3.1 Geometry Toolkit

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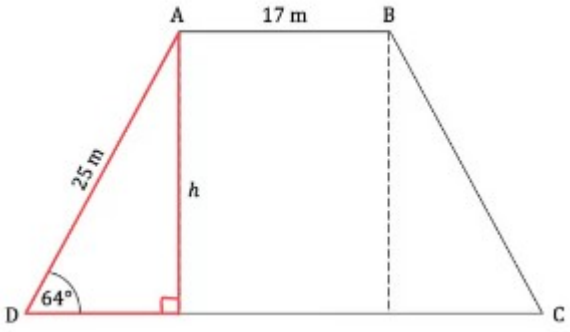
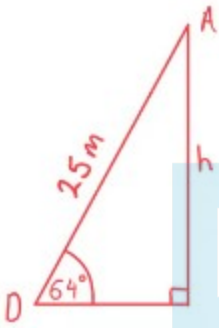
Course	DP IB Maths
Section	3. Geometry & Trigonometry
Topic	3.1 Geometry Toolkit
Difficulty	Medium

To be used by all students preparing for DP IB Maths AI SL
Students of other boards may also find this useful

Question 1



a) Notice the right-angled triangle.



We have

$\theta = 64°$ opp = h hyp = 25 m

$\sin\theta = \frac{\text{opp}}{\text{hyp}}$ from SOH CAH TOA

$\therefore \sin 64° = \frac{h}{25}$

$h = \sin 64° \times 25$

$h = 22.5$ m

b) Base of trapezoid, DC, is equal to AB + 2 (base of right-angled triangle).

$DC = 17 + 2\sqrt{25^2 - 22.5^2}$ (pythagoras)

Area of a trapezoid formula

$A = \frac{1}{2}(a+b)h$ (in formula booklet)

a and b are parrallel sides, h is the height

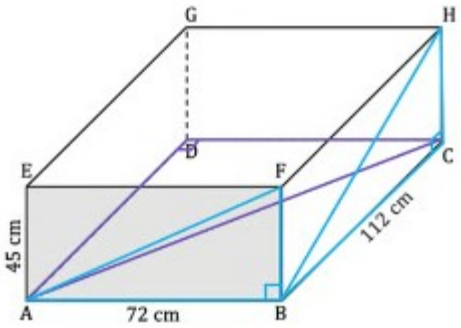
$a = AB = 17$ $b = DC = 17 + 2\sqrt{25^2 - 22.5^2}$ $h = 22.5$

Sub a, b and h into formula.

$A = \frac{1}{2}(17 + 17 + 2\sqrt{25^2 - 22.5^2}) \times 22.5$

$A \approx 628\ m^2$

Question 2



a) Notice the right-angled triangles.

i) $AF = \sqrt{AB^2 + BF^2}$

$AF = \sqrt{72^2 + 45^2}$

$AF \approx 84.9\text{ cm}$

ii) $BH = \sqrt{BC^2 + CH^2}$

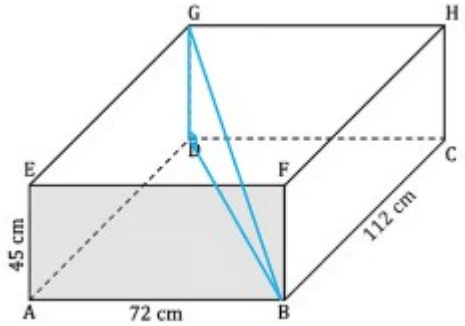
$BH = \sqrt{112^2 + 45^2}$

$BH \approx 121\text{ cm}$

iii) $AC = \sqrt{AD^2 + DC^2}$

$AC = \sqrt{72^2 + 112^2}$

$AC \approx 133\text{ cm}$



b) Notice the right-angled triangle BDG.

$BG = \sqrt{BD^2 + DG^2}$

$BD = AC = 133$ cm $DG = AE = 45$

$BG = \sqrt{133^2 + 45^2}$

$BG \approx 141$ cm

Question 3

a) Distance between two points formula

$d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ (in formula booklet)

$A(4, -6)$ $B(8, 6)$

Sub A and B into formula.

