Exam Papers Practice

3.2 Geometry of 3D Shapes

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Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.2 Geometry of 3D Shapes
Difficulty	Medium

To be used by all students preparing for DP IB Maths AA SL
Students of other boards may also find this useful

Question 1

a) Notice the right-angled triangles.

i) $AF = \sqrt{AB^2 + BF^2}$

$AF = \sqrt{72^2 + 45^2}$

$AF \approx 84.9\text{ cm}$

ii) $BH = \sqrt{BC^2 + CH^2}$

$BH = \sqrt{112^2 + 45^2}$

$BH \approx 121\text{ cm}$

iii) $AC = \sqrt{AD^2 + DC^2}$

$AC = \sqrt{72^2 + 112^2}$

$AC \approx 133\text{ cm}$

b) Notice the right-angled triangle BDG.

$BG = \sqrt{BD^2 + DG^2}$

$BD = AC = 133\text{ cm}$ $\quad DG = AE = 45$

$BG = \sqrt{133^2 + 45^2}$

$BG \approx 141\text{ cm}$

Question 2

a) Volume of a sphere

$V = \frac{4}{3}\pi r^3$ (in formula booklet)

$V = \frac{4}{3}\pi (4)^3$

$V = \frac{256}{3}\pi = 268.08257...$

$V = 2.68 \times 10^2 \text{ mm}^3$ (3sf)

b) Volume of a cylinder

$V = \pi r^2 h$ (in formula booklet)

$\frac{256}{3}\cancel{\pi} = \cancel{\pi} r^2 (16)$

$r = \sqrt{\frac{16}{3}} = 2.3094...$

$r = 2.31$ mm (3sf)

Question 3

a) Volume of a right circular cone

$V = \frac{1}{3}\pi r^2 h$ (in formula booklet)

$V = 120$ $r = 2.8$

Sub V and r into formula and rearrange for h.

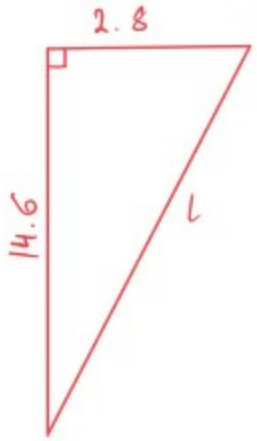
$120 = \frac{1}{3}\pi (2.8)^2 h$

$h = \frac{120}{\frac{1}{3}\pi (2.8)^2}$

$h = 14.616...$

h = 14.6 cm (3sf)

b) Notice the right-angled triangle.



$l = \sqrt{(14.6)^2 + (2.8)^2}$ (pythagoras)

$l = \sqrt{221}$

$l = 14.866...$

l = 14.9 cm (3sf)

c) Curved surface area of a cone formula

$A = \pi r l$ (in formula booklet)

$r = 2.8$ $l = 14.9$

Sub r and l into formula.

$A = \pi (2.8)(14.9)$

$A = 131.067...$

$A = 131\ cm^2$ (3sf)

Question 4

a) Volume of a cylinder formula

$V = \pi r^2 h$ (in formula booklet)

$V = 80$ $r = \frac{6.7}{2} = 3.35$

Sub in V and r into formula and rearrange for h.

$80 = \pi (3.35)^2 h$

$h = \frac{80}{\pi (3.35)^2}$

$h = 2.269...$

$h = 2.27\ cm$ (3sf)

b) Volume of a hemisphere formula.

$V = \frac{2}{3}\pi r^3$ $\left(\frac{V_{sphere}}{2}\right)$

NB the volume of a hemisphere is half the volume of a sphere with the same radius.

$V = 80 \times \frac{1}{4} = 20$

Sub V into formula and rearrange for r.

$20 = \frac{2}{3}\pi r^3$

$r = \sqrt[3]{\frac{20}{\frac{2}{3}\pi}}$

$r = 2.1215...$

r = 2.12 cm (3sf)

Question 5

a) Arc length formula

$l = \frac{\theta}{360} \times 2\pi r$ (in formula booklet)

i) $\theta = 62$ $r = 11.4$

Sub θ and r into formula.

$l = \frac{62}{360} \times 2\pi (11.4)$

$l = 12.3359...$

$l = 12.3$ cm (3sf)

ii) $\theta = 360 - 62 = 298$ $r = 11.4$

Sub θ and r into formula.

$l = \frac{298}{360} \times 2\pi (11.4)$

$l = 59.2923...$

$l = 59.3$ cm (3sf)

b) Sector area formula

$A = \frac{\theta}{360} \times \pi r^2$ (in formula booklet)

$\theta = 62$ $r = 11.4$

Sub θ and r into formula.

$A = \frac{62}{360} \times \pi (11.4)^2$

$A = 70.3151...$

$A = 70.3 \text{ cm}^2$ (3sf)

c) Volume (V) = Cross-sectional area (A) × length (l)

Cross-sectional area is the major sector OAB.

$\therefore V = \underbrace{\frac{\theta}{360} \times \pi r^2}_{\text{sector area}} \times l$

$\theta = 298$ $r = 11.4$ $l = 110$ (1.1 m = 110 cm)

Sub θ, r and l into formula.

$V = \frac{298}{360} \times \pi (11.4)^2 \times 110$

$V = 37\,176.2879...$

$V = 37\,200 \text{ cm}^3$ (3sf)

Question 6

a) Area of a triangle

$A = \frac{1}{2}bh$ (in formula booklet)

$b_{ABV} = 4.2$ $\quad h_{ABV} = \sqrt{(2.1)^2 + (10.6)^2} = \sqrt{116.77}$

$A = \frac{1}{2}(4.2)(\sqrt{116.77})$

$A = 22.6926...$

$A = 22.7 \text{ cm}^2$ (3sf)

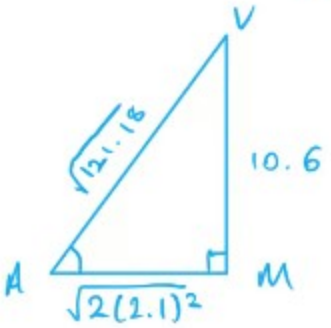
b) Find AV using pythagoras.

$AV = \sqrt{(2.1)^2 + (\sqrt{116.77})^2}$

$AV = \sqrt{121.18} = 11.008...$

$AV = 11.0$ cm (3sf)

c) We have a right-angled triangle VAM.



$\sin V\hat{A}M = \frac{10.6}{\sqrt{121.18}}$

$V\hat{A}M = \sin^{-1}\left(\frac{10.6}{\sqrt{121.18}}\right)$

$V\hat{A}M = 74.3°$ (3sf)