3.1 Graph traversal

Name: ________________________

Class: ________________________

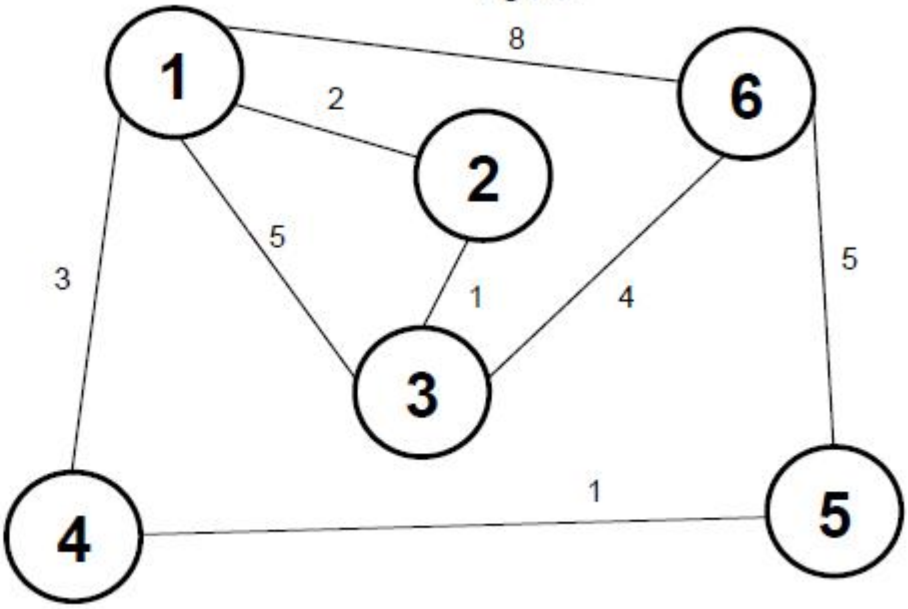
Date: ________________________

Time:	**123 minutes**
Marks:	**87 marks**
Comments:	

Q1.

Figure 1 is a graph that shows the time it takes to travel between six locations in a warehouse. The six locations have been labelled with the numbers 1 - 6. When there is no edge between two nodes in the graph this means that it is not possible to travel directly between those two locations. When there is an edge between two nodes in the graph the edge is labelled with the time (in minutes) it takes to travel between the two locations represented by the nodes.

Figure 1



(a) The graph is represented using an adjacency matrix, with the value 0 being used to indicate that there is no edge between two nodes in the graph.

A value should be written in every cell.

Complete the unshaded cells in **Table 1** so that it shows the adjacency matrix for **Figure 1**.

Table 1

	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

(2)

(b) Instead of using an adjacency matrix, an adjacency list could be used to represent the graph. Explain the circumstances in which it would be more appropriate to use an adjacency list instead of an adjacency matrix.

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(2)

(c) State **one** reason why the graph shown in **Figure 1** is **not** a tree.

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(1)

(d) The graph in **Figure 1** is a weighted graph. Explain what is meant by a **weighted graph**.

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(1)

Figure 2 contains pseudo-code for a version of Djikstra's algorithm used with the graph in **Figure 1**.

`Q` is a priority queue which stores nodes from the graph, maintained in an order based on the values in array `D`. The reordering of `Q` is performed automatically when a value in `D` is changed.

`AM` is the name given to the adjacency matrix for the graph represented in **Figure 1**.

Figure 2

```
Q ← empty queue
FOR C1 ← 1 TO 6
  D[C1] ← 20
  P[C1] ← -1
  ADD C1 TO Q
ENDFOR
D[1] ← 0
WHILE Q NOT EMPTY
  U ← get next node from Q
  remove U from Q
  FOR EACH V IN Q WHERE AM[U, V] > 0
    A ← D[U] + AM[U, V]
    IF A < D[V] THEN
      D[V] ← A
      P[V] ← U
```

```
            ENDIF
          ENDFOR
        ENDWHILE
        OUTPUT D[6]
```

(e) Complete the unshaded cells of **Table 2** to show the result of tracing the algorithm shown in **Figure 2**. Some of the trace, including the maintenance of `Q`, has already been completed for you.

