3.2 Stationary Waves

PHYSICS

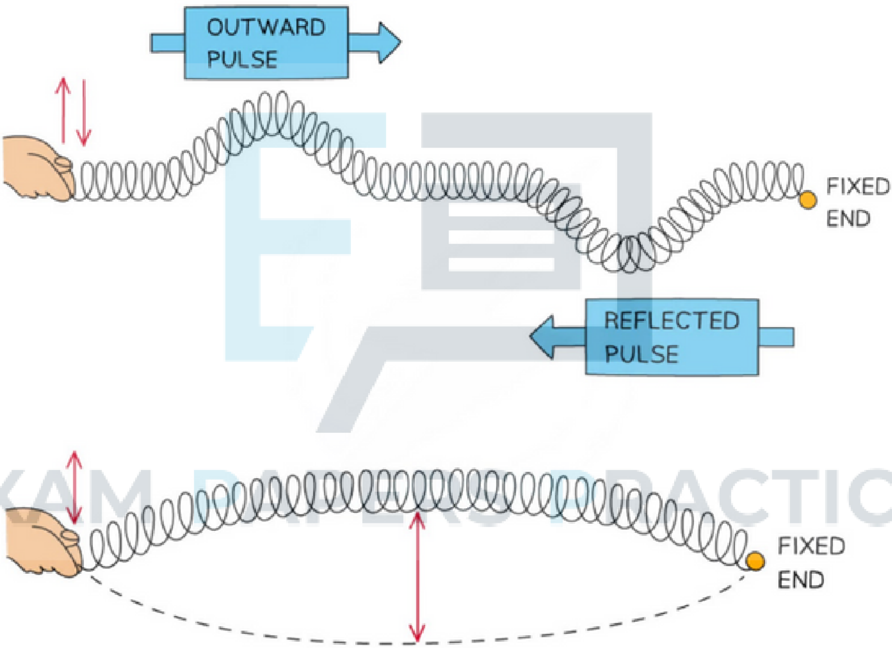
AQA A Level Revision Notes

3.2 Stationary Waves

CONTENTS

Stationary Waves

- **Standing waves** are produced by the superposition of two waves of the same frequency and amplitude travelling in **opposite directions**
- This is usually achieved by a travelling wave and its reflection
  - The superposition produces a wave pattern where the peaks and troughs do not move
- Stationary waves **store** energy, unlike progressive waves which **transfer** energy



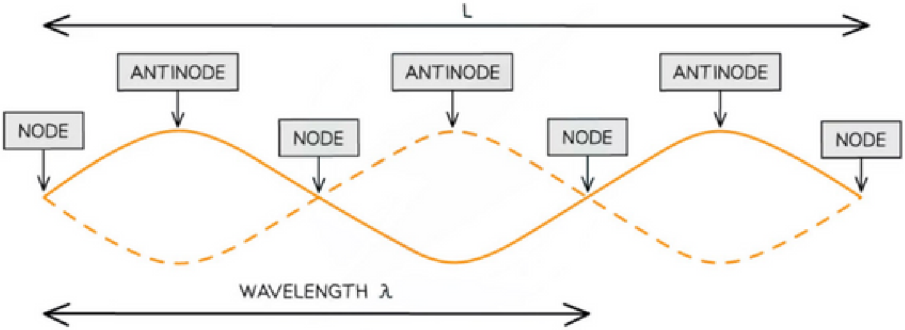
Formation of a stationary wave on a stretched spring fixed at one end

Comparing Progressive & Stationary Waves

Progressive Waves	Stationary Waves
All points have the same amplitude (in turn)	Each point has a different amplitude depending on the amount of superposition
Points exactly a wavelength apart are in phase. The phase of points within one wavelength can be between 0 to 360°	Points between nodes are in phase. Points on either side of a node are out of phase
Energy is transferred along the wave	Energy is stored, not transferred
Does not have nodes or antinodes	Has nodes and antinodes
The wave speed is the speed at which the wave moves through a medium	Each point on the wave oscillates at a different speed. The overall wave does not move

Nodes & Antinodes

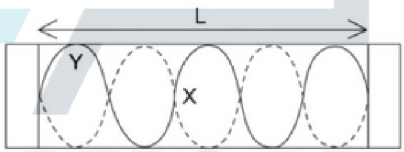
- A stationary wave is made up **nodes** and **antinodes**
  - **Nodes** are regions where there is no vibration
  - **Antinodes** are regions where the vibrations are at their maximum amplitude
- The nodes and antinodes **do not** move along the string
  - Nodes are fixed and antinodes only move in the vertical direction
- The phase difference between two points on a stationary wave are either **in phase** or out of phase
  - Points between nodes are in phase with each other
  - Points that have an **odd** number of nodes between them are out of phase
  - Points that have an **even** number of nodes between them are in phase
- The image below shows the nodes and antinodes on a snapshot of a stationary wave at a point in time



- Where:
  - L is the length of the string
  - One wavelength λ is only a portion of the length of the string

Worked Example

A stretched string is used to demonstrate a stationary wave, as shown in the diagram.