$d = \sqrt{(4-8)^2 + (-6-6)^2}$

$d \approx 12.6$ units

b) Gradient formula

$m = \frac{y_2 - y_1}{x_2 - x_1}$ (in formula booklet)

A(4, -6) B(8, 6)

Sub A and B into formula.

$m = \frac{6 - (-6)}{8 - 4}$ $\therefore m = 3$

Sub A and m into $y - y_1 = m(x - x_1)$

$y - (-6) = 3(x - 4)$

expand both sides

$y + 6 = 3x - 12$

-6

$y = 3x - 18$

c)i) Midpoint formula

$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ (in formula booklet)

$A(4, -6)$ $B(8, 6)$

Sub A and B into formula.

Midpoint $= \left(\frac{4+8}{2}, \frac{-6+6}{2}\right)$

Midpoint = (6, 0)

ii) Perpendicular gradients

$m_{\perp AB} = -\frac{1}{m_{AB}}$

$m_{AB} = 3$ $\therefore m_{\perp AB} = -\frac{1}{3}$

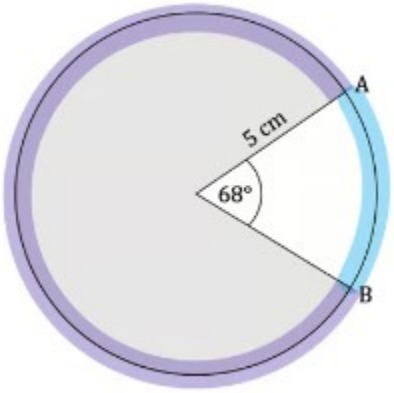
Sub the midpoint and $m_{\perp AB}$ into $y - y_1 = m(x - x_1)$.

$y - 0 = -\frac{1}{3}(x - 6)$

expand RHS

$y = -\frac{1}{3}x + 2$

Question 4



a) Arc length formula

$l = \frac{\theta}{360} \times 2\pi r$ (in formula booklet)

i) Minor arc AB

$\theta = 68$ $r = 5$

Sub θ and r into formula.

$l = \frac{68}{360} \times 2\pi(5)$

$l = 5.93$ cm

ii) Major arc AB

$\theta = 360 - 68 = 292$ $r = 5$

Sub θ and r into formula.

$l = \frac{292}{360} \times 2\pi(5)$

$l = 25.5$ cm

b) Sector area formula

$A = \frac{\theta}{360} \times \pi r^2$ (in formula booklet)

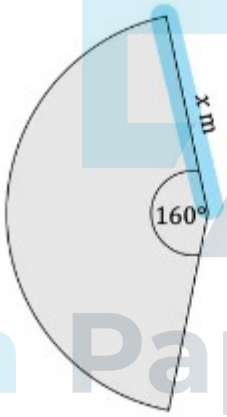
$\theta = 292$ $r = 5$

Sub θ and r into formula.

$A = \frac{292}{360} \times \pi (5)^2$

$A \approx 63.7 \text{ cm}^2$

Question 5



a) Sector area formula

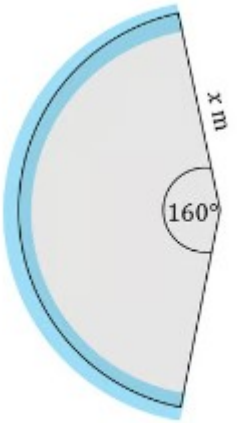
$A = \frac{\theta}{360} \times \pi r^2$ (in formula booklet)

$A = 20$ $\theta = 160$ $r = x$

Sub A and θ into formula and rearrange for x.

$20 = \frac{160}{360} \times \pi x^2$

$x \approx 3.78 \text{ m}$



$x \approx 3.78$ m

b) Arc length formula

$l = \frac{\theta}{360} \times 2\pi r$ (in formula booklet)

$\theta = 160$ $r = 3.78$

Sub θ and r into formula.

