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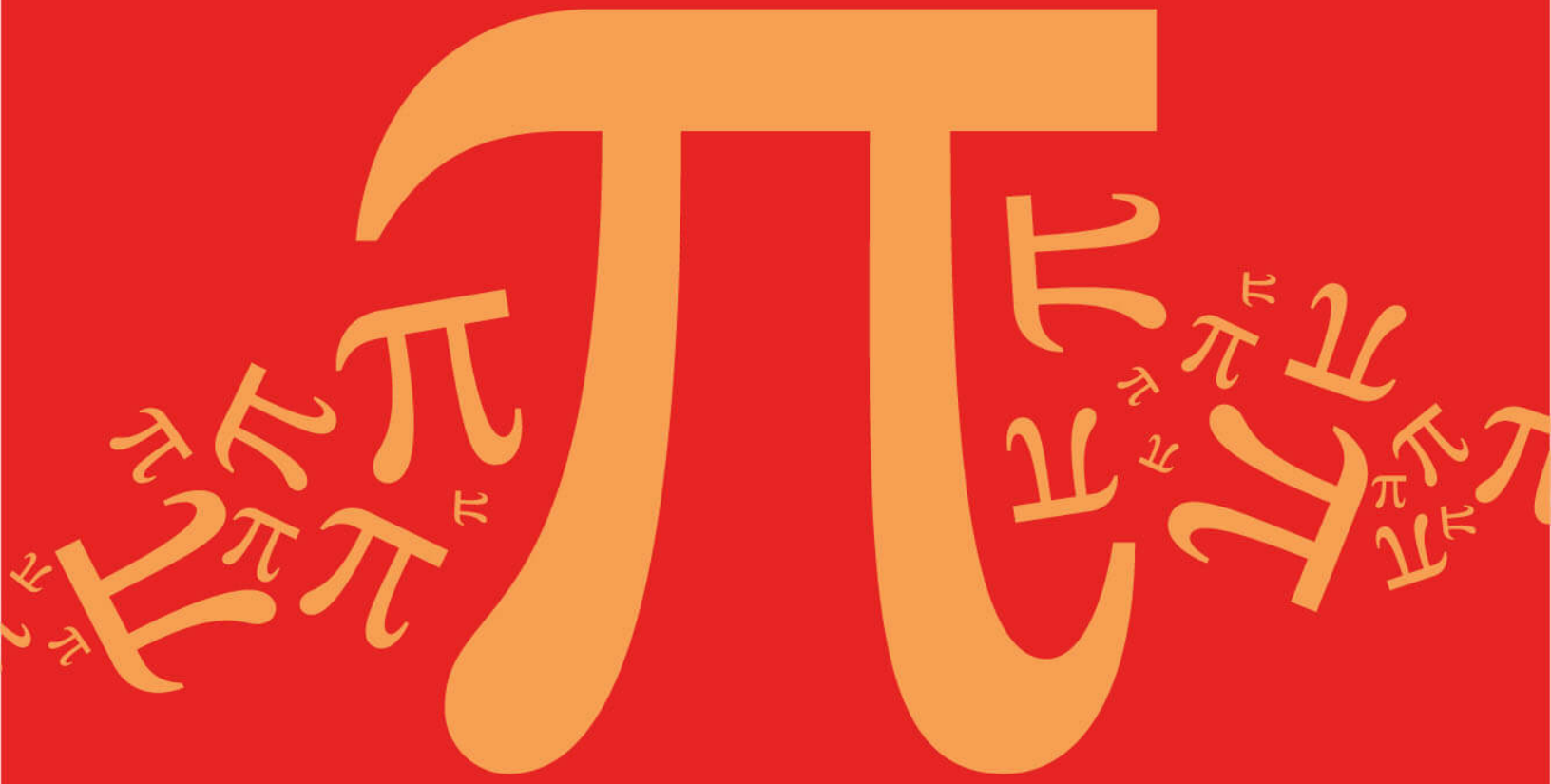
# 3.2 Geometry of 3D Shapes



# IB Maths - Revision Notes

## AA SL

# 3.2.1 3D Coordinate Geometry

## 3D Coordinate Geometry

### How does the 3D coordinate system work?

- In three-dimensional space we can label where any object is using the $x$-$y$-$z$ coordinate system
- In the 3D cartesian system, the $x$- and $y$- axes usually represent lateral space (length and width) and the $z$-axis represents vertical height

### What can we do with 3D coordinates?

- If we have two points with coordinates $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ then we should be able to find:
  - The **midpoint** of the two points
  - The **distance** between the two points
- If the coordinates are labelled A and B then the line segment between them is written with the notation [AB]

### How do I find the midpoint of two points in 3D?

- The midpoint is the **average (middle) point**
  - It can be found by finding the middle of the $x$-coordinates and the middle of the $y$-coordinates
- The coordinates of the midpoint will be

$$\left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}, \frac{z_1+z_2}{2}\right)$$



  - This is given in the formula booklet, you do not need to remember it

### How do I find the distance between two points in 3D?

- The distance between two points with coordinates $(x_1, y_1, z_1)$ and $(x_2, y_2, z_2)$ can be found using the formula

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

  - This is given in the formula booklet, you do not need to remember it

## Worked example

The points $A$ and $B$ have coordinates $(-2, 1, 5)$ and $(4, -3, 2)$ respectively.

i) Calculate the distance of the line segment $AB$.