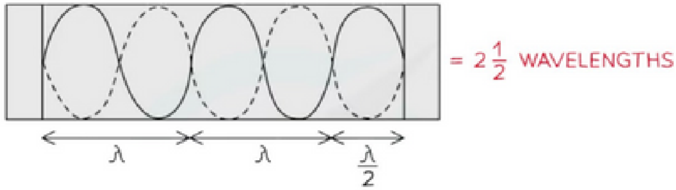
Which row in the table correctly describes the length of L and the name of X and Y?

	Length L	Point X	Point Y
A	5 wavelengths	Node	Antinode
B	$2\frac{1}{2}$ wavelengths	Antinode	Node
C	$2\frac{1}{2}$ wavelengths	Node	Antinode
D	5 wavelengths	Antinode	Node

ANSWER: C

STEP 1

CALCULATE HOW MANY WAVELENGTHS IN THE LENGTH OF THE STRING



THIS RULES OUT A AND D

STEP 2

X IS A POINT OF 0 DISPLACEMENT – A NODE

STEP 3

Y IS A POINT OF MAXIMUM DISPLACEMENT – AN ANTINODE

STEP 4

THE CORRECT ROW IS C

Exam Tip

Make sure you learn the definitions of node and antinode:

- Node = A point of minimum or no disturbance
- Antinode = A point of maximum amplitude

In exam questions, the lengths of the strings will only be in whole or half wavelengths. For example, a wavelength could be made up of 3 nodes and 2 antinodes or 2 nodes and 3 antinodes.

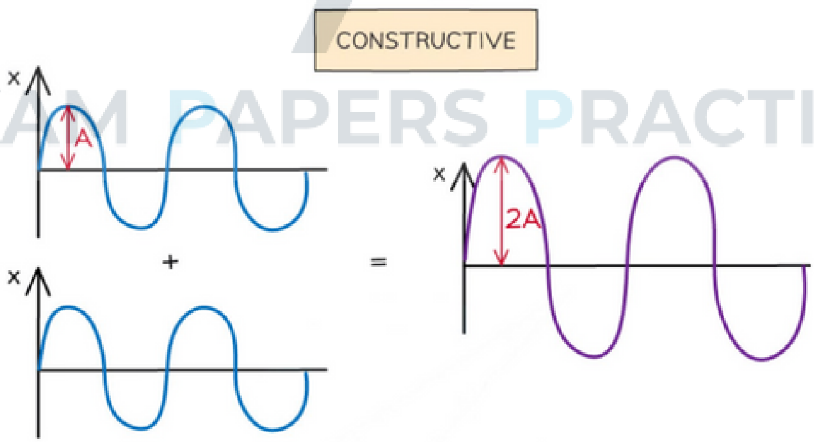
Formation of Stationary Waves

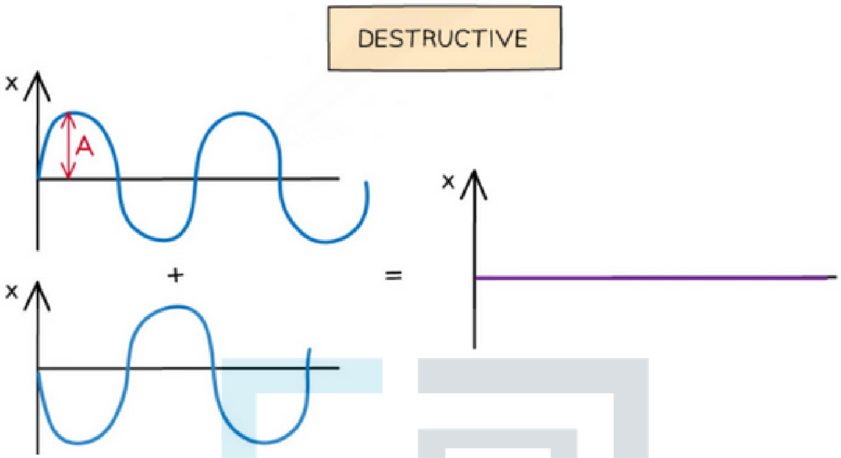
The Principle of Superposition

- The principle of superposition states:

  When two or more waves with the same frequency arrive at a point, the resultant displacement is the sum of the displacements of each wave

- This principle describes how waves that meet at a point in space interact
- When two waves with the same frequency and amplitude arrive at a point, they superpose either:
  - **In phase**, causing **constructive interference.** The peaks and troughs line up on both waves and the resultant wave has double the amplitude
  - In **anti-phase**, causing **destructive interference.** The peaks on one wave line up with the troughs of the other. The resultant wave has no amplitude