Table 2

				D						P					
U	**Q**	**V**	**A**	**1**	**2**	**3**	**4**	**5**	**6**	**1**	**2**	**3**	**4**	**5**	**6**
-	1,2,3,4,5,6	-	-	20	20	20	20	20	20	-1	-1	-1	-1	-1	-1
				0											
1	2,3,4,5,6	2													
		3													
		4													
		6													
2	3,4,5,6	3													
3	4,5,6	6													
4	5,6	5													
5	6	6													
6	-														

(7)

(f) What does the output from the algorithm in **Figure 2** represent?

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(1)

(g) The contents of the array `P` were changed by the algorithm. What is the purpose of the array `P`?

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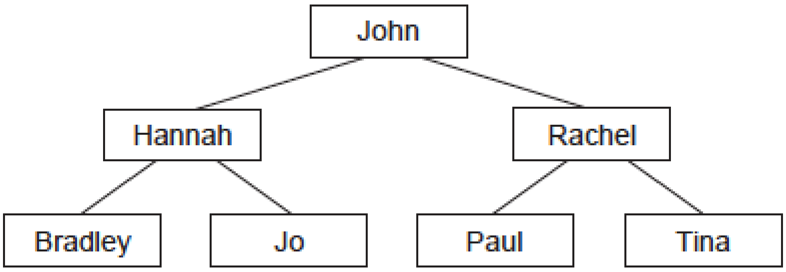
(2)

(Total 16 marks)

Q2.

A binary search tree can be used to represent a list of data so that it can be efficiently searched. **Figure 1** shows an example of a binary search tree:

Figure 1



(a) The tree in **Figure 1** is to be searched for data item "Lisa". The tree does not contain "Lisa".

List the data items that will be examined, in the order that they will be visited, when "Lisa" is searched for.

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(1)

(b) Tick **one** box in the table to indicate the time complexity of the algorithm used to search for data in a binary search tree.

Time Complexity	Tick one box
O(n)	
O(log n)	
$O(n^2)$	

(1)

(c) In **Figure 2** below, show how the tree in **Figure 1** could be represented by a **Start Index**, together with a one-dimensional array of records, each of which contains the fields **Left Pointer**, **Data** and **Right Pointer**:

Figure 2

Start Index

= ________

Index	Left Pointer	Data	Right Pointer
[1]			
[2]			
[3]			
[4]			
[5]			
[6]			
[7]			

(3)

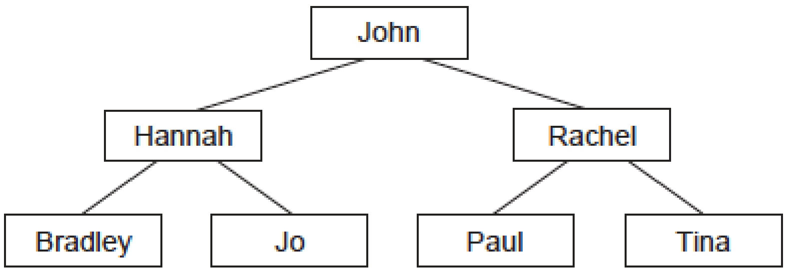
(d) The array shown in **Figure 2** is an example of a static data structure.

Explain the differences between a static data structure and a dynamic data structure, and what the heap is used for with a dynamic data structure.

(3)

(e) An in-order traversal is carried out on the binary tree in **Figure 1** to output the values stored in the nodes of the tree.

Figure 1



(i) Write out the data items from the tree, in the order that they will be output during the traversal.

(1)

(ii) What is the significance of the order that the data items have been output in?

(1)

(f) Graph traversal is a more complex problem than tree traversal. State **one** feature that a graph might have, which a tree cannot have, that makes graph traversal more complex.

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(1)

(Total 11 marks)

Q3.