$l = \frac{160}{360} \times 2\pi(3.78)$

$l \approx 10.6$ m

Question 6

a) Sector area formula

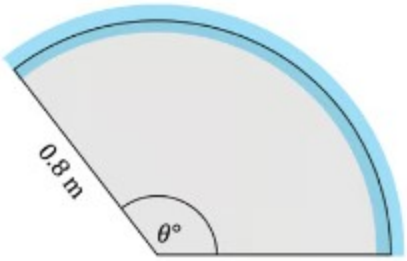
$A = \frac{\theta}{360} \times \pi r^2$ (in formula booklet)

$A = \frac{4}{15}\pi$ $r = 0.8$

Sub A and r into formula and rearrange for θ.

$\frac{4}{15}\pi = \frac{\theta}{360} \times \pi(0.8)^2$

$\theta = 150°$



$\theta = 150°$

b) Arc length formula

$l = \frac{\theta}{360} \times 2\pi r$ (in formula booklet)

$\theta = 150$ $r = 0.8$

Sub θ and r into formula.

$l = \frac{150}{360} \times 2\pi(0.8)$

$l \approx 2.09$ m

Question 7 a) Circle circumference formula

$l = 2\pi r$ (in formula booklet)

Radius of semicircles

$r = \frac{554 - 426}{2}$ $\therefore r = 64$ m

Total distance

$d = 2(426) + 2\pi(64)^*$

$d \approx 1250$ m

*N.B 2 semicircles make a full circle.

b) Total area = rectangle + 2 semicircles

The height of the rectangle is equal to the diameter of the semicircles.

$r = 64$ $\therefore$ height $= 2(64)$
$= 128$ m

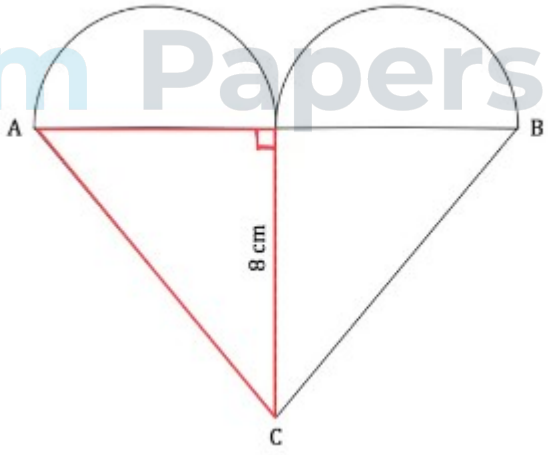
Circle area formula

$A = \pi r^2$ (in formula booklet)

Total area $= (426)(128) + \pi(64)^2$

Total area $\approx$ 67 400 m^2

Question 8

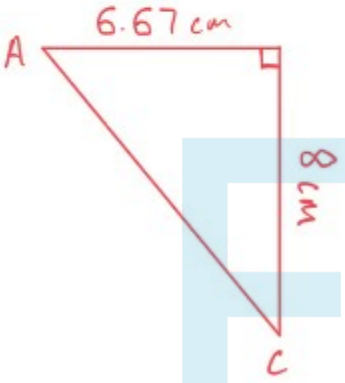


a) Notice the right-angled triangle.

triangle base $= \frac{AB}{2}$

triangle base $= \frac{13.34}{2}$

triangle base $= 6.67$



$\therefore AC = \sqrt{8^2 + 6.67^2}$

$AC \approx 10.4$ cm

b) Total area (A) = triangle + 2 semicircles*

$\therefore A = \frac{1}{2}bh + \pi r^2$

Semicircle radius $(r) = \frac{AB}{4}$

$r = \frac{13.34}{4}$

$b = 13.34 \quad h = 8 \quad r = 3.335$

Sub b, h and r into formula

$A = \frac{1}{2}(13.34)(8) + \pi(3.335)^2$

$A \approx 88.3\text{cm}^2$

*N.B 2 semicircles make a full circle.

c) Number of cookies = $\frac{\text{dough area}}{\text{heart area}}$

Number of cookies = $\frac{1314}{88.3}$

Number of cookies = 14.9

∴ 14 full cookies

N.B Cookie dough can be reformed so no need to account for the irregular shape.