Waves in superposition can undergo constructive or destructive interference

- The principle of superposition applies to all types of waves i.e. transverse and longitudinal, progressive and stationary

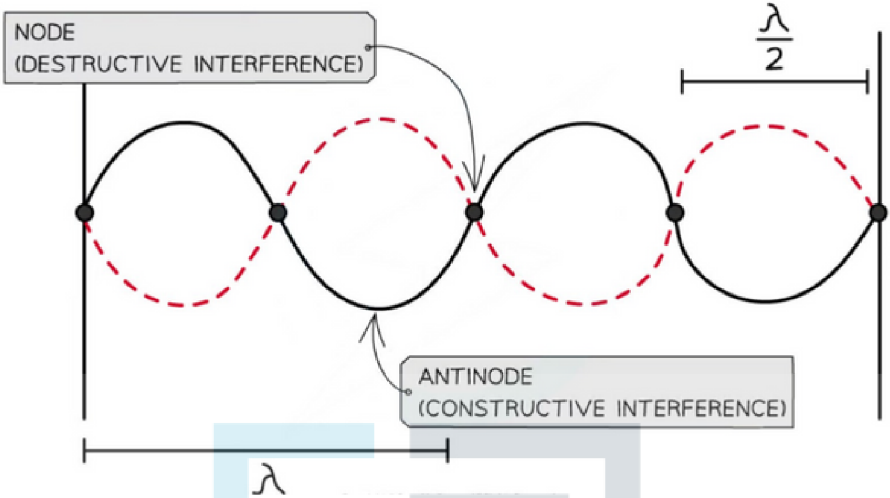
The Formation of Stationary Waves

- A stationary wave is formed when:

  Two waves travelling in opposite directions along the same line with the same frequency superpose

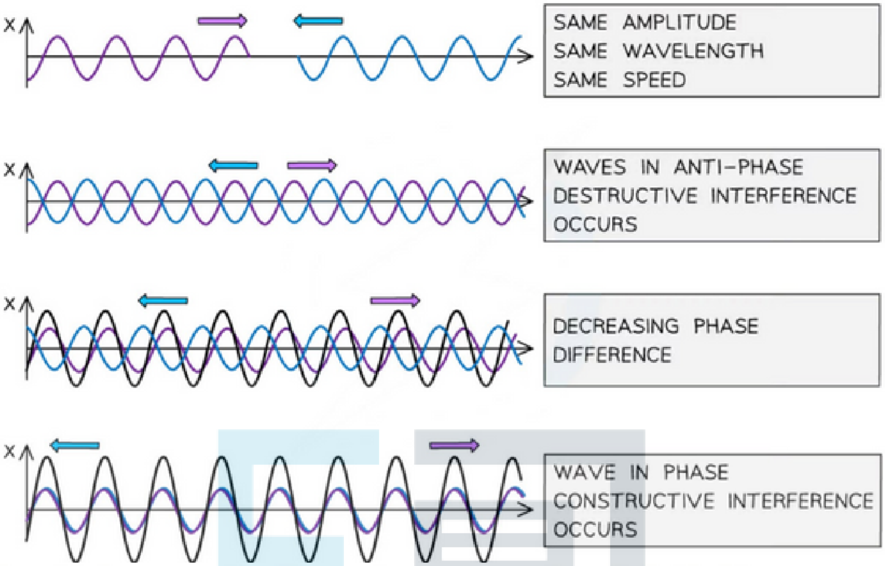
- The waves must have:
  - The same wavelength)
  - A similar amplitude

- As a result of superposition, a resultant wave is produced



Nodes and antinodes are a result of destructive and constructive interference respectively

- At the **nodes**:
  - The waves are in anti-phase meaning destructive interference occurs
  - This causes the two waves to cancel each other out
- At the **antinodes**:
  - The waves are in phase meaning constructive interference occurs
  - This causes the waves to add together
- Each point on the stationary wave has a **different** amplitude (unlike a progressive / travelling wave where each point has the **same** amplitude)



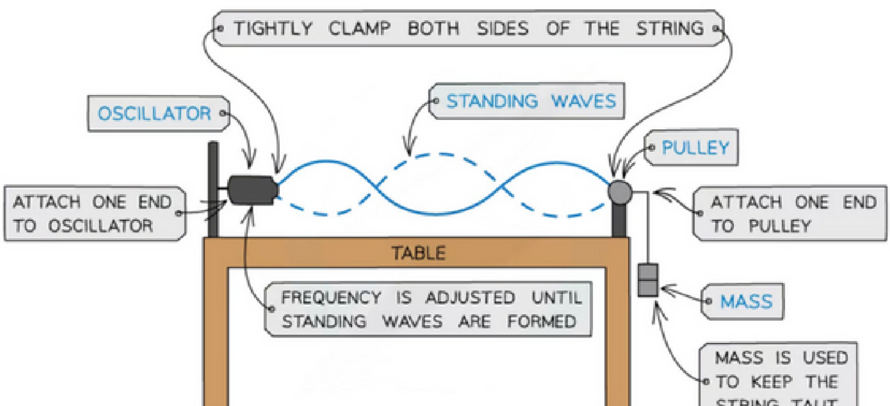
A graphical representation of how stationary waves are formed - the black line represents the resulting wave

