EXAM PAPERS PRACTICE

Matrices

Question Paper

Course	Edexcel Further Maths
Section	Further Statistics
Topic	Matrices
Difficulty	Medium

To be used by all students studying Edexcel Further Mathematics (9FM0)

Students of other boards may also find this useful

Q1.

The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$$

(a) Show that 2 is a repeated eigenvalue of **A** and find the other eigenvalue.

(5)

(b) Hence find three non-parallel eigenvectors of **A**.

(4)

(c) Find a matrix **P** such that **P**−1**AP** is a diagonal matrix.

(2)

(Total for question = 11 marks)

Q2.

$$\mathbf{A} = \begin{pmatrix} 5 & -2 & 5 \\ 0 & 3 & p \\ -6 & 6 & -4 \end{pmatrix} \quad \text{where } p \text{ is a constant}$$

Given that $\begin{pmatrix} 2 \\ 1 \\ -2 \end{pmatrix}$ is an eigenvector for **A**

(a) (i) determine the eigenvalue corresponding to this eigenvector

(1)

(ii) hence show that $\boldsymbol{p} = 2$

(2)

(iii) determine the remaining eigenvalues and corresponding eigenvectors of **A**

(7)

(b) Write down a matrix **P** and a diagonal matrix **D** such that $\mathbf{A} = \mathbf{PDP}^{-1}$

(1)

(c) (i) Solve the differential equation $\dot{u} = ku$, where *k* is a constant.

(2)

With respect to a fixed origin *O*, the velocity of a particle moving through space is modelled by

By considering

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \mathbf{A} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

$$\begin{pmatrix} u \\ v \\ w \end{pmatrix} = \mathbf{P}^{-1} \begin{pmatrix} x \\ y \\ z \end{pmatrix} \text{ so that } \begin{pmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{pmatrix} = \mathbf{P}^{-1} \begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix}$$

(ii) determine a general solution for the displacement of the particle.

(4)

(Total for question = 17 marks)

Q3.

Matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & a \\ -3 & b & 1 \\ 0 & 1 & a \end{pmatrix}$$

where *a* and *b* are integers, such that *a* < *b*

Given that the characteristic equation for **M** is

λ3 – 7λ2 + 13λ + *c* = 0

where *c* is a constant,

(a) determine the values of *a*, *b* and *c*.

(5)

(b) Hence, using the Cayley–Hamilton theorem, determine the matrix **M**–1

(3)

(Total for question = 8 marks)

Q4.

The matrix **M** is given by

$$\mathbf{M} = \begin{pmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ -1 & 0 & 4 \end{pmatrix}$$

(a) Show that 4 is an eigenvalue of **M**, and find the other two eigenvalues.

(4)

(b) For each of the eigenvalues find a corresponding eigenvector.

(4)

(c) Find a matrix **P** such that **P–1MP** is a diagonal matrix.

(2)

(Total for question = 10 marks)

Q5.

(i)

$$\mathbf{M} = \begin{pmatrix} 2 & a & 4 \\ 1 & -1 & -1 \\ -1 & 2 & -1 \end{pmatrix}$$

where a is a constant.

(a) For which values of a does the matrix **M** have an inverse?

(2)

Given that **M** is non-singular,

(b) find M–1 in terms of *a*

(4)

(ii) Prove by induction that for all positive integers *n*,

$$\begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix}$$

(6)

(Total for question = 12 marks)

Q6.

The transformation *P* is an enlargement, centre the origin, with scale factor *k*, where *k* > 0

The transformation *Q* is a rotation through angle *θ* degrees anticlockwise about the origin.

The transformation *P* followed by the transformation *Q* is represented by the matrix

$$\mathbf{M} = \begin{pmatrix} -4 & -4\sqrt{3} \\ 4\sqrt{3} & -4 \end{pmatrix}$$

(a) Determine

(i) the value of *k*,

(ii) the smallest value of *θ*

(4)

A square *S* has vertices at the points with coordinates (0, 0), (*a*, –*a*), (2*a*, 0) and (*a*, *a*) where *a* is a constant.

The square *S* is transformed to the square *S*' by the transformation represented by **M**.

(b) Determine, in terms of *a*, the area of *S*'

(2)

(Total for question = 6 marks)

Q7.

(i) **A** is a 2 by 2 matrix and **B** is a 2 by 3 matrix.

Giving a reason for your answer, explain whether it is possible to evaluate

(a) **AB**

(b) **A** + **B**

(2)

(ii) Given that

$$\begin{pmatrix} -5 & 3 & 1 \\ a & 0 & 0 \\ b & a & b \end{pmatrix}\begin{pmatrix} 0 & 5 & 0 \\ 2 & 12 & -1 \\ -1 & -11 & 3 \end{pmatrix} = \lambda \mathbf{I}$$

where a, b and λ are constants,

(a) determine

- the value of λ
- the value of a
- the value of b

(b) Hence deduce the inverse of the matrix $\begin{pmatrix} -5 & 3 & 1 \\ a & 0 & 0 \\ b & a & b \end{pmatrix}$

(3)

(iii) Given that

$$\mathbf{M} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & \sin\theta & \cos\theta \\ 0 & \cos 2\theta & \sin 2\theta \end{pmatrix} \qquad \text{where } 0 \leqslant \theta < \pi$$

determine the values of θ for which the matrix **M** is singular.

(4)

(Total for question = 9 marks)

Q8.

$$\mathbf{A} = \begin{pmatrix} 4 & -2 \\ 5 & 3 \end{pmatrix}$$

The matrix **A** represents the linear transformation *M*.

