3.2 Geometry of 3D Shapes

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Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.2 Geometry of 3D Shapes
Difficulty	Medium

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful

Question 1

a) Circle circumference formula

$C = 2\pi r$ (in formula booklet)

$r = \frac{1}{2}$ height

$r = \frac{1}{2}(2286)$

$r = 1143$

Sub r into formula.

$C = 2\pi(1143)$

$C = 7181.68...$

$C = 7180$ (3sf)

$C = 7.18 \times 10^3$ mm

b) Surface area of a sphere formula

$A = 4\pi r^2$ (in formula booklet)

$r = 1143$

Sub r into formula.

$A = 4\pi(1143)^2$

$A = 16\ 417\ 322.32...$

$A = 16\ 400\ 000$ (3sf)

$A = 1.64 \times 10^7 mm^2$

c) Volume of a sphere formula

$V = \frac{4}{3}\pi r^3$ (in formula booklet)

$r = 1143$

Sub r into formula.

$V = \frac{4}{3}\pi(1143)^3$

$V = 6\ 254\ 999\ 804.97$

$V = 6\ 250\ 000\ 000$ (3sf)

$V = 6.25 \times 10^9\ mm^3$

Question 2 a) Volume of a right circular cone

$V = \frac{1}{3}\pi r^2 h$ (in formula booklet)

$V = 120$ $r = 2.8$

Sub V and r into formula and rearrange for h.

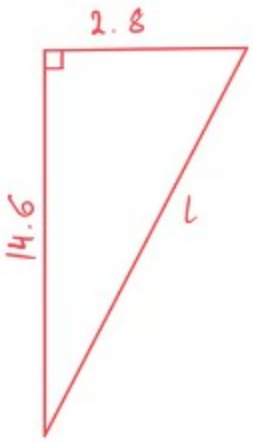
$120 = \frac{1}{3}\pi (2.8)^2 h$

$h = \frac{120}{\frac{1}{3}\pi (2.8)^2}$

$h = 14.616...$

$h = 14.6\ cm$ (3sf)

b) Notice the right-angled triangle.



$l = \sqrt{(14.6)^2 + (2.8)^2}$ (pythagoras)

$l = \sqrt{221}$

$l = 14.866...$

$l = 14.9$ cm (3sf)

c) Curved surface area of a cone formula

$A = \pi r l$ (in formula booklet)

$r = 2.8$ $l = 14.9$

Sub r and l into formula.

$A = \pi (2.8)(14.9)$

$A = 131.067...$

$A = 131$ cm^2 (3sf)

Question 3

a) Volume of a cylinder formula

$V = \pi r^2 h$ (in formula booklet)

$V = 80$ $\quad r = \frac{6.7}{2} = 3.35$

Sub in V and r into formula and rearrange for h.

$80 = \pi (3.35)^2 h$

$h = \frac{80}{\pi (3.35)^2}$

$h = 2.269...$

$h = 2.27$ cm (3sf)

b) Volume of a hemisphere formula.

$V = \frac{2}{3} \pi r^3$ $\quad \left(\frac{V_{sphere}}{2}\right)$

NB the volume of a hemisphere is half the volume of a sphere with the same radius.

$V = 80 \times \frac{1}{4} = 20$

Sub V into formula and rearrange for r.

$20 = \frac{2}{3} \pi r^3$

$r = \sqrt[3]{\frac{20}{\frac{2}{3}\pi}}$

$r = 2.1215...$

$r = 2.12$ cm (3sf)

Question 4

a) Arc length formula

$l = \frac{\theta}{360} \times 2\pi r$ (in formula booklet)

i) $\theta = 62$ $r = 11.4$

Sub θ and r into formula.

$l = \frac{62}{360} \times 2\pi (11.4)$

$l = 12.3359...$

$l = 12.3$ cm (3sf)

ii) $\theta = 360 - 62 = 298$ $r = 11.4$

Sub θ and r into formula.