Examples of Stationary Waves

Stretched Strings

- Vibrations caused by stationary waves on a stretched string produce **sound**
  - This is how stringed instruments, such as guitars or violins, work

- This can be demonstrated by a length of string under tension fixed at one end and vibrations made by an oscillator:

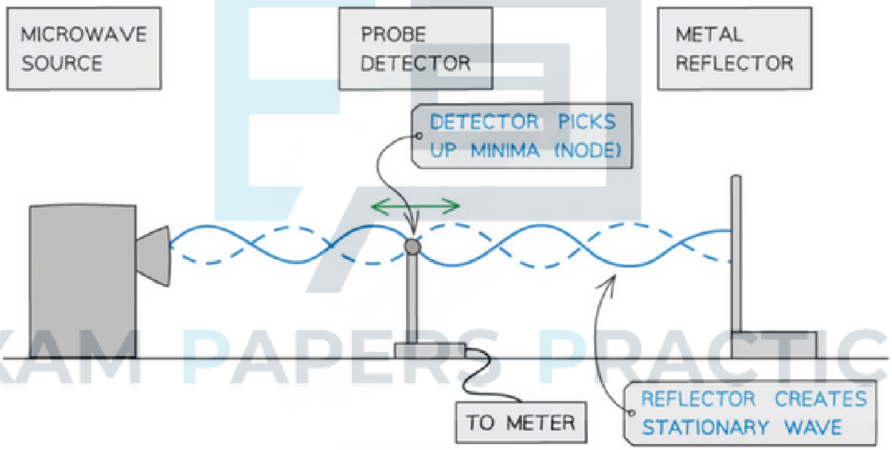


Stationary wave on a stretched string

- At specific frequencies, known as **resonant frequencies**, a whole number of half wavelengths will fit on the length of the string
- As the resonant frequencies of the oscillator are achieved, standing waves with different numbers of minima (nodes) and maxima (antinodes) form

Microwaves

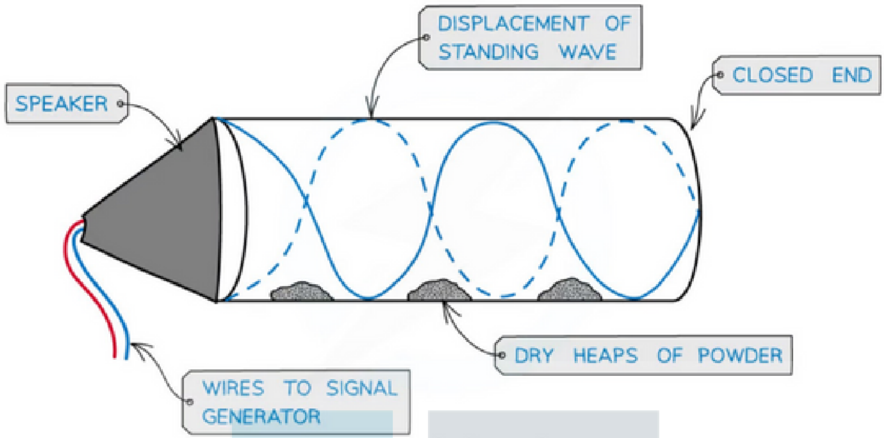
- A microwave source is placed in line with a reflecting plate and a small detector between the two
- The reflector can be moved to and from the source to vary the stationary wave pattern formed
- By moving the detector, it can pick up the minima (nodes) and maxima (antinodes) of the stationary wave pattern



Using microwaves to demonstrate stationary waves

Sound Waves

- Sound waves can be produced as a result of the formation of stationary waves inside an air column
    - This is how musical instruments, such as clarinets and organs, work
- This can be demonstrated by placing a fine powder inside the air column and a loudspeaker at the open end
- At certain frequencies, the powder forms evenly spaced heaps along the tube, showing where there is zero disturbance as a result of the nodes of the stationary wave



Stationary wave in an air column

- In order to produce a stationary wave, there must be a minima (node) at one end and a maxima (antinode) at the end with the loudspeaker

Exam Tip

Always refer back to the experiment or scenario in an exam question e.g. the wave produced by a **loudspeaker** reflects at the end of a **tube**. This reflected wave, with the same frequency, overlaps the initial wave to create a stationary wave.

