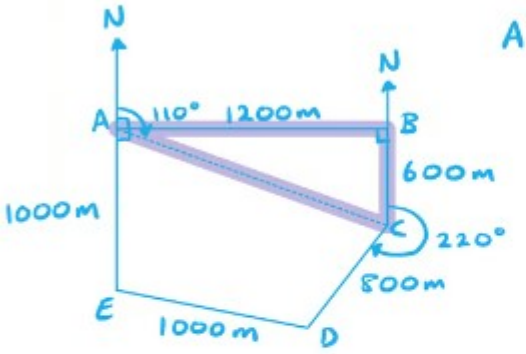
Question 2

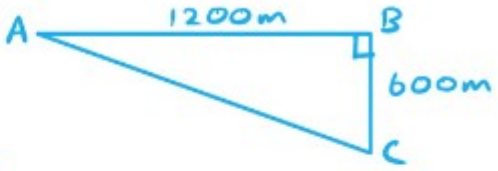




(b) USE DIAGRAM CONSTRUCTED IN PART (a)

AC = HYPOTENUSE OF RIGHTANGLED TRIANGLE ABC





USING PYTHAGORAS' THEOREM

$a^2 + b^2 = c^2$

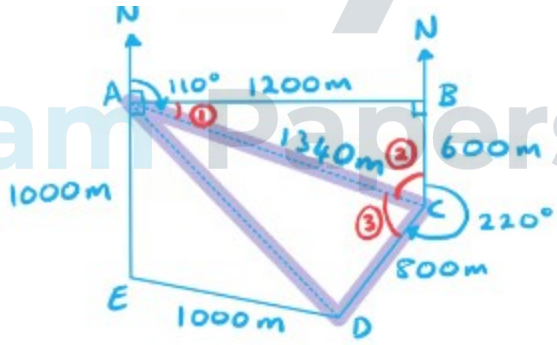
$AB^2 + BC^2 = AC^2$

$AC^2 = 1200^2 + 600^2$

$AC = \sqrt{1200^2 + 600^2}$

$AC = 1341.640786$

AC = 1340 m (3sf)



NEED TO FIND MORE INFO FOR TRIANGLE ACD

USE BEARINGS TO LABEL MORE ANGLES ON DIAGRAM

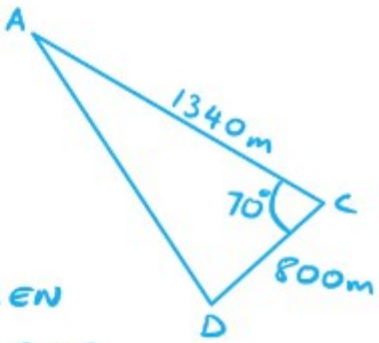
[4]

(c) USE DIAGRAM CONSTRUCTED IN PART (a)

① $110 - 90 = 20°$

② $180 - (90 + 20) = 70°$

③ $360 - (220 + 70) = 70°$



WE NOW HAVE ANGLE BETWEEN TWO SIDES SO CAN USE COSINE RULE

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$AD^2 = AC^2 + CD^2 - 2(AC)(CD)\cos C$$

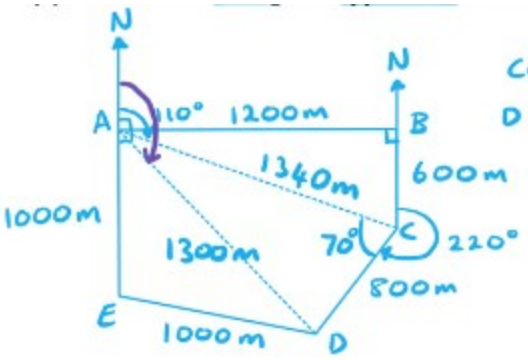
SUB IN VALUES

$$AD^2 = 1340^2 + 800^2 - 2(1340)(800)\cos(70)$$

$$AD = \sqrt{1340^2 + 800^2 - 2(1340)(800)\cos(70)}$$

$$AD = 1304.72557$$

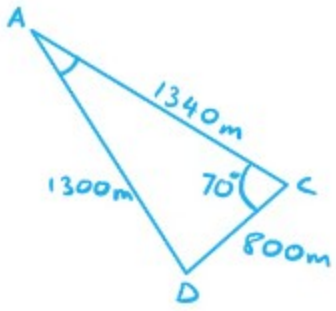
AD = 1300 m (3sf)