Prove that, for the linear transformation *M*, there are no invariant lines.

(5)

(Total for question = 5 marks)

Q9.

$$\mathbf{M} = \begin{pmatrix} a & 2 & -3 \\ 2 & 3 & 0 \\ 4 & a & 2 \end{pmatrix} \quad \text{where } a \text{ is a constant}$$

(a) Show that **M** is non-singular for all values of *a*.

(2)

(b) Determine, in terms of *a*, **M**–1

(4)

(Total for question = 6 marks)

Q10.

$$\mathbf{M} = \begin{pmatrix} 3 & a \\ 0 & 1 \end{pmatrix} \quad \text{where a is a constant}$$

(a) Prove by mathematical induction that, for $n \in \mathbb{N}$

$$\mathbf{M}^n = \begin{pmatrix} 3^n & \frac{a}{2}(3^n - 1) \\ 0 & 1 \end{pmatrix}$$

(6)

Triangle *T* has vertices *A*, *B* and *C*.

Triangle *T* is transformed to triangle *T'* by the transformation represented by **M**n where $n \in \mathbb{N}$

Given that

- triangle *T* has an area of 5 cm2
- triangle *T'* has an area of 1215 cm2
- vertex *A*(2, –2) is transformed to vertex *A'*(123, –2)

(b) determine

(i) the value of *n*

(ii) the value of *a*

(5)

(Total for question = 11 marks)

Q11.

$$\mathbf{M} = \begin{pmatrix} 1 & k & -2 \\ 2 & -4 & 1 \\ 1 & 2 & 3 \end{pmatrix}$$

where *k* is a constant.

(a) Show that, in terms of *k*, a characteristic equation for **M** is given by

$$\lambda^3 - (2k + 13)\lambda + 5(k + 6) = 0$$

(3)

Given that det **M** = 5

(b) (i) find the value of *k*

(ii) use the Cayley-Hamilton theorem to find the inverse of **M**. (7)

(Total for question = 10 marks)

Q12.

$$\mathbf{M} = \begin{pmatrix} 2 & -1 & 1 \\ 3 & k & 4 \\ 3 & 2 & -1 \end{pmatrix} \quad \text{where } k \text{ is a constant}$$

(a) Find the values of *k* for which the matrix **M** has an inverse.

(2)

(b) Find, in terms of *p*, the coordinates of the point where the following planes intersect

$2x - y + z = p$

$3x - 6y + 4z = 1$

$3x + 2y - z = 0$

(5)

(c) (i) Find the value of *q* for which the set of simultaneous equations

$$2x - y + z = 1$$

$$3x - 5y + 4z = q$$

$$3x + 2y - z = 0$$

can be solved.

(ii) For this value of *q*, interpret the solution of the set of simultaneous equations geometrically.

(4)

(Total for question = 11 marks)

Q13.

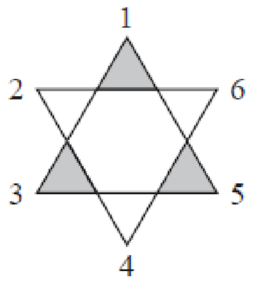


Figure 3

Figure 3 shows a plane shape made up of a regular hexagon with an equilateral triangle joined to each edge and with alternate equilateral triangles shaded.

The symmetries of this shape are the rotations and reflections of the plane that preserve the shape and its shading.

The symmetries of the shape can be represented by permutations of the six vertices labelled 1 to 6 in Figure 3. The set of these permutations with the operation of composition form a group, *G*.

(a) Describe geometrically the symmetry of the shape represented by the permutation

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 4 & 5 & 6 & 1 & 2 \end{pmatrix}$$

(2)

(b) Write down, in similar two-line notation, the remaining elements of the group *G*.

(4)

(c) Explain why each of the following statements is false, making your reasoning clear.

(i) *G* has a subgroup of order 4

(ii) *G* is cyclic.

(2)

Diagram 1 shows an unshaded shape with the same outline as the shape in Figure 3.

(d) Shade the shape in Diagram 1 in such a way that the group of symmetries of the resulting shaded shape is isomorphic to the cyclic group of order 6.

(2)

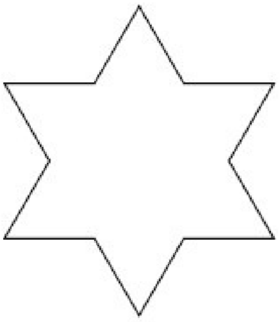
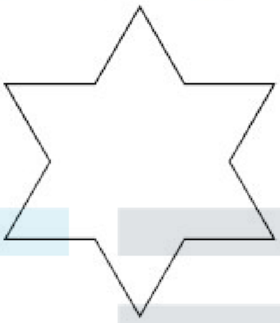


Diagram 1

Spare copy of Diagram 1



Only use this diagram if you need to redraw your answer to part (*d*).

(Total for question = 10 marks)

Q14.

(In this question you must show all stages of your working.)

A college offers only three courses: Construction, Design and Hospitality.

Each student enrols on just one of these courses.

In 2019, there was a total of 1110 students at this college.

There were 370 more students enrolled on Construction than Hospitality.

Construction **increased** by 1.25%
Design **increased** by 2.5%
Hospitality **decreased** by 2%

In 2020, the total number of students at the college increased by 0.27% to 2 significant figures.

(a) (i) Define, for each course, a variable for the number of students enrolled on that course in 2019.

(ii) Using your variables from part (a)(i), write down **three** equations that model this situation.

(4)

(b) By forming and solving a matrix equation, determine how many students were enrolled on each of the three courses in 2019.

(4)

(Total for question = 8 marks)