$l = \frac{298}{360} \times 2\pi (11.4)$

$l = 59.2923...$

$l = 59.3$ cm (3sf)

b) Sector area formula

$A = \frac{\theta}{360} \times \pi r^2$ (in formula booklet)

$\theta = 62$ $r = 11.4$

Sub θ and r into formula.

$A = \frac{62}{360} \times \pi (11.4)^2$

$A = 70.3151...$

$A = 70.3 \text{ cm}^2$ (3sf)

c) Volume (V) = Cross-sectional area (A) × length (l)

Cross-sectional area is the major sector OAB.

$\therefore V = \underbrace{\frac{\theta}{360}}_{\text{sector area}} \times \pi r^2 \times l$

$\theta = 298 \quad r = 11.4 \quad l = 110 \quad (1.1\text{ m} = 110\text{ cm})$

Sub θ, r and l into formula.

$V = \frac{298}{360} \times \pi (11.4)^2 \times 110$

$V = 37\,176.2879...$

$V = 37\,200\text{ cm}^3$ (3sf)

Question 5

a) Surface area of a cylinder formula

$^*A = \underbrace{2\pi rh}_{\text{curved surface area}} + \underbrace{2\pi r^2}_{2 \times \text{circular ends}}$

$r = \frac{28}{2} = 14 \quad h = 37$

Sub r and h into formula.

$A = 2\pi (14)(37) + 2\pi (14)^2$

$A = 1428\pi$

$A = 4486.19...$

$A = 4490\text{ cm}^2$ (3sf)

* Curved surface area and circle area formula are in the formula booklet.

b) Surface area of a cuboid formula

$A = 2lw + 2lh + 2wh$

$A = 4490 \quad w = 28 \quad h = 37 \quad l = x$

Sub A, w and h into formula and solve for x on your GDC.

$4490 = 2x(28) + 2x(37) + 2(28)(37)$

$x = 18.6\text{ cm}$

Question 6

a) Volume of a sphere formula

$V = \frac{4}{3}\pi r^3$ (in formula booklet)

$r = 1.84$

Sub r into formula.

$V = \frac{4}{3}\pi(1.84)^3$

$V = 26.094...$

$V = 26.1\ m^3$ (3sf)

b) Let V_c = cooled volume and

r_c = cooled radius.

Method 1

$$0.99 = \frac{V_c}{V} = \frac{\frac{4}{3}\pi r_c^3}{\frac{4}{3}\pi (1.84)^3}$$

$$r_c = \sqrt[3]{0.99} \times 1.84$$

$$r_c = 1.8338...$$

$r_c = 1.83$ m (3sf)

Method 2

$$V_c = 26.1 \times 0.99$$
$$\approx 25.8 \text{ m}^3$$

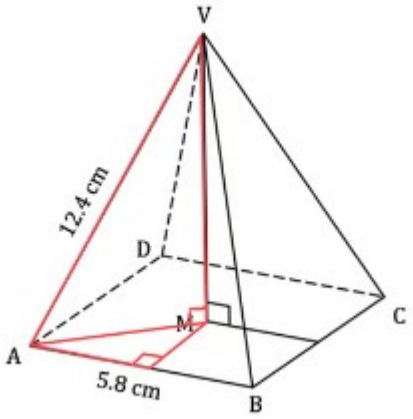
$$25.8 = \frac{4}{3}\pi r_c^3$$

$$r_c = \sqrt[3]{\frac{25.8}{\frac{4}{3}\pi}}$$

$$r_c = 1.8338...$$

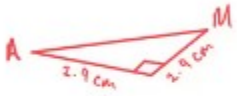
$r_c = 1.83$ m (3sf)

Question 7



a) Notice the right-angled triangles.

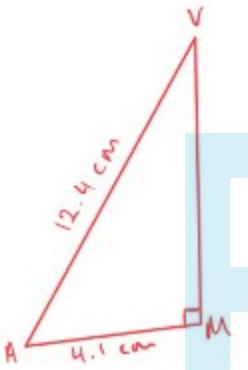
First, we need to find AM.