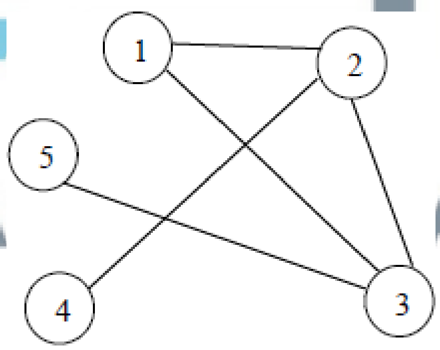
The Cat transportation company (CTC) is a business that specialises in preparing cats for cat shows.

They need to take five cats to the AQA cat show. They will transport the cats in their van. CTC owns only one van.

They cannot put all the cats in their van at the same time because some of the cats get stressed when in the company of some of the other cats. The cats would not therefore arrive in top condition for the cat show if they were all in the van at the same time.

The graph in **Figure 1** shows the relationships between the five cats (labelled 1 to 5). If there is an edge between two cats in the graph then they **cannot** travel in the van together at the same time.

Figure 1



(a) Explain why the graph in **Figure 1** is **not** a tree.

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(1)

(b) Represent the graph shown in **Figure 1** as an adjacency list by completing **Table 1**
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Table 1

Vertex (in Figure 1)	Adjacent vertices
1	

2	
3	
4	
5	

(2)

(c) **Table 2** shows how the graph in **Figure 1** can be represented as an adjacency matrix.

Table 2

Vertex (in Figure 1)	1	2	3	4	5
1	0	1	1	0	0
2	1	0	1	1	0
3	1	1	0	0	1
4	0	1	0	0	0
5	0	0	1	0	0

Explain the circumstances in which it is more appropriate to represent a graph using an adjacency list instead of an adjacency matrix.

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(2)

(d) **Figure 2** shows an algorithm, written in pseudo-code, that CTC use.

Figure 2

```
NoOfCats ← 5
Cat[1] ← 1
FOR A ← 2 TO NoOfCats
  B ← 1
  C ← 1
  WHILE B < A DO
    IF M[A, B] = 1
      THEN
        IF Cat[B] = C
          THEN
            B ← 1
```

```
                C ← C + 1
              ELSE B ← B + 1
            ENDIF
          ELSE B ← B + 1
        ENDIF
      ENDWHILE
      Cat[A] ← C
    ENDFOR
```

The two-dimensional array, M, is used to store the adjacency matrix shown in **Table 2**.

Complete **Table 3** to show the result of tracing the algorithm in **Figure 2**.

Table 3

				Cat				
NoOfCats	**A**	**B**	**C**	**1**	**2**	**3**	**4**	**5**

(6)

(e) Explain the purpose of the algorithm in **Figure 2**.

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(1)

(f) After a cat show, CTC needs to return the cats to their owners. They can have all the cats in the van at the same time because the show is now finished.

CTC likes to plan the return journey so that the shortest possible distance is travelled by the van. This is an example of an intractable problem.

What is meant by an intractable problem?

(2)

(g) What approach might a programmer take if asked to solve an intractable problem?