Question 9

a) Perimeter = arc + 2(radius)

Arc length formula

$l = \frac{\theta}{360} \times 2\pi r$ (in formula booklet)

$\theta = 60$ $r = 15$

Sub θ and r into formula.

$l = \frac{60}{360} \times 2\pi(15)$

Perimeter = $\frac{60}{360} \times 2\pi(15) + 2(15)$

Perimeter ≈ 45.7 cm

b) Crust area (A_c) = Pizza area (A_p) - Toppings area (A_T)

Toppings radius (r_T) = Pizza radius (r_p) - crust width

$\therefore r_T = 15 - 2.2$

$= 12.8\text{ cm}$

Sector area formula

$A = \frac{\theta}{360} \times \pi r^2$ (in formula booklet)

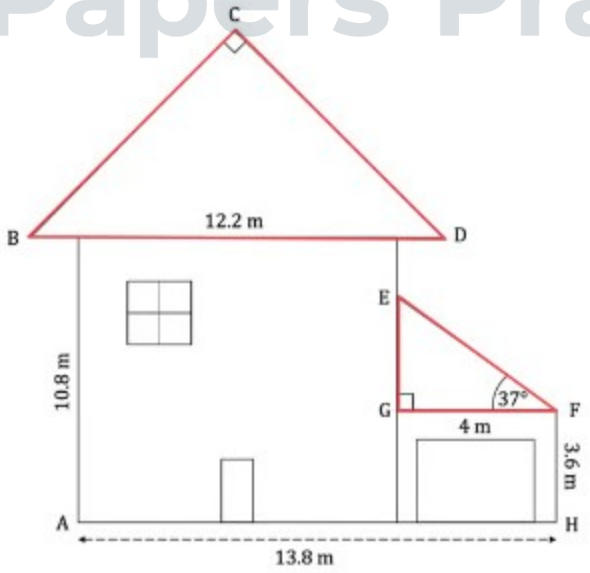
$\theta = 60$ $r_p = 15$ $r_T = 12.8$

Sub θ, r_p and r_T into formula to find A_c.

$A_c = \frac{60}{360} \times \pi(15)^2 - \frac{60}{360} \times \pi(12.8)^2$

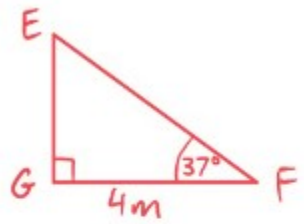
$A_c \approx 32\text{ cm}^2$

Question 10



a) Notice the right-angled triangles

i)

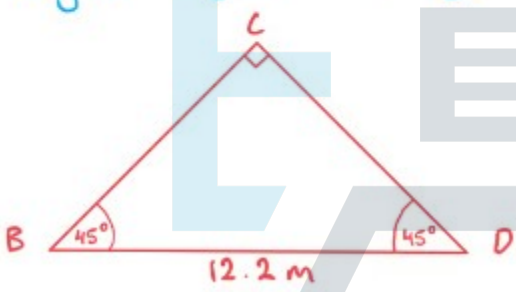


$\therefore \tan 37° = \frac{EG}{4}$

$EG = \tan 37° \times 4$

$EG \approx 3.01\,m$

ii) Base angles of an isosceles right-angled triangle equal $45°$.



$\sin 45° = \frac{BC}{12.2}$

$BC = \sin 45° \times 12.2$

$BC \approx 8.63\,m$

b) Total area (A) = House + Roof + Garage

House area = rectangle

= height × base

Height = 10.8 base = 13.8 - 4
= 9.8

Roof area = triangle

$= \frac{1}{2}(BC)(CD)$

$BC = CD \approx 8.63$

Garage area = trapezoid.

$= \frac{1}{2}(FH + (EG + FH)(GF))$

$FH = 3.6$ $EG \approx 3.01$ $GF = 4$

$\therefore A = (10.8)(9.8) + \frac{1}{2}(8.63)^2 + \frac{1}{2}(3.6 + (3.6 + 3.01))(4)$

$A \approx 163.5\ m^2$