Formula for the distance of a line segment:

in formula booklet

$$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$$

$A: (-2, 1, 5)$ $\quad$ $B: (4, -3, 2)$

$x_1$ $y_1$ $z_1$ $\quad$ $x_2$ $y_2$ $z_2$

Substitute:

$$d = \sqrt{(-2 - 4)^2 + (1 - (-3))^2 + (5 - 2)^2}$$

$$= \sqrt{(-6)^2 + 4^2 + 3^2}$$

$$= \sqrt{36 + 16 + 9}$$

$$= \sqrt{61}$$

$$d = 7.81 \text{ units (3sf)}$$



ii) Find the midpoint of $[AB]$.

Formula for the midpoint of a line segment:

$$MP = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2}\right)$$

in formula booklet

$A: (-2, 1, 5)$ $B: (4, -3, 2)$

$x_1$ $y_1$ $z_1$ $x_2$ $y_2$ $z_2$

Substitute:

$$MP = \left(\frac{-2+4}{2}, \frac{1+(-3)}{2}, \frac{5+2}{2}\right)$$

$$= \left(\frac{2}{2}, -\frac{2}{2}, \frac{7}{2}\right)$$

$$MP = (1, -1, 3.5)$$

# 3.2.2 Volume & Surface Area

## Volume of 3D Shapes

### What is volume?

- The volume of a 3D shape is a measure of how much 3D space it takes up
  - A 3D shape is also called a **solid**
- You need to be able to calculate the volume of a number of common shapes

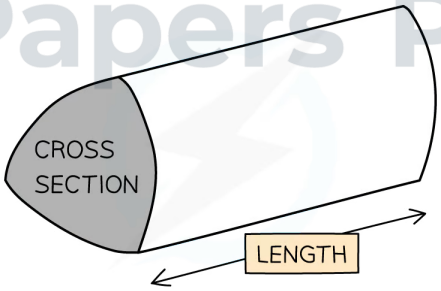
### How do I find the volume of cuboids, prisms and cylinders?

- A prism is a 3-D shape that has two identical **base** shapes connected by parallel **edges**
  - A prism has the same base shape all the way through
  - A **prism** takes its name from its base
- To find the **volume** of any prism use the formula:

$$\text{Volume of a prism} = Ah$$

  - Where ***A*** is the area of the cross section and ***h*** is the base height
    - *h* could also be the length of the prism, depending on how it is oriented
  - This is in the formula booklet in the **prior learning** section at the beginning
  - The base could be any shape so as long as you know its area and length you can calculate the volume of any prism






- Note two special cases:
  - To find the volume of a cuboid use the formula:

$$\text{Volume of a cuboid} = \text{length} \times \text{width} \times \text{height}$$

$$V = lwh$$

- The volume of a **cylinder** can be found in the same way as a prism using the formula:

$$\text{Volume of a cylinder} = \pi r^2 h$$

  - where $r$ is the radius, $h$ is the height (or length, depending on the orientation
  - Note that a cylinder is technically not a prism as its base is not a polygon, however the method for finding its volume is the same
- Both of these are **in the formula booklet** in the **prior learning** section

### How do I find the volume of pyramids and cones?

- In a **right-pyramid** the apex (the joining point of the triangular faces) is vertically above the centre of the base
  - The base can be any shape but is usually a square, rectangle or triangle
- To calculate the volume of a **right-pyramid** use the formula

$$V = \frac{1}{3}Ah$$

  - Where $A$ is the area of the base, $h$ is the height
  - Note that the height must be **vertical to the base**
- **A right cone** is a circular-based pyramid with the vertical height joining the apex to the centre of the circular base
- To calculate the volume of a **right-cone** use the formula

$$V = \frac{1}{3}\pi r^2 h$$

  - Where $r$ is the radius, $h$ is the height
- These formulae are both given in the formula booklet