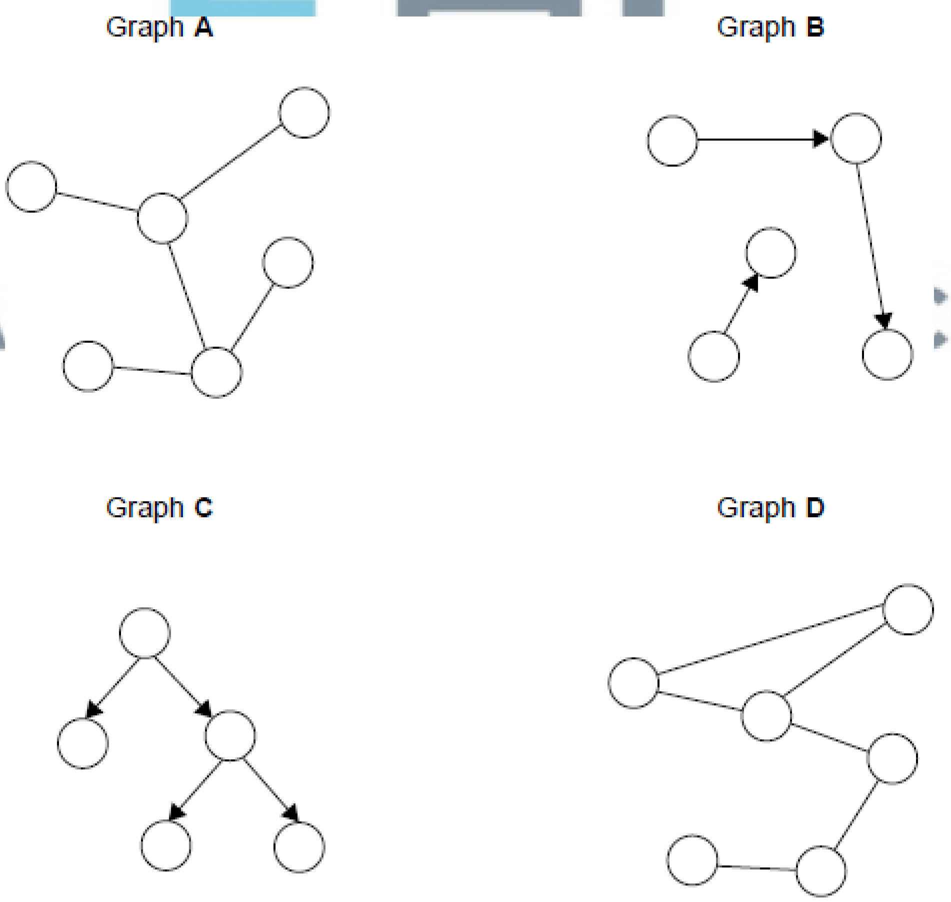
(2)

(Total 16 marks)

Q4.

(a) **Figure 1** shows four graphs, labelled with the letters **A** to **D**.

Figure 1



Complete **Table 1** below. In the **Correct letter (A-D)** column write the appropriate letter from **A** to **D** to indicate which graph in **Figure 1** matches the description in the **Description** column.

Do **not** use the same letter more than once. You will not need to use all of the letters.

Table 1

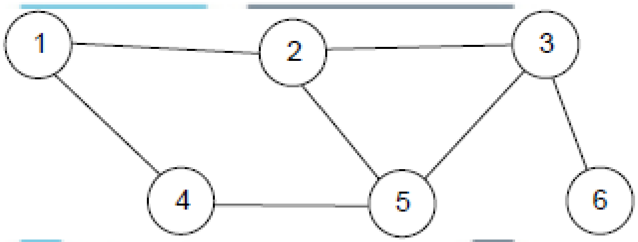
Description	**Correct letter (A-D)**
A graph that is not connected	
A graph that is a tree	

(2)

(b) It is possible to represent a computer network as a graph, with each vertex representing a router and each edge representing a communications link.

Figure 2 is a graph representation of a medium-sized computer network that consists of 6 routers and 7 communications links. The routers have been numbered from 1 to 6.

Figure 2



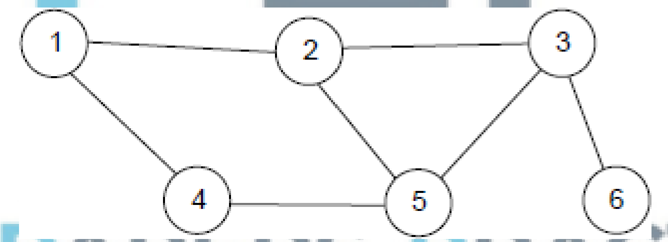
Complete **Table 2** below to show how the graph in **Figure 2** would be stored using an adjacency matrix.

Table 2

(2)

(c) **Figure 2** is repeated here.

Figure 2 (Repeated)



A simple method of determining the shortest path through a network from one router to another is to perform a breadth first search of the graph representation of the network.