CONTINUE LABELLING DIAGRAM WITH NEW VALUES [4]

BEARING FROM A TO D = BEARING FROM A TO C + $D\hat{A}C$

d) $D\hat{A}C$ CAN BE FOUND USING SINE RULE



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin D\hat{A}C}{DC} = \frac{\sin A\hat{C}D}{AD}$$

SUB IN VALUES

$$\frac{\sin D\hat{A}C}{800} = \frac{\sin 70}{1300}$$

$$\sin D\hat{A}C = \frac{\sin 70}{1300} \times 800$$

$$D\hat{A}C = \sin^{-1}\left(\frac{\sin 70}{1300} \times 800\right)$$

$$D\hat{A}C = 35.32912271$$

BEARING FROM A TO D = 110 + 35.3... = 145.3...

BEARINGS ARE ALWAYS GIVEN AS 3 FIGURES

BEARING A TO D = 145°

Question 3 (a) AREA = $\frac{1}{2}ab\sin C$

$$A = \frac{1}{2}(AC)(CB)\sin(A\hat{C}B)$$

SUB IN VALUES

$$A = \frac{1}{2}(21)(15)\sin(75)$$

$$A = 152.133...$$

AREA = 152 km² (3sf)

b) AB = SIDE OPPOSITE GIVEN ANGLE

USING COSINE RULE $a^2 = b^2 + c^2 - 2bc\cos A$

$$AB^2 = AC^2 + CB^2 - 2(AC)(CB)\cos(A\hat{C}B)$$

SUB IN VALUES

$$AB^2 = 21^2 + 15^2 - 2(21)(15)\cos(75)$$

$$AB = \sqrt{21^2 + 15^2 - 2(21)(15)\cos(75)}$$

$$AB = 22.4264\ldots$$

AB = 22.4 km (3sf)

c) $C\hat{A}B$ = ANGLE OPPOSITE SIDE CB

TWO PAIRS OF OPPOSITE SIDES AND ANGLES

= SINE RULE $\frac{\sin B}{b} = \frac{\sin A}{a}$

$$\frac{\sin(C\hat{A}B)}{CB} = \frac{\sin(A\hat{C}B)}{AB}$$

SUB IN VALUES

$$\frac{\sin(C\hat{A}B)}{15} = \frac{\sin(75)}{22.4264\ldots} \text{ (USE ANSWER FROM b)}$$

$$\sin(C\hat{A}B) = \frac{\sin(75)}{22.4264\ldots} \times 15$$

$$C\hat{A}B = \sin^{-1}\left(\frac{\sin(75)}{22.4264\ldots} \times 15\right)$$

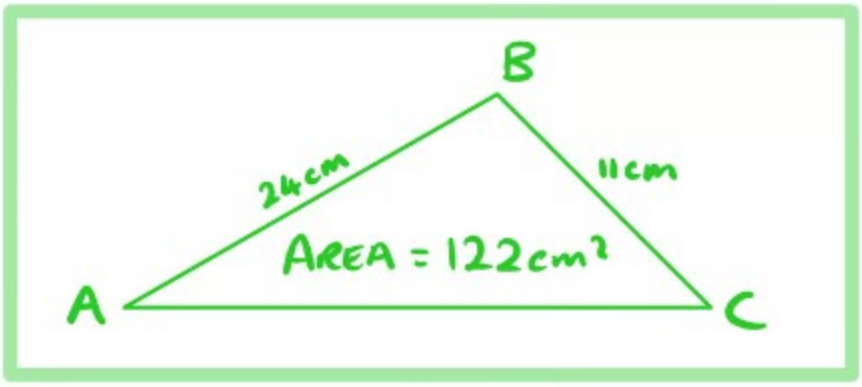
$$C\hat{A}B = 40.2454\ldots$$

$C\hat{A}B = 40.2°$ (3SF)

