



EXAM PAPERS PRACTICE

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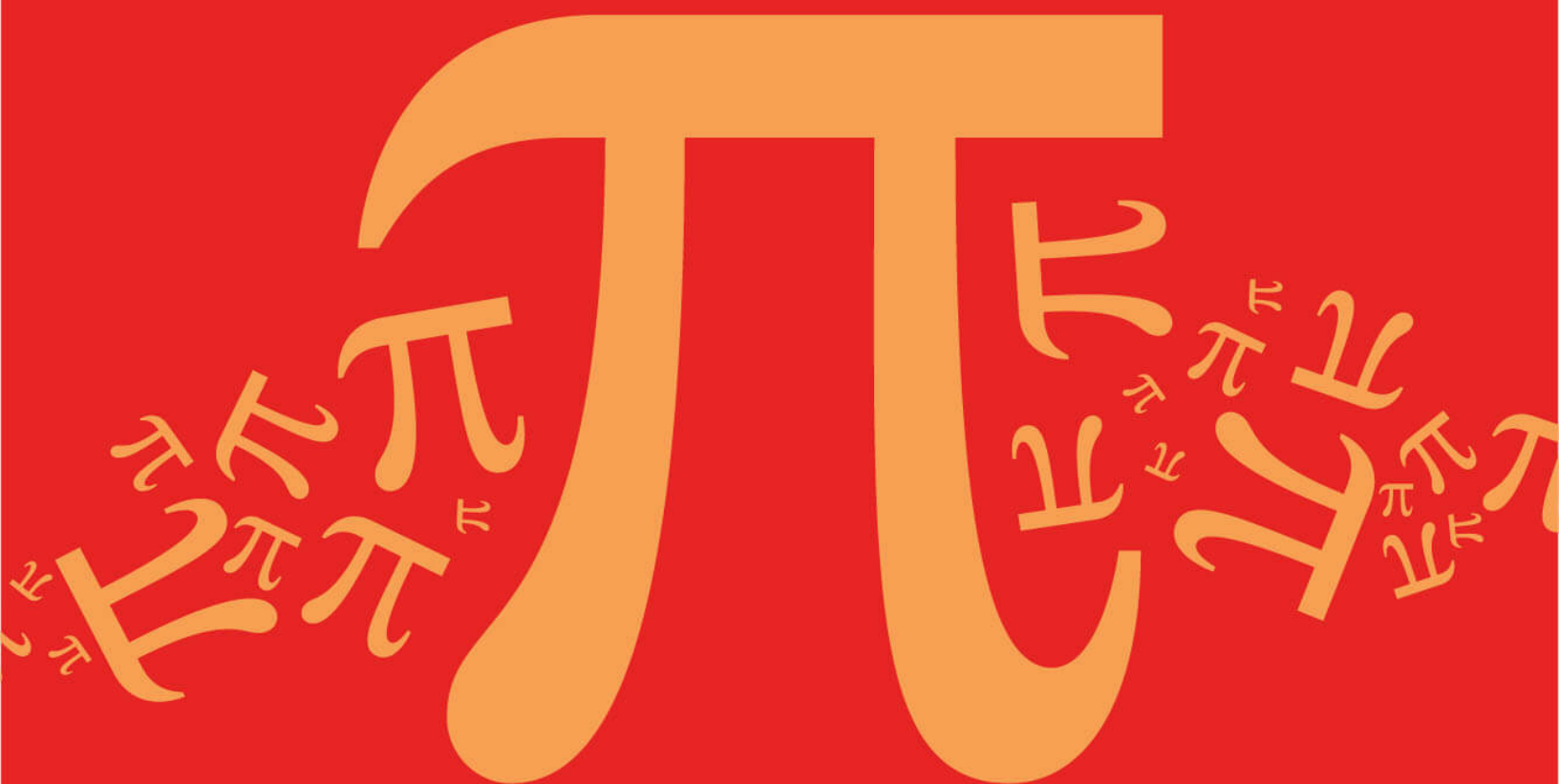
Practice questions created by actual examiners and assessment experts

Detailed mark scheme

Suitable for all boards

Designed to test your ability and thoroughly prepare you

3.2 Geometry of 3D Shapes



IB Maths - Revision Notes

3.2.1 3D Coordinate Geometry

3D Coordinate Geometry

How does the 3D coordinate system work?