The algorithm in **Figure 3** can be used to perform a breadth first search of a graph. It makes use of two subroutines, `PutVertexIntoQueue` and `GetVertexFromQueue`, which are explained below the algorithm.

Figure 3

```
Procedure ShortestRoute(S, D)
    PutVertexIntoQueue(S)
    Discovered[S]  ←  True
    Found  ←  False
    While Queue is Not Empty And Found = False Do
          V = GetVertexFromQueue
          For each vertex U which is adjacent to V Do
              If Discovered[U] = False And Found = False Then
                PutVertexIntoQueue(U)
                Discovered[U]  ←  True
                Parent[U]  ←  V
```

```
                If U = D Then Found  ←  True
              EndIf
          EndFor
    EndWhile
    If Found = True Then
          C  ←  D
          Output D
          Repeat
              C  ←  Parent[C]
              Output C
          Until C = S
    EndIf
EndProcedure
```

- `PutVertexIntoQueue` is a subroutine that adds a vertex to the rear of a queue.
- `GetVertexFromQueue` is a subroutine that returns the name of the vertex at the front of the queue and removes it from the queue.

Complete the trace table below to show how the `Discovered` and `Parent` arrays, the variable `Found` and the queue contents are updated, together with what output is produced by the algorithm when it is called using `ShortestRoute(1, 6)`.

Before the algorithm is carried out, all cells in the `Discovered` array are set to the value `False` and the queue is empty.

The values of the variables `S, D, V, U` and `C` have already been entered into the table for you.

The letter `F` has been used as an abbreviation for `False`. You should use T as an abbreviation for `True`.

						Discovered						Parent							
S	D	V	U	C	Queue Front Rear	1	2	3	4	5	6	1	2	3	4	5	6	Found	Output
						F	F	F	F	F	F								
1	6																		
		1	2																
			4																
		2	1																
			3																
			5																
		4	1																
			5																
		3	2																
			5																
			6																
				6															
				3															
				2															
				1															

(6)

(d) Explain why it is useful to find the shortest path through the network.

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(1)

(Total 11 marks)

Q5.

A computer program is being developed to allow commuters to plan journeys on the London Underground railway network which connects together over 250 stations.

The program needs to store a representation of the network so that the **shortest route** (ie shortest distance) between any two stations can be found.

Figure 1 is a map of central London, showing the location of ten of the stations on the London Underground. The locations of the underground railway lines are not shown. Note that nine of the stations are indicated by the symbol but Charing Cross has a different symbol because it is a combined underground and overground station.