3.2.3 Harmonics

Harmonics

- Stationary waves can have different wave patterns, known as **harmonics**
  - These depend on the **frequency** of the vibration and the **situation** in which they are created
- These harmonics can be observed on a string with two fixed ends
- As the frequency is increased, more harmonics begin to appear

Harmonics on a String

- When a stationary wave, such as a vibrating string, is fixed at both ends, the simplest wave pattern is a single loop made up of two nodes and an antinode
  - This is called the **first harmonic** or **fundamental frequency**

- The particular frequencies (i.e. resonant frequencies) of stationary waves formed depend on the length of the string *L* and the wave speed *v*
- The frequencies can be calculated from the string length and wave equation
- For a string of length *L*, the wavelength of the lowest harmonic is 2*L*
  - This is because there is only one loop of the stationary wave, which is a half wavelength

- Therefore, the frequency is equal to:

$$f_1 = \frac{v}{\lambda} = \frac{v}{2L}$$

- The second harmonic has **three nodes** and **two antinodes**
- The wavelength is *L* and the frequency is equal to:

$$f_2 = \frac{v}{\lambda} = \frac{v}{L}$$

$$f_2 = 2f_1$$

- The third harmonic has **four nodes** and **three antinodes**
- The wavelength is 2L / 3 and the frequency is equal to:

$$f_3 = \frac{v}{\lambda} = \frac{3v}{2L}$$

$$f_3 = 3f_1$$

- The nth harmonic has n antinodes and n + 1 nodes
- The wavelengths and frequencies of the first three harmonics can be summarised as follows:

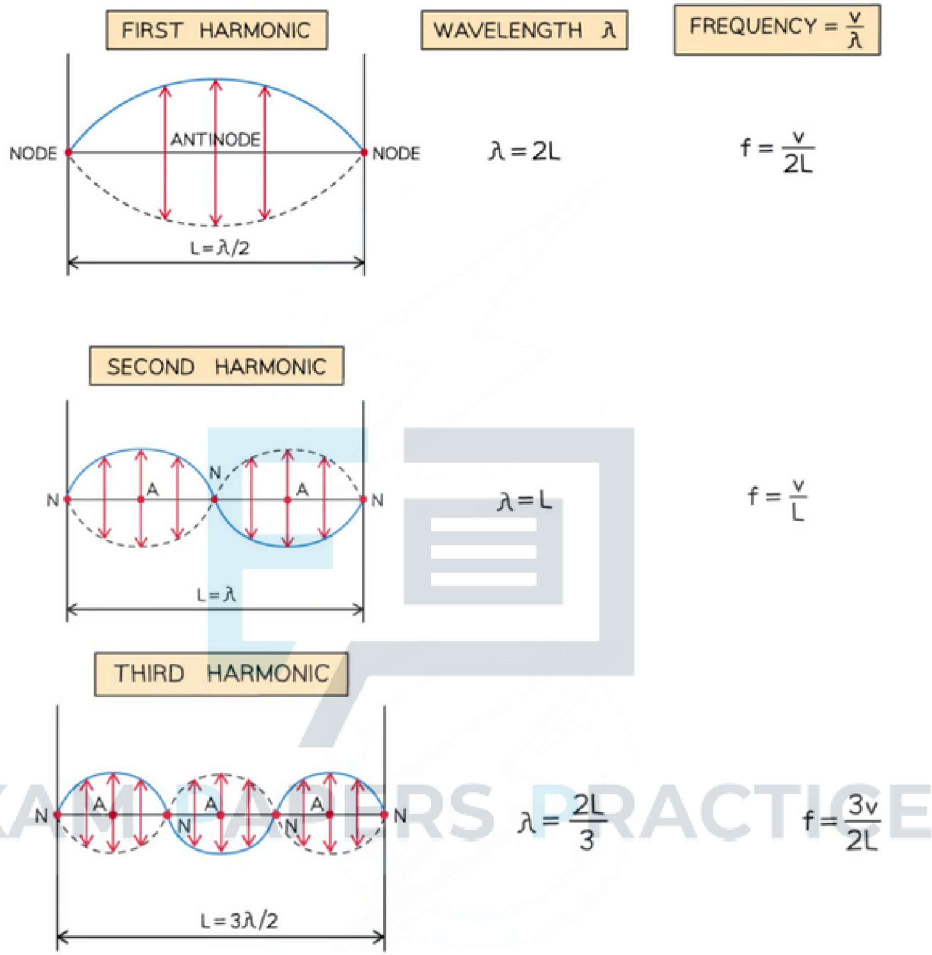


Diagram showing the first three modes of vibration of a stretched string with corresponding frequencies

Worked Example

A stationary wave made from a string vibrating in the third harmonic has a frequency of 150 Hz. Calculate the frequency of the fifth harmonic