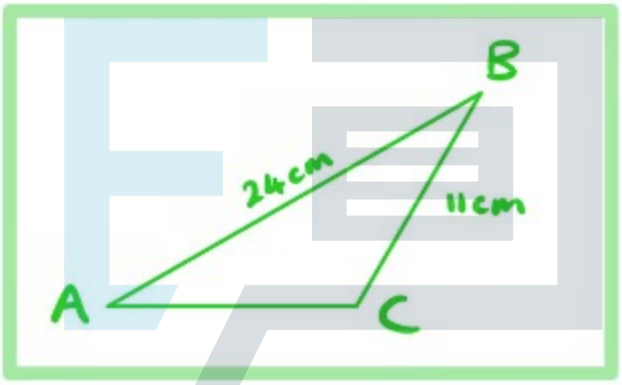
$C\hat{A}B = 40.3$ USING AB = 22.4

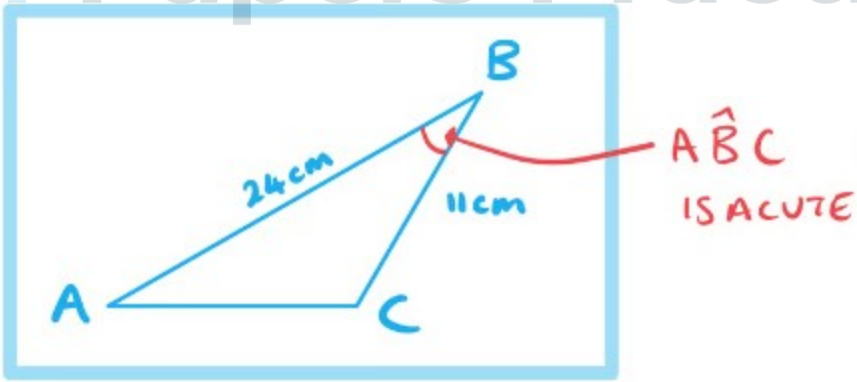
Question 4

a) TWO POSSIBLE VALID DIAGRAMS



OR





b) (i) AREA = $\frac{1}{2}abSINC$ $A = \frac{1}{2}(AB)(BC)SIN(A\hat{B}C)$

SUB IN VALUES AND REARRANGE

$122 = \frac{1}{2}(24)(11)SIN(A\hat{B}C)$

$SIN(A\hat{B}C) = \frac{122}{132}$

$A\hat{B}C = SIN^{-1}\left(\frac{122}{132}\right) = 67.55439...$

$A\hat{B}C = 67.6°$ (3sf)

(ii) TWO SIDES GIVEN, USE COSINE RULE FOR THIRD SIDE $a^2 = b^2 + c^2 - 2bc\cos A$

$AC^2 = AB^2 + BC^2 - 2(AB)(BC)\cos A\hat{B}C$

$AC^2 = 24^2 + 11^2 - 2(24)(11)\cos 67.55439...$ (From (i))

$AC = \sqrt{24^2 + 11^2 - 2(24)(11)\cos 67.55439...}$

$AC = 22.25772561$

$AC = 22.3$ cm (3sf)

Question 5

(a) AB = SIDE OPPOSITE GIVEN ANGLE

USING COSINE RULE $a^2 = b^2 + c^2 - 2bc\cos A$

$BD^2 = AB^2 + AD^2 - 2(AB)(AD)\cos(D\hat{A}B)$

SUB IN VALUES

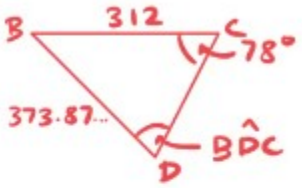
$BD^2 = 246^2 + 257^2 - 2(246)(257)\cos(96)$

$BD = \sqrt{246^2 + 257^2 - 2(246)(257)\cos(96)}$

$BD = 373.8743064$

$BD = 374$ m (3sf)