Figure 1

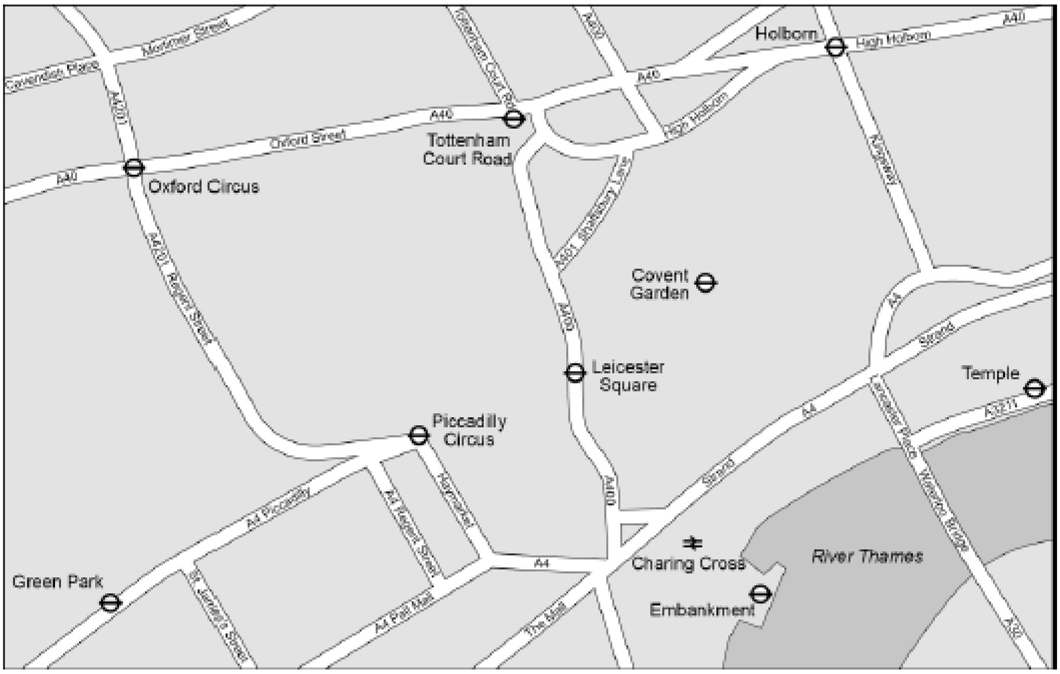

Figure 2 is a map of part of the underground railway network, showing the same ten stations. This map does not show the streets above ground but instead shows the underground railway lines that connect the stations together.

Figure 2

Due to copyright restrictions we are unable to show this image. Please use the link below to find the appropriate section of the tube map.

Standard Tube map - Transport for London

Figure 2 can be used in conjunction with a table of distances between adjacent stations to calculate the shortest route between any two stations on the network.

The map of the entire underground railway network (**not** just the parts shown in **Figure 1** and **Figure 2**) together with the full table of distances can be represented logically as a graph.

(a) The representation of the underground railway network as a graph is an abstraction.

Explain what an abstraction is.

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(1)

(b) Write a detailed description of:

- how the underground railway network and table of distances could be represented as a graph, **and**,

- how this representation could be implemented as either an adjacency matrix **or** an adjacency list (describe **one** of these alternatives only), using array(s) in a programming language that does not have a built-in data structure for graphs.

Your implementation should store all the details that are required to calculate the shortest distance between any two stations, but you do not need to describe how the shortest distance would be worked out.

In your answer you will be assessed on your ability to use good English, and to organise your answer clearly in complete sentences, using specialist vocabulary where appropriate.

You may use diagrams to help clarify your description, but as you are being assessed on your ability to use good English, you must ensure that all diagrams are fully explained.

(8)

(Total 9 marks)

Q6.

A graph can be drawn to represent a maze. In such a graph, each graph vertex represents one of the following:

- the entrance to or exit from the maze
- a place where more than one path can be taken
- a dead end.

Edges connect the vertices according to the paths in the maze.

Diagram 1 shows a maze and **Diagram 2** shows one possible representation of this maze.
Position 1 in **Diagram 1** corresponds to vertex 1 in **Diagram 2** and is the entrance to the maze. Position 7 in **Diagram 1** is the exit to the maze and corresponds to vertex 7.

Dead ends have been represented by the symbol in **Diagram 2**.

Diagram 3 shows a simplified undirected graph of this maze with dead ends omitted.