Step 1: Calculate the frequency of the first harmonic

$$f_3 = 3 f_1$$

$$f_1 = f_3 \div 3 = 150 \div 3 = 50 \text{ Hz}$$

Step 2: Calculate the frequency of the fifth harmonic

$$f_5 = 5 f_1$$

$$f_5 = 5 \times 50 = 250 \text{ Hz}$$

Exam Tip

Make sure to match the correct wavelength with the harmonic asked for in the question:

- The **first** harmonic (or n = 1) is the lowest frequency with half or quarter of a wavelength
- The **second** harmonic (or n = 2) is a full wavelength

Frequency of the First Harmonic

- The speed of a wave travelling along a string with two fixed ends is given by:

$$v = \sqrt{\frac{T}{\mu}}$$

- Where:
  - T = tension in the string (N)
  - μ = mass per unit length of the string (kg m^{-1})

- For the first harmonic of a stationary wave of length L, the wavelength, $\lambda = 2L$
- Therefore, according to the wave equation, the speed of the stationary wave is:

$$\mathbf{v = f\lambda = f \times 2L}$$

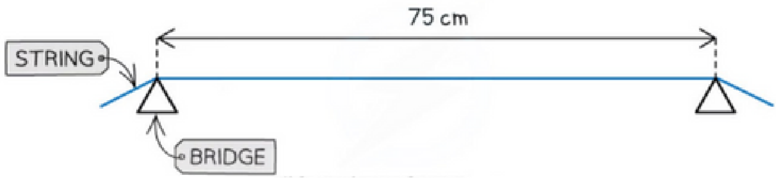
- Combining these two equations leads to the frequency of the first harmonic:

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

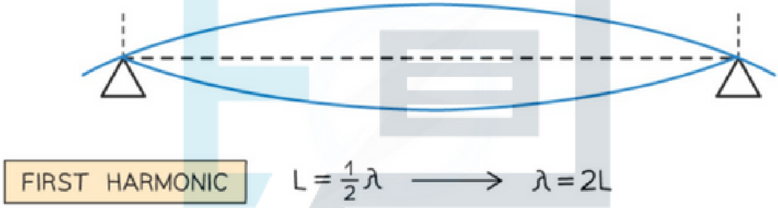
- Where:
  - f = frequency (Hz)
  - L = the length of the string (m)

Worked Example

A guitar string of mass 3.2 g and length 90 cm is fixed onto a guitar. The string is tightened to a tension of 65 N between two bridges at a distance of 75 cm.



Calculate the frequency of the first harmonic produced when the string is plucked.



Step 1: Write down the known quantities

- Tension, T = 65 N
- String length, L = 75 cm = 75×10^{-2} m
- Mass per unit length, $\mu = \frac{3.2 \times 10^{-3}}{90 \times 10^{-2}} = 3.56 \times 10^{-3}$ kg m^{-1}

Step 2: Write down the required equation

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}}$$

Step 3: Calculate the frequency of the first harmonic

$$f = \frac{1}{2 \times (75 \times 10^{-2})} \sqrt{\frac{65}{3.56 \times 10^{-3}}} = 90.139$$

The first harmonic occurs at frequency, f = 90 Hz

3.2.4 Required Practical: Investigating Stationary Waves

Required Practical: Investigating Stationary Waves

Aims of the Experiment

- The overall aim of the experiment is to measure how the frequency of the first harmonic is affected by changing one of the following variables:
  - The length of the string
  - The tension in the string
  - Strings with different values of mass per unit length

Variables:

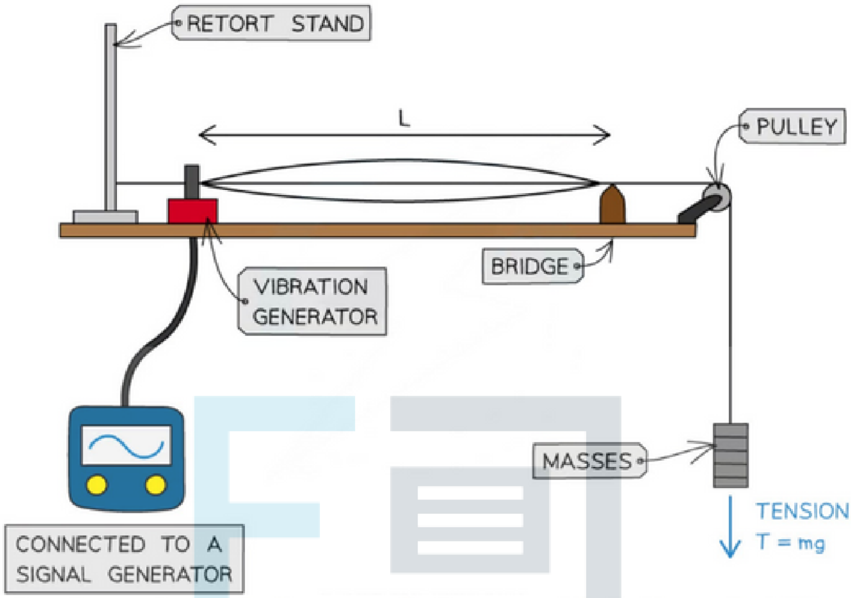
- Independent variable = either length, tension, or mass per unit length
- Dependent variable = frequency of the first harmonic
- Control variables
  - If length is varied = same masses attached (tension), same string (mass per unit length)
  - If tension is varied = same length of the string, same string (mass per unit length)
  - If mass per unit length is varied = same masses attached (tension), same length of the string