(b) AREA = $\frac{1}{2}$ ab SIN C SO FIRST NEED TO CALCULATE $D\hat{B}C$



SINE RULE TO FIND $B\hat{D}C$

$$\frac{\sin B\hat{D}C}{312} = \frac{\sin 78}{373.87...}$$ ← ANSWER FROM (a)

$$B\hat{D}C = \sin^{-1}\left(312 \times \frac{\sin 78}{373.87...}\right)$$

$B\hat{D}C = 54.71304365$ 54.685... IF USING 374m

$D\hat{B}C = 180 - 54.173... - 78 = 47.286...$

$AREA_{ABCD} = AREA_{ABD} + AREA_{DBC}$

$AREA_{ABCD} =$

$\frac{1}{2}(246)(257)\sin 96 + \frac{1}{2}(312)(373.87...)\sin 47.286...$

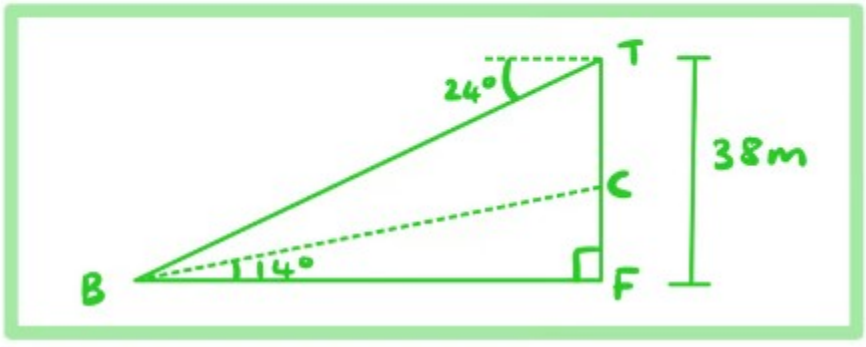
$= 74292.27283$ USING VALUES TO 3SF AREA = 74315

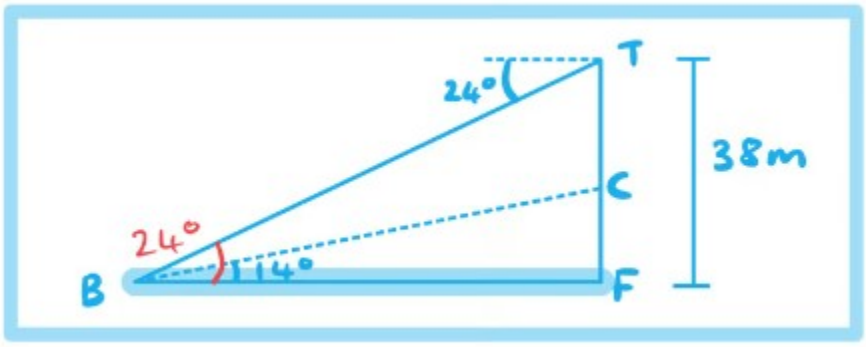
AREA = 74300 m^2 (3sf)