Diagram 1

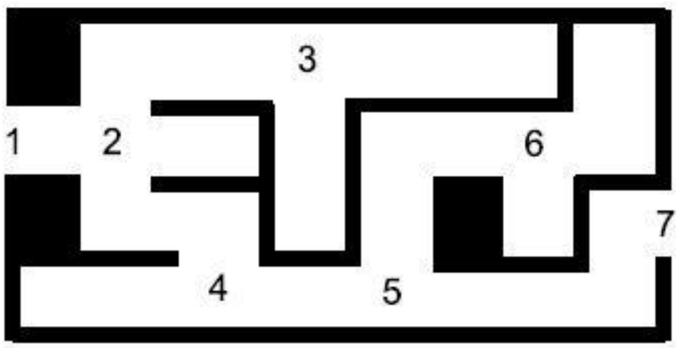
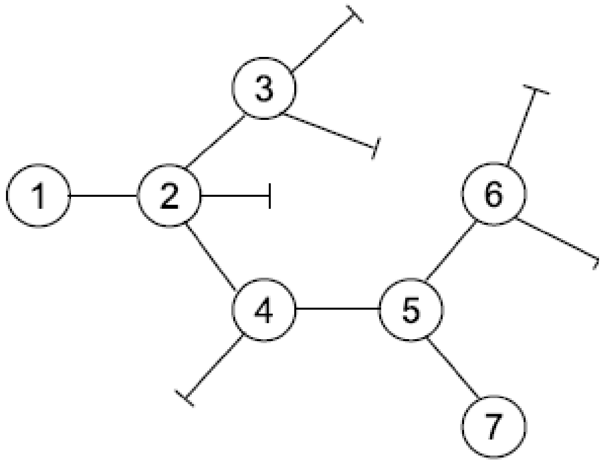
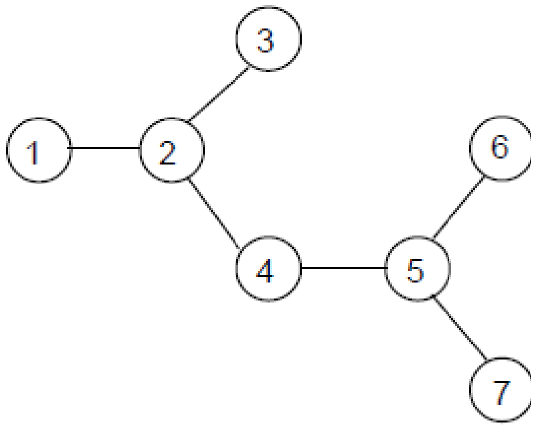


Diagram 2

Diagram 3



Representation of maze
including dead ends



Graph representing maze
with dead ends omitted

(a) The graph in **Diagram 3** is a tree.

State **one** property of the graph in **Diagram 3** that makes it a tree.

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(1)

(b) The graphs of some mazes are not trees.

Describe a feature of a maze that would result in its graph **not** being a tree.

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____
(1)

(c) Complete the table below to show how the graph in **Diagram 3** would be stored using an adjacency matrix.

(2)

(d) (i) What is a *recursive routine?*

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(1)

(ii) To enable the use of recursion a programming language must provide a stack.

Explain what this stack will be used for and why a stack is appropriate.

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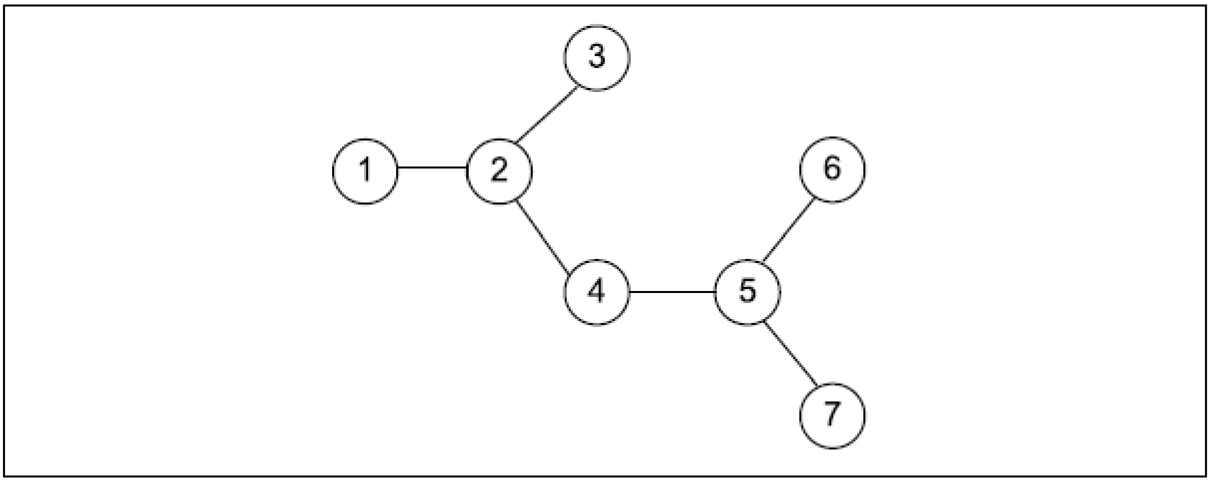
(2)

Diagram 3 is repeated here so that you can answer Question (e) without having to turn pages.