- In three-dimensional space we can label where any object is using the x - y - z coordinate system
- In the 3D cartesian system, the x - and y - axes usually represent lateral space (length and width) and the z -axis represents vertical height

What can we do with 3D coordinates?

- If we have two points with coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) then we should be able to find:
 - The **midpoint** of the two points
 - The **distance** between the two points
- If the coordinates are labelled A and B then the line segment between them is written with the notation [AB]

How do I find the midpoint of two points in 3D?

- The midpoint is the **average (middle) point**
 - It can be found by finding the middle of the x -coordinates and the middle of the y -coordinates
- The coordinates of the midpoint will be

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

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- This is given in the formula booklet, you do not need to remember it

How do I find the distance between two points in 3D?

- The distance between two points with coordinates (x_1, y_1, z_1) and (x_2, y_2, z_2) can be found using the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

- This is given in the formula booklet, you do not need to remember it



Worked example

The points A and B have coordinates $(-2, 1, 5)$ and $(4, -3, 2)$ respectively.

- i) Calculate the distance of the line segment AB .

Formula for the distance of a line segment:

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

↙ in formula booklet

$$A: (-2, 1, 5) \quad B: (4, -3, 2)$$

$\begin{matrix} \uparrow & \uparrow & \uparrow \\ x_1 & y_1 & z_1 \end{matrix} \quad \begin{matrix} \uparrow & \uparrow & \uparrow \\ x_2 & y_2 & z_2 \end{matrix}$

Substitute:

$$\begin{aligned} d &= \sqrt{(-2 - 4)^2 + (1 - (-3))^2 + (5 - 2)^2} \\ &= \sqrt{(-6)^2 + 4^2 + 3^2} \\ &= \sqrt{36 + 16 + 9} \\ &= \sqrt{61} \end{aligned}$$

$d = 7.81 \text{ units (3 sf)}$

- ii) Find the midpoint of $[AB]$.

Formula for the midpoint of a line segment:

$$MP = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$$

↙ in formula booklet

$$A: (-2, 1, 5) \quad B: (4, -3, 2)$$

$\begin{matrix} \uparrow & \uparrow & \uparrow \\ x_1 & y_1 & z_1 \end{matrix} \quad \begin{matrix} \uparrow & \uparrow & \uparrow \\ x_2 & y_2 & z_2 \end{matrix}$

Substitute:

$$\begin{aligned} MP &= \left(\frac{-2 + 4}{2}, \frac{1 + (-3)}{2}, \frac{5 + 2}{2} \right) \\ &= \left(\frac{2}{2}, -\frac{2}{2}, \frac{7}{2} \right) \end{aligned}$$

$MP = (1, -1, 3.5)$

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3.2.2 Volume & Surface Area

Volume of 3D Shapes

What is volume?

- The volume of a 3D shape is a measure of how much 3D space it takes up
 - A 3D shape is also called a **solid**
- You need to be able to calculate the volume of a number of common shapes

How do I find the volume of cuboids, prisms and cylinders?

- A prism is a 3-D shape that has two identical **base** shapes connected by parallel **edges**
 - A prism has the same base shape all the way through
 - A **prism** takes its name from its base
- To find the **volume** of any prism use the formula:

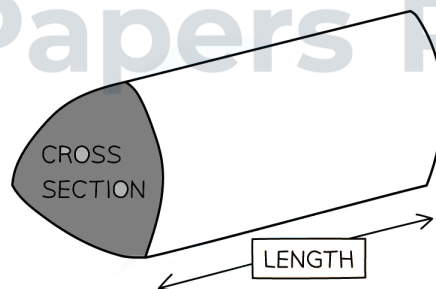
$$\text{Volume of a prism} = Ah$$

- Where **A** is the area of the cross section and **h** is the base height
 - **h** could also be the length of the prism, depending on how it is oriented
- This is in the formula booklet in the **prior learning** section at the beginning
- The base could be any shape so as long as you know its area and length you can calculate the volume of any prism