### How do I find the volume of a sphere?

- To calculate the volume of a **sphere** use the formula

$$V = \frac{4}{3}\pi r^3$$

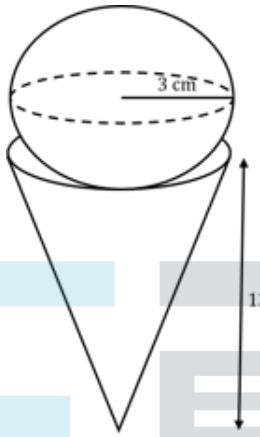
  - Where $r$ is the radius
    - the line segment from the centre of the sphere to the surface
  - This formula is given in the formula booklet

#### Exam Tip

- Remember to make use of the formula booklet in the exam as all the volume formulae you need will be here
  - Formulae for basic 3D objects (cuboid, cylinder and prism) are in the **prior learning** section
  - Formulae for other 3D objects (pyramid, cone and sphere) are in the **Topic 3: Geometry** section

## Worked example

A dessert can be modelled as a right-cone of radius 3 cm and height 12 cm and a scoop of ice-cream in the shape of a sphere of radius 3 cm. Find the total volume of the ice-cream and cone.




Volume of a sphere: $V = \frac{4}{3}\pi r^3$ (In formula booklet)

Substitute: $r = 3 \Rightarrow V = \frac{4}{3}\pi \times 3^3$

$= 36\pi$



Volume of a right cone: $V = \frac{1}{3}\pi r^2 h$ (In formula booklet)

Substitute: $r = 3,\ h = 12 \Rightarrow V = \frac{1}{3}\pi(3)^2(12)$

$= 36\pi$

Total Volume $= 72\pi\ cm^3$

**Total Volume = 226 $cm^3$ (3sf)**

# Surface Area of 3D Shapes

## What is surface area?

- The surface area of a 3D shape is the sum of the areas of all the **faces** that make up a shape
  - A **face** is one of the flat or curved surfaces that make up a 3D shape
  - It often helps to consider a 3D shape in the form of its 2D net

## How do I find the surface area of cuboids, pyramids and prisms?

- Any prisms and pyramids that have polygons as their bases have only flat faces
  - The surface area is simply found by adding up the areas of these flat faces
  - Drawing a 2D net will help to see which faces the 3D shape is made up of

## How do I find the surface area of cylinders, cones and spheres?

- Cones, cylinders and spheres all have curved faces so it is not always as easy to see their shape
  - The net of a **cylinder** is made up of two identical circles and a rectangle
  - The rectangle is the curved surface area and is harder to identify
  - The length of the rectangle is the same as the circumference of the circle
  - The area of the **curved surface area** is

$$A = 2\pi rh$$

    - where $r$ is the radius, $h$ is the height
  - This is given in the formula book in the prior learning section
  - The area of the **total surface area of a cylinder** is

$$A = 2\pi rh + 2\pi r^2$$

  - This is **not** given in the formula book, however it is easy to put together as both the area of a circle and the area of the curved surface area are given
- The net of a **cone** consists of the circular base along with the curved surface area
  - The area of the **curved surface area** is

$$A = \pi rl$$

    - Where $r$ is the radius and $l$ is the **slant height**
  - This is **given in the formula book**
    - Be careful not to confuse the slant height, $l$, with the vertical height, $h$
    - Note that $r$, $h$ and $l$ will create a **right-triangle** with $l$ as the hypotenuse
  - The area of the **total surface area of a cone** is

$$A = \pi rl + \pi r^2$$

- This is **not** given in the formula book, however it is easy to put together as both the area of a circle and the area of the curved surface area are given
- To find the surface area of a **sphere** use the formula