Equipment List

Apparatus	Purpose
Signal generator	Used to operate the vibration generator and measure the frequency of the first harmonic
Vibration generator	Connected to the signal generator to produce the stationary wave
Retort stand	To provide a stable fixed end on the table
G clamp or 2 kg mass	To place on the retort stand to stabilise apparatus
2.0 m of string	Used to observe the stationary wave
Pulley	To allow the masses to hang vertically, and introduces less friction than the edge of the table
Wooden bridge	To provide the other fixed end which can vary the length of the string
Mass hanger + 100 g masses	To hang from the pulley to vary the tension in the string
Metre ruler	To measure the length of the string
Top-pan balance	To measure the mass of the string

- Resolution of measuring equipment:
  - Metre ruler = 1 mm
  - Signal generator ~ 10 nHz
  - Top-pan balance = 0.005 g

Method



The setup of apparatus required to measure the frequency of the first harmonic at different values of length, tension, or mass per unit length

This method is an example of the procedure for varying the length of the string with the frequency – this is just one possible relationship that can be tested

1. Set up the apparatus by attaching one end of the string to the vibration generator and pass the other end over the bench pulley and attach the mass hanger
2. Adjust the position of the bridge so that the length *L* is measured from the vibration generator to the bridge using a metre ruler
3. Turn on the signal generator to set the string oscillating
4. Increase the frequency of the vibration generator until the first harmonic is observed and read the frequency that this occurs at
5. Repeat the procedure with different lengths
6. Repeat the frequency readings at least two more times and take the average of these measurements
7. Measure the tension in the string using $T = mg$
   - Where m is the amount of mass attached to the string and g is the gravitational field strength on Earth (9.81 N kg^{-1})
8. Measure the mass per unit length of the string μ = mass of string ÷ length of string
   - Simply take a known length of the string (1 m is ideal) and measure its mass on a balance

- An example of a table with some possible string lengths might look like this:

LENGTH OF THE STRING

FREQUENCY OF THE FIRST HARMONIC

L / m	f / Hz 1st READING	f / Hz 2nd READING	f / Hz 3rd READING	f / Hz MEAN
0.2				
0.4				
0.6				
0.8				
1.0				
1.2				
1.4				
1.6				

Before conducting an experiment, a table must be set up to detail of what measurements are to be made

Analysing the Results

- For the first harmonic, wavelength, $\lambda = 2L$
- So, the speed of the stationary wave is:

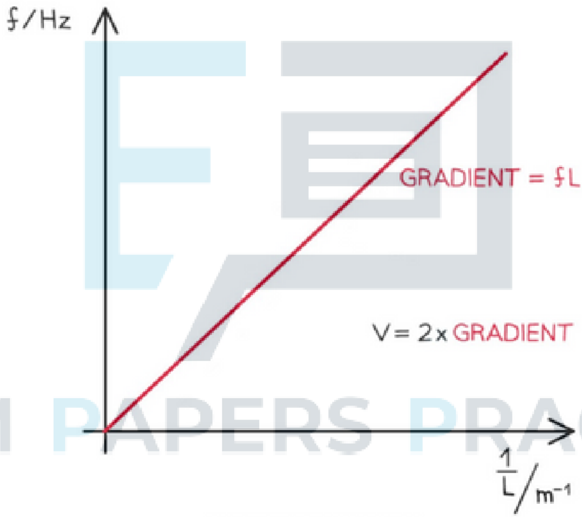
$$v = f\lambda = f \times 2L$$

- Rearranging for frequency, *f*:

$$f = \frac{v}{2L} = \frac{v}{2}\left(\frac{1}{L}\right)$$

- Comparing this to the equation of a straight line: $y = mx$
  - $y = f$ (Hz)
  - $x = 1/L$ (m^{-1})
  - Gradient = $v/2$ ($m\ s^{-1}$)

1. Plot a graph of the mean values of f against $1/L$
2. Draw a line of best fit and calculate the gradient
3. Work out the wave speed, which will be 2 × gradient:



If the frequency is plotted again the inverse of the length, the velocity is twice the gradient of the graph

- Verify the wave speed of the travelling waves using the equation:

$$v = \sqrt{\frac{T}{\mu}}$$

- Where:
  - T = tension (N)
  - μ = mass per unit length (kg m^{-1})