$AM = \sqrt{2.9^2 + 2.9^2}$ $\left(\frac{5.8}{2} = 2.9\right)$

$AM = \sqrt{16.82}$ $(AM^2 = 16.82)$

Use AM to find VM.



$VM = \sqrt{12.4^2 - 16.82}$

$VM = \sqrt{136.94}$

$VM = 11.7021...$

VM = 11.7 cm (3sf)

b) Volume of a right pyramid formula

$V = \frac{1}{3}Ah$ (in formula booklet)

where A is the area of the base.

$A = 5.8^2$ $h = 11.7$

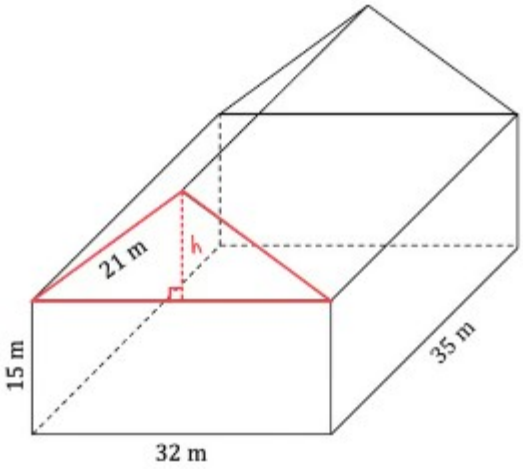
Sub A and h into formula.

$V = \frac{1}{3}(5.8^2)(11.7)$

$V = 131.196$

$V = 131\ cm^3$ (3sf)

Question 8



Area = 2 (warehouse ends) + 2 (warehouse sides)
+ 2 (roof slanted sides) + 2(roof ends)

Roof ends are isosceles triangles.

Area of a triangle formula

$A = \frac{1}{2}bh$ (in formula booklet)

b is the base, h is the perpendicular height

$b = 32$ $h = \sqrt{21^2 - 16^2}$ (pythagoras)

Sub b and h into formula and sum all the surfaces.

$A = 2(15 \times 32) + 2(15 \times 35) + 2(35 \times 21)$
$+ 2\left(\frac{1}{2} \times 32 \times \sqrt{21^2 - 16^2}\right)$

$A = 3915.247...$

$A = 3915\ m^2$ (nearest m^2)

Question 9

a) Distance between two points formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ (in formula booklet)

A(11, 14, 4) S(0, 0, 0)

Sub A and S into formula.

$d_A = \sqrt{11^2 + 14^2 + 4^2}$

$d_A = \sqrt{333}$

$d_A = 18.2$ km (3sf)

B(4, 17, 3) S(0, 0, 0)

Sub B and S into formula.

$d_B = \sqrt{4^2 + 17^2 + 3^2}$

$d_B = \sqrt{314}$

$d_B = 17.7$ km. (3sf)

∴ Plane A is farthest from Sharp airport.

N.B No need to sub S into formula as all values are zero.

b) Distance between two points formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ (in formula booklet)

A(11, 14, 4) B(4, 17, 3)

Sub A and B into formula.

$d = \sqrt{(11-4)^2 + (14-17)^2 + (4-3)^2}$

$d = \sqrt{59}$

$d = 7.68$ km (3sf)

c) Distance between two points formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ (in formula booklet)

A (-8, 20, 5) S (0, 0, 0) K (-15, 1, 0)

Distance to Sharp airport

Sub A and S into formula.

$d_S = \sqrt{(-8)^2 + 20^2 + 5^2}$

$d_S = \sqrt{489}$

$d_S = 22.1$ km (3sf)

Distance to Kit airport

Sub A and K into formula.

$d_K = \sqrt{(-8 - (-15))^2 + (20 - 1)^2 + 5^2}$

$d_K = \sqrt{435}$

$d_K = 20.9$ km (3sf)

∴ The pilot should land at Kit airport.