$$A = 4\pi r^2$$

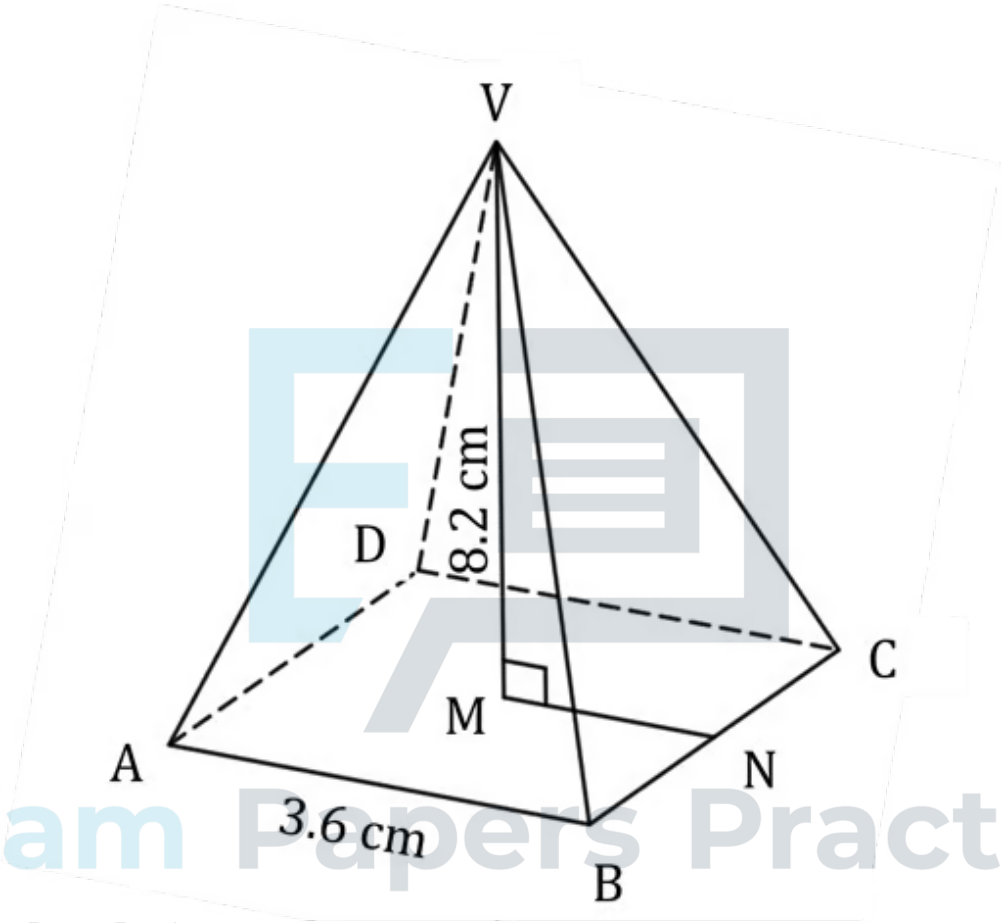
  - where $r$ is the radius (line segment from the centre to the surface)
  - This is given in the formula booklet, you do not have to remember it

### Exam Tip

- Remember to make use of the formula booklet in the exam as all the area formulae you need will be here
  - Formulae for basic 2D shapes (parallelogram, triangle, trapezoid, circle, curved surface of a cylinder) are in the **prior learning** section
  - Formulae for other 2D shapes (curved surface area of a cone and surface area of a sphere ) are in the **Topic 3: Geometry** section

## Worked example

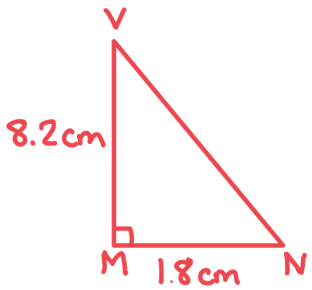
In the diagram below *ABCD* is the square base of a right pyramid with vertex *V*. The centre of the base is *M*. The sides of the square base are 3.6 cm and the vertical height is 8.2 cm.






i) Use the Pythagorean Theorem to find the distance VN.

Sketch the triangle MNV:




M is the midpoint
so MN = 3.6 ÷ 2

By the Pythagorean Theorem:

$$VN^2 = \sqrt{VM^2 + MN^2}$$

$$= \sqrt{8.2^2 + 1.8^2}$$

$$= \sqrt{70.48}$$

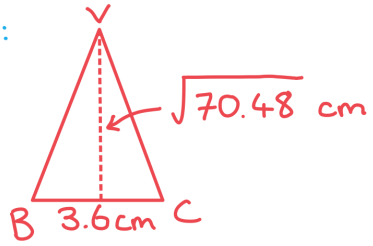
**VN = 8.40 cm (3sf)**



ii) Calculate the area of the triangle ABV.

Area $\Delta ABV$ = area $\Delta BCV$

Sketch $\Delta BCV$:




Area of a triangle $= \frac{1}{2}bh$

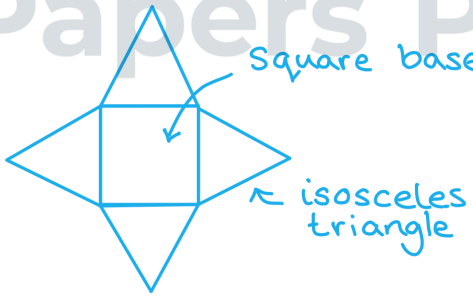
Substitute $b = 3.6$, $h = \sqrt{70.48}$

$$\text{Area} = \frac{1}{2}(3.6)(\sqrt{70.48})$$
$$= 15.111\ldots \text{ cm}^2$$

**Area $\Delta ABV$ = 15.1 cm²**

iii) Find the surface area of the right pyramid.

Considering the net may help:




$$\text{Surface area} = \text{area square} + 4(\text{area triangle})$$
$$SA = 3.6^2 + 4(15.111\ldots)$$
$$= 73.405\ldots \text{ cm}^2$$

**SA = 73.4 cm² (3sf)**