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- Note two special cases:
 - To find the volume of a cuboid use the formula:

$$\text{Volume of a cuboid} = \text{length} \times \text{width} \times \text{height}$$

$$V = lwh$$



- The volume of a **cylinder** can be found in the same way as a prism using the formula:

$$\text{Volume of a cylinder} = \pi r^2 h$$

- where r is the radius, h is the height (or length, depending on the orientation)
- Note that a cylinder is technically not a prism as its base is not a polygon, however the method for finding its volume is the same
- Both of these are in the **formula booklet** in the **prior learning** section

How do I find the volume of pyramids and cones?

- In a **right-pyramid** the apex (the joining point of the triangular faces) is vertically above the centre of the base
 - The base can be any shape but is usually a square, rectangle or triangle
- To calculate the volume of a **right-pyramid** use the formula

$$V = \frac{1}{3} Ah$$

- Where A is the area of the base, h is the height
- Note that the height must be **vertical to the base**
- A **right cone** is a circular-based pyramid with the vertical height joining the apex to the centre of the circular base
- To calculate the volume of a **right-cone** use the formula

$$V = \frac{1}{3} \pi r^2 h$$

- Where r is the radius, h is the height
- These formulae are both given in the formula booklet

How do I find the volume of a sphere?

- To calculate the volume of a **sphere** use the formula

$$V = \frac{4}{3} \pi r^3$$

- Where r is the radius
 - the line segment from the centre of the sphere to the surface
- This formula is given in the formula booklet

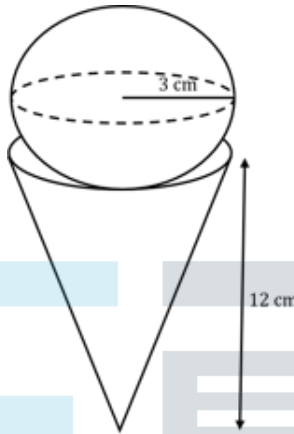
Exam Tip

- Remember to make use of the formula booklet in the exam as all the volume formulae you need will be here
 - Formulae for basic 3D objects (cuboid, cylinder and prism) are in the **prior learning** section
 - Formulae for other 3D objects (pyramid, cone and sphere) are in the **Topic 3: Geometry** section



Worked example

A dessert can be modelled as a right-cone of radius 3 cm and height 12 cm and a scoop of ice-cream in the shape of a sphere of radius 3 cm. Find the total volume of the ice-cream and cone.



Volume of a sphere: $V = \frac{4}{3} \pi r^3$ (In formula booklet)

Substitute: $r = 3 \Rightarrow V = \frac{4}{3} \pi \times 3^3$
 $= 36\pi$

Volume of a right cone: $V = \frac{1}{3} \pi r^2 h$ (In formula booklet)

Substitute: $r = 3, h = 12 \Rightarrow V = \frac{1}{3} \pi (3)^2 (12)$
 $= 36\pi$

Total Volume = $72\pi \text{ cm}^3$

Total Volume = 226 cm^3 (3sf)

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Surface Area of 3D Shapes

What is surface area?

- The surface area of a 3D shape is the sum of the areas of all the **faces** that make up a shape
 - A **face** is one of the flat or curved surfaces that make up a 3D shape
 - It often helps to consider a 3D shape in the form of its 2D net

How do I find the surface area of cuboids, pyramids and prisms?

- Any prisms and pyramids that have polygons as their bases have only flat faces
 - The surface area is simply found by adding up the areas of these flat faces
 - Drawing a 2D net will help to see which faces the 3D shape is made up of

How do I find the surface area of cylinders, cones and spheres?