(e) A recursive routine can be used to perform a depth-first search of the graph that represents the maze to test if there is a route from the entrance (vertex 1) to the exit (vertex 7).

The recursive routine in the diagram below is to be used to explore the graph in **Diagram 3**. It has two parameters, `V` (the current vertex) and `EndV` (the exit vertex).

```
Procedure DFS(V, EndV)
    Discovered[V] ← True
    If V = EndV Then Found ← True
    For each vertex U which is connected to V Do
        If Discovered [U] = False Then DFS(U, EndV)
    EndFor
    CompletelyExplored[V] ← True
EndProcedure
```

Complete the trace table below to show how the `Discovered` and `CompletelyExplored` flag arrays and the variable `Found` are updated by the algorithm when it is called using `DFS(1,7)`.

The details of each call and the values of the variables `V`, `U` and `EndV` have already been entered into the table for you. The letter `F` has been used as an abbreviation for `False`. You should use `T` as an abbreviation for `True`.

				Discovered							CompletelyExplored							
Call	V	U	EndV	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	Found
	-	-		F	F	F	F	F	F	F	F	F	F	F	F	F	F	F
DFS(1,7)	1	2	7															
DFS(2,7)	2	1	7															
		3	7															
DFS(3,7)	3	2	7															
DFS(2,7)	2	4	7															
DFS(4,7)	4	2	7															
		5	7															
DFS(5,7)	5	4	7															
		6	7															
DFS(6,7)	6	5	7															
DFS(5,7)	5	7	7															
DFS(7,7)	7	5	7															
DFS(5,7)	5	-	7															
DFS(4,7)	4	-	7															
DFS(2,7)	2	-	7															
DFS(1,7)	1	-	7															

(5)

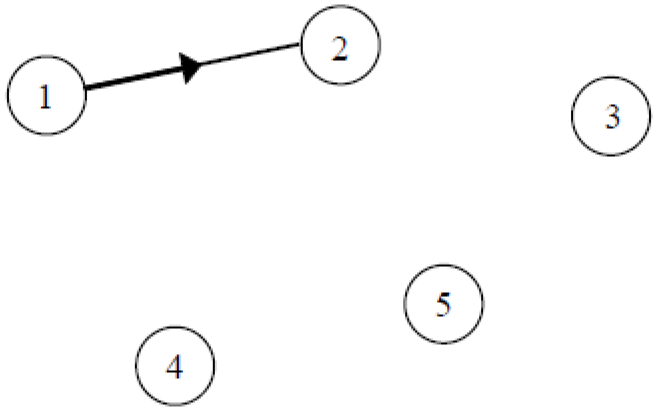
(Total 12 marks)

Q7.

The table below shows an adjacency matrix representation of a directed graph (digraph).

		1	**2**	**3**	**4**	**5**
From	**1**	0	1	0	1	0
	2	0	0	1	1	0
	3	0	0	0	0	0
	4	0	0	0	0	1
	5	0	1	0	0	0

(a) Complete this unfinished diagram of the directed graph.



(2)

(b) Directed graphs can also be represented by an adjacency list.

Explain under what circumstances an adjacency matrix is the most appropriate method to use to represent a directed graph, and under what circumstances an adjacency list is more appropriate.

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(2)

(c) A tree is a particular type of graph.

What properties must a graph have for it to be a tree?

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(2)

(d) Data may be stored as a binary tree.

Show how the following data may be stored as a binary tree for subsequent processing in alphabetic order by drawing the tree. Assume that the first item is the root of the tree and the rest of the data items are inserted into the tree in the order given.

Data items: Jack, Bramble, Snowy, Butter, Squeak, Bear, Pip

(3)

(e) A binary tree such as the one created in part (d) could be represented using one array of records or, alternatively, using three one-dimensional arrays.

Describe how the data stored in the array(s) could be structured for **one** of these two possible methods of representation.

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(3)

(Total 12 marks)