Question 6

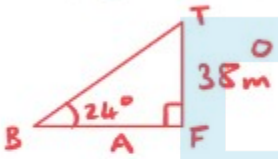
(a) DEPRESSION = DOWN FROM HORIZONTAL

ELEVATION = UP FROM HORIZONTAL





(b) RIGHT ANGLED TRIG USING PARALLEL SEA AND DEPRESSION GIVES $T\hat{B}F = 24°$

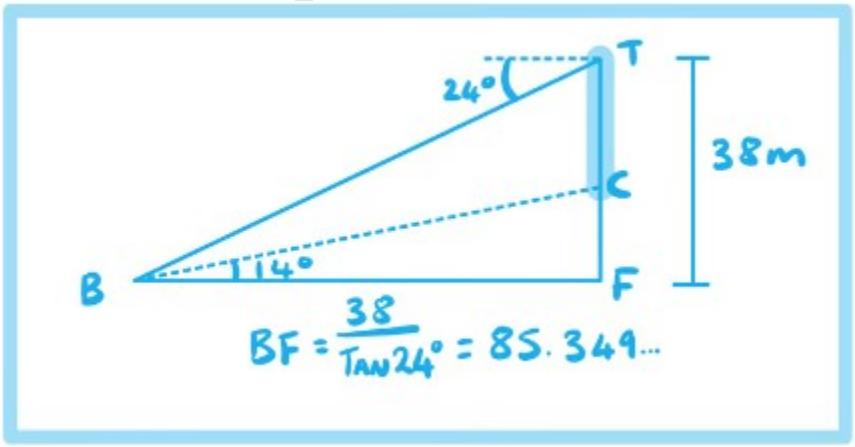


$\text{TAN}\,\theta = \frac{O}{A}$ $\quad \text{TAN}\,T\hat{B}F = \frac{TF}{BF}$

$$BF = \frac{TF}{\text{TAN}\,T\hat{B}F}$$

$$BF = \frac{38}{\text{TAN}\,24} = 85.34939741$$

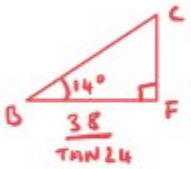
$$BF = 85.3\ \text{m}$$



(c) TO FIND CLIMB DISTANCE CT

$CT = 38 - FC$

USE RIGHT ANGLED TRIG ON BFC



$FC = \tan 14° \times \frac{38}{\tan 24°}$ ← BF FROM (b) TO RETAIN ACCURACY

$FC = 21.2799948$

$CT = 38 - 21.2799948$

$CT = 16.7200052$

CT = 16.7 m (3sf)

Question 7

(a) THREE SIDES FINDING ANGLE = COSINE RULE

$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

$\cos Y\hat{Z}W = \frac{WZ^2 + YZ^2 - WY^2}{2(WZ)(YZ)}$

$Y\hat{Z}W = \cos^{-1}\left(\frac{4.2^2 + 5.4^2 - 5.8^2}{2(4.2)(5.4)}\right)$

$Y\hat{Z}W = 73.13465266$

$Y\hat{Z}W = 73.1°$ (3sf)

b) $\text{AREA} = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(XW + WZ)(YZ)\sin(Y\hat{Z}W)$

$\text{AREA}_{XYZ} = \frac{1}{2}(5.6 + 4.2)(5.4)\sin(73.13465266)$

$\text{AREA}_{XYZ} = 25.32193494$

$\text{AREA}_{XYZ} = 25.3\ \text{cm}^2\ (3\text{sf})$

c) $\text{AREA} = \frac{1}{2}ab\sin C$

$\text{AREA}_{XYW} = \text{AREA}_{XYZ} - \text{AREA}_{WYZ}$

$\text{AREA}_{XYW} = \text{AREA}_{XYZ} - \frac{1}{2}(4.2)(5.4)\sin(Y\hat{Z}W)$

USING VALUES FOR AREA_{XYZ} AND $Y\hat{Z}W$ FROM (a) AND (b) TO KEEP ACCURACY

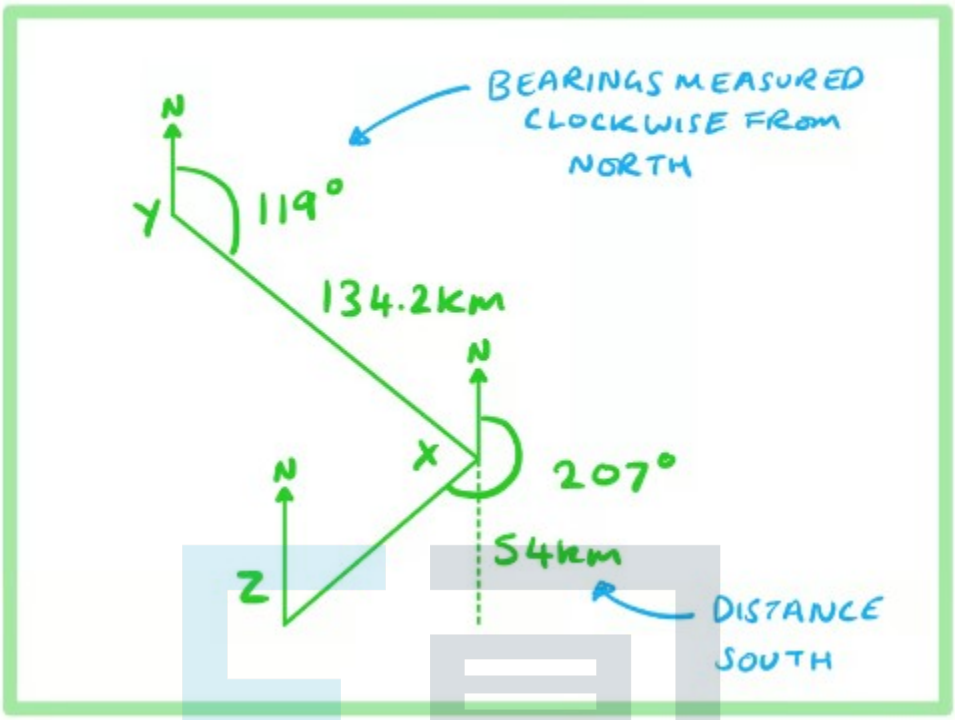
$\text{AREA}_{XYW} = 25.321\ldots - \frac{1}{2}(4.2)(5.4)\sin(73.134\ldots)$