- Cones, cylinders and spheres all have curved faces so it is not always as easy to see their shape
 - The net of a **cylinder** is made up of two identical circles and a rectangle
 - The rectangle is the curved surface area and is harder to identify
 - The length of the rectangle is the same as the circumference of the circle
 - The area of the **curved surface area** is

$$A = 2\pi rh$$

- where r is the radius, h is the height
- This is given in the formula book in the prior learning section
- The area of the **total surface area of a cylinder** is

$$A = 2\pi rh + 2\pi r^2$$

- This is **not** given in the formula book, however it is easy to put together as both the area of a circle and the area of the curved surface area are given

Copyright © 2024 Exam Papers Practice. The net of a **cone** consists of the circular base along with the curved surface area

© 2024 Exam Papers Practice. The area of the **curved surface area** is

$$A = \pi rl$$

- Where r is the radius and l is the **slant height**
- This is **given in the formula book**
 - Be careful not to confuse the slant height, l , with the vertical height, h
 - Note that r , h and l will create a **right-angled triangle** with l as the hypotenuse
- The area of the **total surface area of a cone** is

$$A = \pi rl + \pi r^2$$

- This is **not** given in the formula book, however it is easy to put together as both the area of a circle and the area of the curved surface area are given
- To find the surface area of a **sphere** use the formula

$$A = 4\pi r^2$$

- where r is the radius (line segment from the centre to the surface)
- This is given in the formula booklet, you do not have to remember it