- Assess the uncertainties in the measurements of length and the frequency and carry out calculations to determine the uncertainty in the wave speed

Evaluating the Experiment

Systematic errors:

- An oscilloscope can be used to verify the signal generator's readings
- The signal generator should be left for about 20 minutes to stabilise
- The measurements would have a greater resolution if the length used is as large as possible, or as many half-wavelengths as possible
  - This means measurements should span a suitable range, for example, 20 cm intervals over at least 1.0 m

Random errors:

- The sharpness of resonance leads to the biggest problem in deciding when the first harmonic is achieved
  - This can be resolved by adjusting the frequency while looking closely at a node. This is a technique to gain the largest response
  - Looking at the amplitude is likely to be less reliable since the wave will be moving very fast

- When taking repeat measurements of the frequency, the best procedure is as follows:
  - Determine the frequency of the first harmonic when the largest vibration is observed and note down the frequency at this point
  - Increase the frequency and then gradually reduce it until the first harmonic is observed again and note down the frequency of this
  - If taking three repeat readings, repeat this procedure again
  - Average the three readings and move onto the next measurement

Safety Considerations

- Use a rubber string instead of a metal wire, in case it snaps under tension
- If using a metal wire, wear goggles to protect the eyes in case it snaps
- Stand well away from the masses in case they fall onto the floor
- Place a crash mat or any soft surface under the masses to break their fall

Worked Example

A student investigates the relationship between the frequency of the stationary waves on a wire and the tension in the wire. The tension is varied by adding masses to a hanger which is attached to a pulley over one end of a table. The student records the following data:

- Mass of the wire = 0.16 g
- Length of the wire weighed = 1.0 m
- Distance between the fixed ends = 0.4 m

Mass on hanger (kg)	Frequency (Hz) 1st reading	Frequency (Hz) 2nd reading	Frequency (Hz) mean
0.20	140	135	
0.40	195	200	
0.60	240	240	
0.80	280	278	
1.00	310	310	
1.20	340	340	
1.40	365	368	
1.60	392	390	

Calculate the frequency using the relation between frequency, length, tension and mass per unit length. Evaluate the percentage uncertainty in these values.

Step 1: Calculate the mean values of frequency

- The mean readings are: 138 Hz; 198 Hz; 240 Hz; 279 Hz; 310 Hz; 340 Hz; 367 Hz; 391 Hz

Step 2: Work out the tensions and mass per unit length

- Use the equation to calculate the frequency for a mass of 0.20 kg:

$$\mu = \frac{0.16 \times 10^{-3}}{1.0} = 0.16 \times 10^{-3} \text{ kg m}^{-1}$$

$$T = mg = 0.20 \times 9.81 = 1.96 \text{ N}$$

$$f = \frac{1}{2L}\sqrt{\frac{T}{\mu}} = \frac{1}{2 \times 0.4}\sqrt{\frac{1.96}{0.16 \times 10^{-3}}} = 138.3 = 140 \text{ Hz (2 s.f.)}$$

- Working in the same way, the frequencies for the remaining lengths are: 200 Hz; 240 Hz; 280 Hz; 310 Hz; 340 Hz; 370 Hz; 390 Hz

Step 3: Determine the uncertainties in each quantity

Mass on hanger

- The maximum uncertainty is likely to be 0.005 kg
- Therefore, % uncertainty in measurement of 0.2 kg is $\frac{0.005}{0.20} \times 100\% = 2.5\%$
  - Since each mass has an uncertainty of 0.005 kg, these are added together as the masses increase, so each mass has the same percentage uncertainty i.e. $\frac{0.01}{0.40} \times 100\% = 2.5\%$

Length

- Resolution of a metre ruler is 1 mm, but errors of up to 1 cm are possible in this set up
- Therefore, % uncertainty in measurement of 0.4 m is $\frac{0.01}{0.4} \times 100\% = 2.5\%$

Mass per unit length

- The resolution of a metre ruler is 1 mm and the resolution of a top-pan balance is 0.005 g
- Therefore the uncertainty in the mass per unit length:
    - $\frac{\Delta\mu}{\mu} = \frac{\Delta m}{m} + \frac{\Delta l}{l}$
    - $\Delta\mu = 0.16 \times \left(\frac{0.005}{0.16} + \frac{0.001}{1.0}\right) = 0.005$ g/m
- Therefore, % uncertainty in measurement of 0.16 g/m is $\frac{0.005}{0.16} \times 100\% = 3\%$

Step 4: Determine the uncertainties in the mean frequency values

- Since tension (proportional to the mass on the hanger) and mass per unit length are to the power of $\frac{1}{2}$, their % uncertainties are halved
- Maximum % uncertainty in calculated frequencies would be the % uncertainty in each value added together:
    - % uncertainty in frequency = 2.5 % + 1.25% + 1.5% = 5.25%