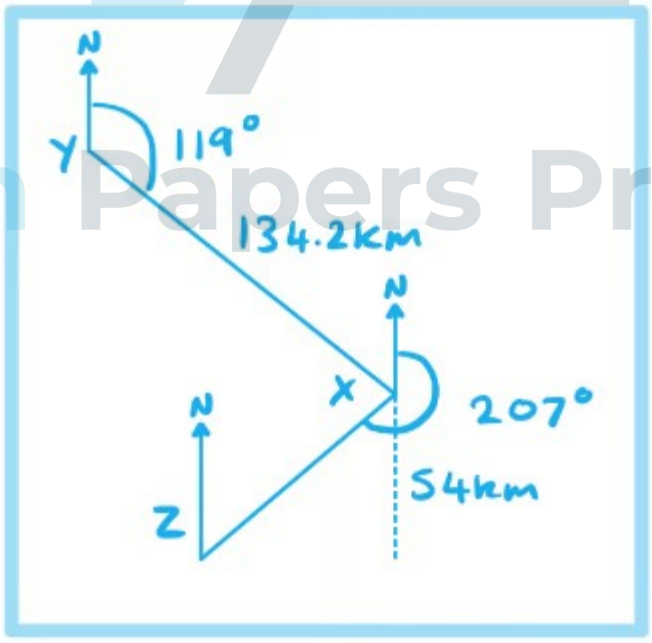
$\text{AREA}_{XYW} = 14.46967711$

$\text{AREA}_{XYW} = 14.5\ \text{cm}^2\ (3\text{sf})$

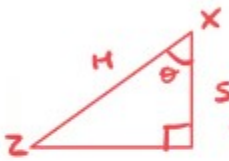
Question 8

(a)





(b) DISTANCE XZ USES RIGHT ANGLED TRIG



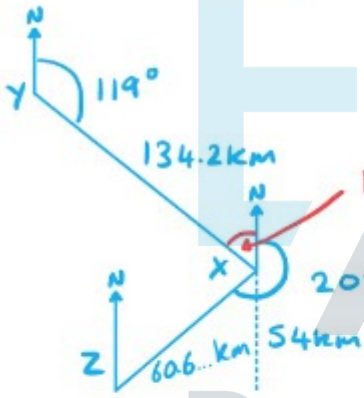
$\theta = 207 - 180 = 27°$

$\cos\theta = \frac{A}{H}$ ← XZ

$XZ = \frac{54}{\cos 27°} = 60.60561683$

XZ = 60.6 km (3sf)

c) USE BEARINGS TO FIND $Y\hat{X}Z$



$180 - 119 = 61°$

$Y\hat{X}Z = 360 - 207 - 61$
$= 92°$

USE COSINE RULE TO FIND YZ

$a^2 = b^2 + c^2 - 2bc\cos A$

$YZ^2 = YX^2 + XZ^2 - 2(YX)(XZ)\cos(Y\hat{X}Z)$

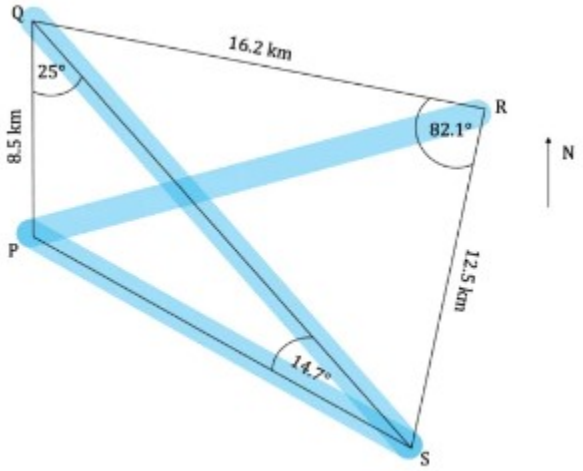
$YZ^2 = (134.2)^2 + \left(\frac{54}{\cos 27}\right)^2 - 2(134.2)\left(\frac{54}{\cos 27}\right)\cos(92)$