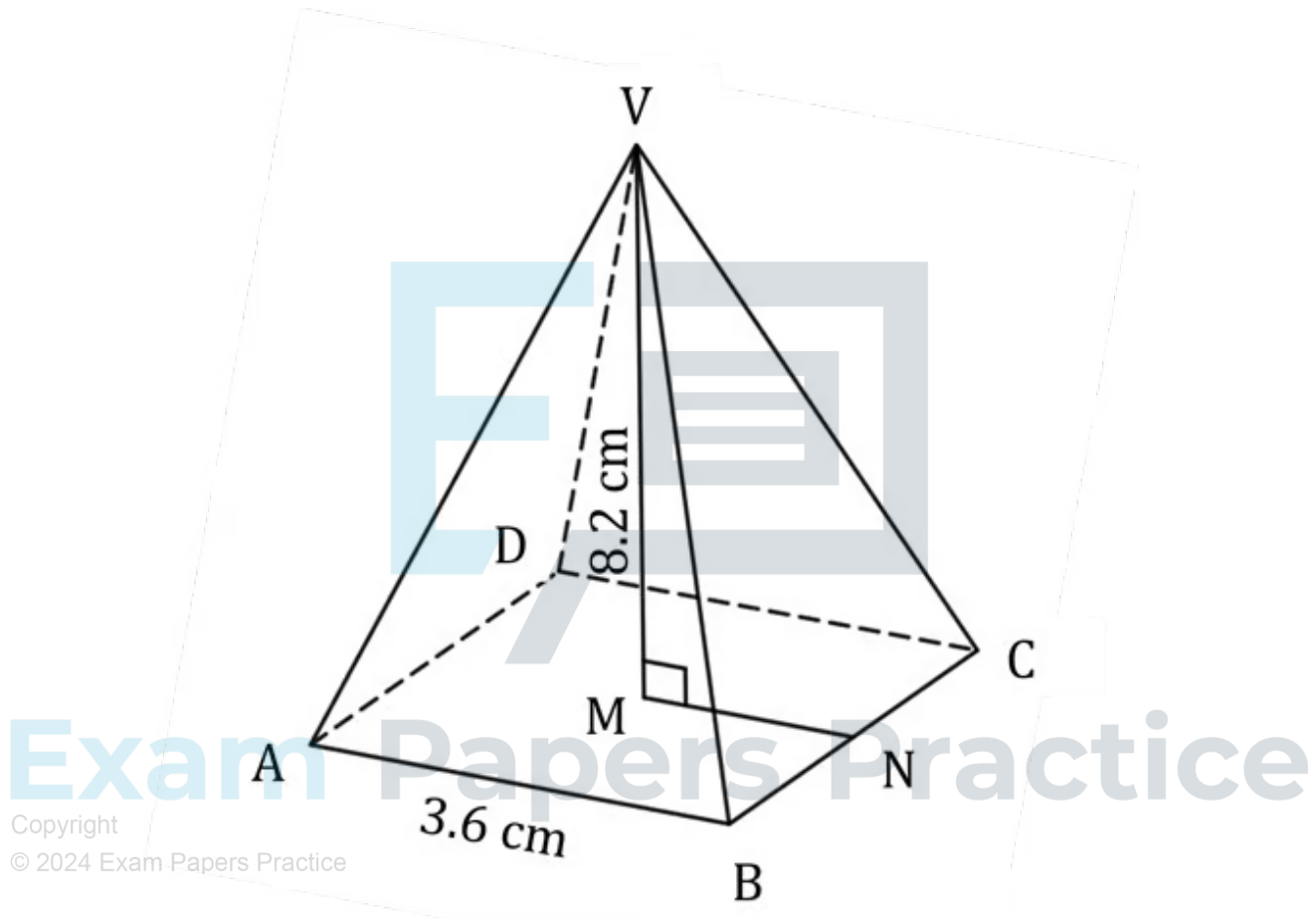
Exam Tip

- Remember to make use of the formula booklet in the exam as all the area formulae you need will be here
 - Formulae for basic 2D shapes (parallelogram, triangle, trapezoid, circle, curved surface of a cylinder) are in the **prior learning** section
 - Formulae for other 2D shapes (curved surface area of a cone and surface area of a sphere) are in the **Topic 3: Geometry** section



 **Worked example**

In the diagram below $ABCD$ is the square base of a right pyramid with vertex V . The centre of the base is M . The sides of the square base are 3.6 cm and the vertical height is 8.2 cm.

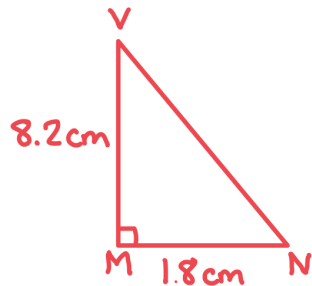


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- i) Use the Pythagorean Theorem to find the distance VN .



Sketch the triangle MNV:



M is the midpoint
so $MN = 3.6 \div 2$

By the Pythagorean Theorem:

$$\begin{aligned} VN^2 &= \sqrt{VM^2 + MN^2} \\ &= \sqrt{8.2^2 + 1.8^2} \\ &= \sqrt{70.48} \end{aligned}$$

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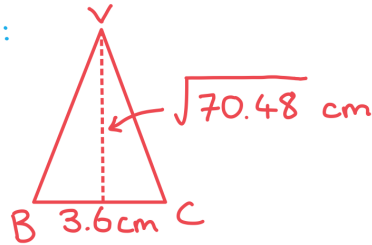
$$VN = 8.40 \text{ cm (3sf)}$$

- ii) Calculate the area of the triangle ABV.



$$\text{Area } \triangle ABV = \text{area } \triangle BCV$$

Sketch $\triangle BCV$:



$$\text{Area of a triangle} = \frac{1}{2}bh$$

$$\text{Substitute } b = 3.6, h = \sqrt{70.48}$$

$$\begin{aligned} \text{Area} &= \frac{1}{2}(3.6)(\sqrt{70.48}) \\ &= 15.111... \text{ cm}^2 \end{aligned}$$

$$\text{Area } \triangle ABV = 15.1 \text{ cm}^2$$

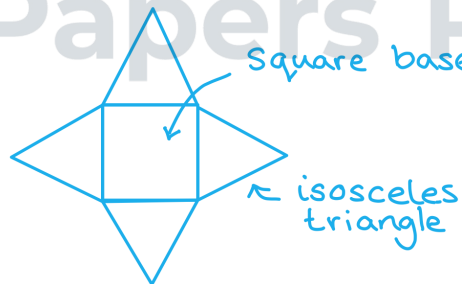
iii) Find the surface area of the right pyramid.

Considering the net may help:

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$$\text{Surface area} = \text{area square} + 4(\text{area triangle})$$

$$\begin{aligned} SA &= 3.6^2 + 4(15.111...) \\ &= 73.405... \text{ cm}^2 \end{aligned}$$

$$SA = 73.4 \text{ cm}^2 \text{ (3sf)}$$