$YZ = \sqrt{(134.2)^2 + \left(\frac{54}{\cos 27}\right)^2 - 2(134.2)\left(\frac{54}{\cos 27}\right)\cos(92)}$

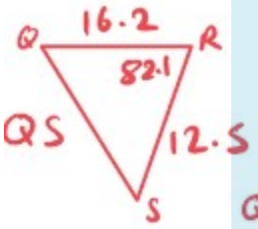
$YZ = 149.1655963$

YZ = 149 km (3sf)

Question 9



FOR EACH DISTANCE, FIRST FIND CORRESPONDING TRIANGLE TO USE

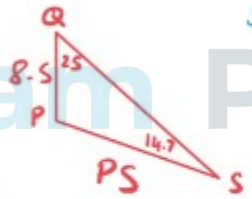


COSINE RULE $a^2 = b^2 + c^2 - 2bc\cos A$

$QS^2 = (16.2)^2 + (12.5)^2 - 2(16.2)(12.5)\cos(82.1)$

$QS = \sqrt{(16.2)^2 + (12.5)^2 - 2(16.2)(12.5)\cos(82.1)}$

$QS = 19.05321387$ km



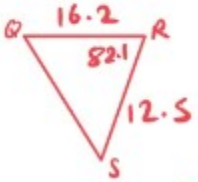
SINE RULE $\frac{a}{\sin A} = \frac{b}{\sin B}$

$\frac{PS}{\sin 25} = \frac{8.5}{\sin 14.7}$

$PS = \frac{8.5}{\sin 14.7} \times \sin 25$

$PS = 14.15622762$ km

TO FIND PR FIRST FIND $S\hat{Q}R$ USING SINE RULE

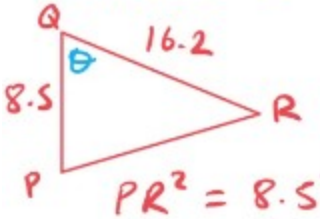


$\frac{\sin S\hat{Q}R}{12.5} = \frac{\sin 82.1}{QS}$

$S\hat{Q}R = \sin^{-1}\left(\frac{\sin 82.1}{19.05321387} \times 12.5\right)$

$S\hat{Q}R = 40.52885831$

THEN USE $\theta = P\hat{Q}S + S\hat{Q}R$ TO FIND PR



$\theta = 25 + 40.52885831$

$= 65.52885831$

$PR^2 = 8.5^2 + 16.2^2 - 2(8.5)(16.2)\cos\theta$

$PR = \sqrt{8.5^2 + 16.2^2 - 2(8.5)(16.2)\cos(65.528\ldots)}$

$PR = 14.85293632$ km

TOTAL DISTANCE = QS + SP + PR

$19.05321387 + 14.15622762 + 14.85293632$

TOTAL = 48.06237781 km

TOTAL = 48.1 km (3sf)

Question 10

(a) DRAW DIAGRAM

$\sin\theta = \frac{O}{H}$

$MN = \frac{10}{\sin 21.6}$

$MN = 27.16471892$

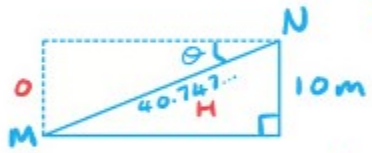
MN = 21.2 m (3sf)

(b) $LN = 1.5 \times MN$

$LN = 1.5 \times 27.16471892$

$LN = 40.74707837$

DRAW DIAGRAM DEPRESSION IS DOWN FROM HORIZONTAL



$\sin\theta = \frac{O}{H}$ ← LN

$\theta = \sin^{-1}\left(\frac{10}{LN}\right) = \sin^{-1}\left(\frac{10}{40.74707837}\right)$

$\theta = 14.20644114$

$\theta = 14.2^\circ$ (3sf)